Match Circuit And Field Resistances For Optimal Magnetics Design

by Dennis Feucht, Innovatia Laboratories, Cayo, Belize

Design of transformers and coupled inductors (transductors) can be optimized for maximum power transfer by matching the resistance of the circuit to that of the magnetic field of the core. Utilization of the core is maximized by optimizing two parameters: saturation field intensity, $\overline{H}$, at the core operating-point, and allowable core power loss density, p_c. By maximizing both, core energy transfer is also maximized. $\overline{H}$ drives the core into saturation to the desired extent and p_c drives the magnetic-field-density ripple amplitude, $\hat{B}_{\sim}$, to the value of a given power-loss density in the core at a given frequency, f_s, and core size.

The core is not oversized if it dissipates the maximum acceptable magnetic power loss while driven to the limit of saturation. Both of these maxima can be achieved simultaneously, resulting in the smallest required core volume for a given power transfer through it.

For linear magnetics, the core energy density (w) is

$$w = \frac{W}{V} = \int H \cdot dB = \int\left(\frac{B}{\mu}\right) \cdot dB = \frac{1}{2} \cdot \left(\frac{B^2}{\mu}\right).$$

where W is energy, V is core volume, H is magnetic field intensity, B is magnetic field density, and μ is core permeability. Then, the energy stored in a core of volume V is

$$W_L = \frac{B^2 \cdot V}{2 \cdot \mu},$$

and the energy transferred during each switching cycle of the converter is

$$\Delta W_L = \frac{V}{2 \cdot \mu} \cdot (\Delta B^2) = \frac{V}{\mu} \cdot \Delta B \cdot \overline{B} = \frac{V}{\mu} \cdot \Delta B \cdot (\mu \cdot \overline{H}) = [\Delta B \cdot \overline{H}] \cdot V .$$

where ΔB is the peak-to-peak change in B within a switching cycle and $\overline{B}$ is the static or operating-point B.

For linear magnetics, μ is constant and the incremental and total-variable μ are equal. The above expression can be used in transductor design by linearizing nonlinear magnetics around an operating-point $(\overline{H}, \overline{B})$ with excursions of $\Delta B/2$ each side of $\overline{B}$, where $\Delta B/2$ is relatively small or *incremental*. Then, the full variation around $\overline{B}$ can be expressed as

$$\Delta B = \left(\overline{B} + \frac{\Delta B}{2}\right) - \left(\overline{B} - \frac{\Delta B}{2}\right)$$

and

$$\Delta(B^2) = \Delta B^2 = \left(\overline{B} + \frac{\Delta B}{2}\right)^2 - \left(\overline{B} - \frac{\Delta B}{2}\right)^2 = 2 \cdot \Delta B \cdot \overline{B} .$$

From the expression for ΔW_L, maximum transfer of energy is achieved when the core is driven with as large a ΔB as the thermal limit allows and with as large of an $\overline{H}$ as saturation allows. Then, the rate at which core energy is transferred from input to output depends on f_s.

Current Waveform Matched To Core Material

The number of winding turns, *N*, is the key transductor parameter because it relates circuit and field quantities. Saturation and power-loss constraints can both be expressed as limits on *N*. At a given power dissipation and f_s, $\hat{B}_\sim$ is determined. For a given core cross-sectional area, *A*,

$$N_\lambda = \frac{\Delta\lambda}{\Delta\phi} = \frac{V_p \cdot t_{on}}{\Delta B \cdot A} = \frac{V_p \cdot (D/f_s)}{(2\cdot\hat{B}_\sim)\cdot A}$$

where $\Delta\lambda$ is the circuit flux change, $\Delta\phi$ is the corresponding field flux change, V_p is the voltage applied to the primary winding, t_{on} is the on-time during which V_p is applied, *D* is the duty ratio (cycle), and $\Delta B = 2\cdot\hat{B}_\sim$.

