Exclusive Technology Feature

ISSUE: September 2011

Current-Loop Control In Switching Converters

Part 1: Historical Overview

by Dennis Feucht, Innovatia Laboratories, Cayo, Belize

Peak or valley current control of switching converters is well established in engineering practice. Yet the irony of current-loop control is that, after decades, its theory is still undergoing refinement. This is the result, in part, of the complexity of the seemingly simple current-loop controller circuit. Its typical circuit diagram has few parts, yet the current-feedback loop is nonlinear and switched, having discrete-time behavior.

This series of articles reviews current-loop control history, clarifies established concepts, presents some problems with the existing theories or models of the current-control loop, and then offers what might be the first truly unified model of current control. We begin with some historical perspective.

History Of Model Progression

The history of continuous-conduction mode (CCM) current-loop model development roughly follows the list of references used in this series. The first notable model is the low-frequency averaged (lf-avg) model developed in detail by Erickson and Macsimović in their classic text *Fundamentals of Power Electronics*.[1] The lf-avg model was based on the assumption that power converter operation can be described in terms of continuous electrical variables and did not take digital or sampling effects into account.

After the development of that model, Ray Ridley at Virginia Tech discovered that the current-loop comparator functioned as a sample and hold or zero-order hold for the inductor current. From this discovery, Ridley developed a sampled-loop model in the *s*-domain transfer function of the current loop. This work was published in his landmark paper about the *sampled-loop* or “continuous-time” model: “A New, Continuous-Time Model For Current-Mode Control.”[2]

The older lf-avg model correctly predicts the quasi-static behavior of the loop while the sampled-loop model extends it to include dynamic effects, including *subharmonic* oscillation. This is a commonly observed behavior whereby the duty ratio, D, alternates every cycle and each successive switching cycle has the other D. This instability is predicted in Ridley’s sampled-loop model by a resonance at half the switching frequency that causes the loop to have a peak in the gain at a negative phase margin, resulting in a discrete-time oscillation.

About the time the sampled-loop model was developed, the making of an *early unified* model was worked out by Tymerski and Li as an exact state-space model that is then approximated and uses average inductor current and sampling.[3] This model offers justification for Ridley’s choice of discrete-time inductor current equations and placement of the sampling block, H_e, in the feedback path.

A couple of years later, R.D. Middlebrook and his student F. Dong Tan published a paper on unification of the average current of the lf-avg model with the sampling effects of the sampled-loop model: “A Unified Model for Current-Programmed Converters.”[4] In this paper, Middlebrook and Tan moved the sampling block from the feedback path to the forward path (where the current comparator is that actually does the sampling). However, their choice of a transfer function for the PWM generator, F_m— another forward-path block in the loop—was taken from the lf-avg model and was not a result of a more-general dynamic model. The incremental (small-signal) model of the current loop is shown below and will be explained more fully later in this article series.

© 2011 How2Power. All rights reserved.

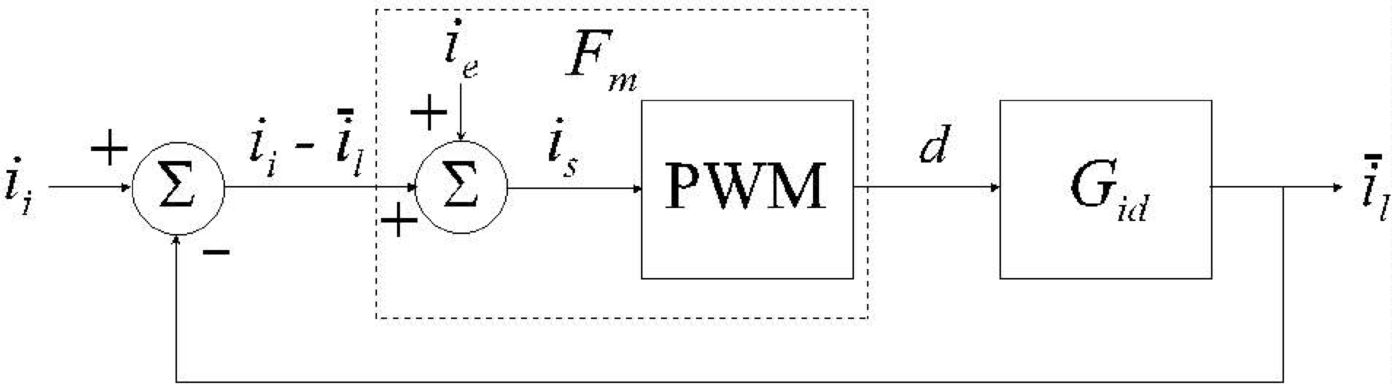

Figure. The small-signal model of the current loop in