For a maximum field flux amplitude of

$$\hat{\phi}_\sim = \hat{B}_\sim \cdot A$$

then N_λ is the minimum allowable number of turns for maximum core power loss for a given circuit flux change, $\Delta\lambda$.

The core must also be driven to the maximum specified saturation, as quantified by k_{sat}, the fractional saturation of a core, or *L(I)/L(0 A)* where *L* is inductance. For a given maximum $\overline{H}$, this results in a maximum operating-point field-referred current, $N\bar{i}$, of

$$N\bar{i} = \overline{H}\cdot l = N_i \cdot I_p$$

where *l* is the core magnetic path length and I_P is the winding on-time current amplitude. The number of turns, N_i, relates the field current to the circuit current:

$$N_i = \frac{N\bar{i}}{I_p}.$$

To satisfy both equations for N_i and N_λ, the two turns expressions must be equal ($N_\lambda = N_i$), and either can be used to calculate *N*. However, both equations must hold simultaneously and this specifies the circuit current for a given winding voltage, V_p. From this emerges the notion that the static circuit terminal resistance, V_p/I_p, is fixed for optimal core utilization.

Core materials differ in their ratios of maximum acceptable static $\overline{H}$ and ripple $\hat{H}_\sim$ values. These parameters can be directly related to the current ripple of the winding current waveform. The average ripple factor for current, γ, is defined as

$$\gamma = \frac{\Delta i/2}{\bar{i}} = \frac{\hat{i}_\sim}{\bar{i}}$$

where the bar indicates average and $\hat{i}_\sim$ is the current ripple (~) amplitude or peak (^) corresponding to $\hat{B}_\sim$.

The average ripple factor is a waveform parameter optimally related to core material parameters as

$$\gamma = \frac{\hat{H}_\sim}{\overline{H}}.$$

An optimal *γ* for a given core material means that current waveforms are best matched to the magnetic material so that the power-loss and saturation fractional design margins are the same and in the optimal case are both at their design maximums. That is why inductors, which typically have small ripple relative to average current and thus low-γ, use low-γ material. Meanwhile, transformers, which have zero average current, use high-γ material.

The choice of core material specifies γ and along with core volume, determines the per-turn-squared or field-referred inductance $\mathcal{L}$, where

$$\mathcal{L} = k_{sat} \cdot \mathcal{L}(0\ \mathrm{A}).$$

For maximum power transfer,

$$\gamma_{opt} = \frac{\hat{H}_{\sim}}{\overline{H}} = \frac{\hat{B}_{\sim}/\mu}{N\bar{i}/l} \cdot \frac{A}{A} = \frac{\hat{B}_{\sim} \cdot A}{\left(\frac{\mu \cdot A}{l}\right) \cdot N\bar{i}} = \frac{\hat{\phi}_{\sim}}{\mathcal{L} \cdot N\bar{i}}$$

where $\hat{B}_{\sim}$ has the maximum design power-loss value. For a 25.7-mm (0.5-in.) OD core of Mn-Zn ferrite (and in particular the Magnetics Inc. type K material) operating at 150 kHz,

$$\gamma_{opt}(\mathrm{Mn\text{-}Zn}) \approx 0.4,$$

while for same-size iron-powder (Fe-pwd) cores (and in particular the Micrometals 26 material) at f_s = 150 kHz,

$$\gamma_{opt}(\mathrm{Fe\text{-}pwd}) \approx 0.1$$

Optimal Number Of Turns

The equations for N_λ and N_i impose bounds on the acceptable values of N:

$$N_\lambda \le N \le N_i.$$

N can also be constrained by winding window area as a corresponding N_w, the turns that can occupy the window area allotted for the winding at the allowed current density, taking wire packing factor into account. The bounds on N are then

$$N_\lambda \le N \le \min\{N_i, N_w\}.$$

We will proceed assuming that N_w is not a limitation. When the range for N is reduced to one value, N is optimized for maximum core transfer energy;

$$N_{opt} = N_\lambda = N_i.$$

Under this optimality condition,

$$N = \frac{V_p \cdot (D/f_s)}{(2 \cdot \hat{B}_{\sim}) \cdot A} = \frac{N\bar{i}}{I_p}.$$

This can be rearranged to make the average winding power, $\overline{p}_p$, explicit:

$$\overline{p}_p = D \cdot V_p \cdot I_p = V_p \cdot \bar{i}_p = N\bar{i} \cdot (2 \cdot \hat{B}_{\sim}) \cdot A \cdot f_s.$$

Ideally, with no winding-to-core power loss, this would also be the core transfer power. Under the optimality condition, I_p is no longer a free design parameter but is determined by choice of the core, V_p and t_{on}. The basic core limitations are made explicit in the above equation and can be expressed in field operating-point quantities as follows:

$$\overline{p}_p = V_p \cdot \bar{i}_p = N\bar{i} \cdot (2 \cdot \hat{B}_\sim) \cdot A \cdot f_s = (\overline{H} \cdot l) \cdot (\Delta B) \cdot A \cdot f_s = [\Delta B \cdot \overline{H}] \cdot V \cdot f_s = \Delta W_L \cdot f_s .$$

Core materials can be easily evaluated for power density by finding only ΔB and $\overline{H}$ for them.

Resistance-Matching Circuit And Field

Optimally, for a given choice of V_p, the value of I_p is determined. This implies that a resistance associated with the winding terminals is also determined. The turns equations can be expressed differently to make this winding resistance explicit. The transductor will have a static winding-terminal resistance of

$$R_{ckt} = \frac{V_p}{I_p}$$

The N_λ turns formula can be expressed differently by introducing field inductance $\mathcal{L}$ (a property of a given core) and current waveform parameter, γ;

$$\Delta\phi = \mathcal{L} \cdot \Delta(Ni) = \mathcal{L} \cdot N \cdot \Delta i_p = \mathcal{L} \cdot N \cdot (2 \cdot \gamma \cdot I_p)$$

where I_p is the average on-time current amplitude. Then

$$N = N_\lambda = \frac{\Delta\lambda}{\Delta\phi} = \frac{V_p \cdot t_{on}}{\mathcal{L} \cdot (2 \cdot \gamma \cdot I_p) \cdot N} .$$

Solving for N,

$$N^2 = \frac{V_p / I_p}{\mathcal{L} \cdot (2 \cdot \gamma) / t_{on}} = \frac{R_{ckt}}{R_{fld}} .$$

The denominator, R_{fld}, is an expression that can be interpreted as the circuit terminal resistance, R_{ckt}, referred to the field and is the steady-state resistance of the field. Ordinarily, the circuit resistance across a secondary winding can be referred to the primary winding by the turns ratio of n^2, but here, the field is referred to the winding by N^2. Consequently, a turns ratio, $n = N_p/N_s$, can be considered a composition of two winding-field referrals (winding s to field to winding p), with the field as the intermediary between windings.

Substituting for γ_{opt} in R_{fld},

$$R_{fld} = \frac{\mathcal{L} \cdot (2 \cdot \gamma)}{t_{on}} = \frac{\mathcal{L}}{t_{on}} \cdot \frac{2 \cdot \hat{\phi}_\sim}{\mathcal{L} \cdot N\bar{i}} = \frac{\Delta\phi}{N\bar{i}} \cdot \frac{1}{t_{on}}$$

where for optimality, $\Delta\phi$ and $N\bar{i}$ are at their design maximums, which are implicit within γ_{opt}. The term R_{fld} can be derived alternatively from R_{ckt}:

$$R_{fld} = \frac{R_{ckt}}{N^2} = \frac{V_p / N}{N \cdot I_p} = \frac{\text{field voltage}}{\text{field current}} = \frac{\Delta\phi / \Delta t}{N \cdot (\Delta i_p / 2 \cdot \gamma)} = \left(\frac{\Delta\phi}{N \cdot \Delta i_p} \right) \cdot \frac{2 \cdot \gamma}{t_{on}} = \mathcal{L} \cdot \frac{2 \cdot \gamma}{t_{on}}$$

where the final equivalence follows from Magnetic Ohm's Law, or $\lambda = L \cdot i$ referred to the field:

$$\phi = \mathcal{L} \cdot Ni .$$

A form of R_{fld} making explicit the design-constraining parameters is

$$R_{fld} = \frac{\Delta\phi}{N\bar{i}} \cdot \frac{1}{t_{on}} = \frac{\Delta\phi}{N \cdot \bar{i}_p} \cdot f_s$$

where $N\bar{i} = N \cdot I_p$. In this expression for R_{fld}, both limiting parameters appear, and when f_s is chosen, R_{fld} is determined.

As an example of how these equations apply to circuit design, consider a converter with a 5 V input, f_s = 150 kHz and nominal D = 0.5. Then t_{on} = 3.33 μs. The magnetic core is a Fe-pwd Micrometals T50-26D. Its characteristics, taken from the core catalog, are: $\mathcal{L}(0)$ = 72 nH, k_{sat} = 0.7, and $N\bar{i}$ = 75 A. Then

$$\gamma_{opt} = \frac{\hat{B}_{\sim} \cdot A}{\mathcal{L} \cdot N\bar{i}} = \frac{(17\,\text{mT}) \cdot (14.8\,\text{mm}^2)}{[(0.7) \cdot (72\,\text{nH})] \cdot (75\,\text{A})} = 0.066\,.$$

For maximum power transfer,

$$N = \frac{\Delta\lambda}{\Delta\phi} = \frac{V_g \cdot t_{on}}{(2 \cdot \hat{B}_{\sim}) \cdot A} = \frac{16.67\,\mu\text{V} \cdot \text{s}}{(2 \cdot 17\,\text{mT}) \cdot (14.8\,\text{mm}^2)} = 33.12 \to 33\,.$$

Then to satisfy the saturation condition,

$$I_p = \frac{N\bar{i}}{N} = \frac{75\,\text{A}}{33} = 2.27\,\text{A}\,.$$

With $\gamma << 1$, the converter is operating deep into continuous-conduction mode (CCM). A 20-AWG wire will handle this (mostly static) current and will fill 56% of the T50D winding window, about the right fraction for a primary winding. The steady-state circuit resistance is

$$R_{ckt} = \frac{V_p}{I_p} = \frac{5\,\text{V}}{2.27\,\text{A}} = 2.20\,\Omega\,.$$

The field resistance is

$$R_{fld} = \frac{\mathcal{L} \cdot (2 \cdot \gamma)}{t_{on}} = \frac{(0.7) \cdot (72\,\text{nH}) \cdot (2 \cdot 0.0666)}{3.33\,\mu\text{s}} = 2.01\,\text{m}\Omega$$

The field resistance is much smaller than the circuit resistance and they are matched with

$$N = \sqrt{\frac{R_{ckt}}{R_{fld}}} = \sqrt{\frac{2.20\,\Omega}{2.01\,\text{m}\Omega}} = 33\,.$$

The circuit-field conversion parameter, N, matches the resistance of the circuit to the field to achieve optimal core utilization and maximum power transfer. No smaller core using this material with the same circuit parameters will deliver as much power.

If the resulting winding current does not fit the circuit requirements, then either a different core must be chosen or else the maximum transfer-power condition abandoned. It is not uncommon to design suboptimally in that Fe-pwd cores, for instance, will often have higher saturation than is needed to achieve design goals. In these cases, utilization is not maximized, yet the circuit design goals are satisfied.

About the Author

Dennis Feucht has been involved in power electronics for 25 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been doing current-loop converter modeling and converter optimization.

For more on magnetics design, see the How2Power Design Guide, select the Advanced Search option, go to Search by Design Guide Category, and select “Magnetics” in the Design Area category.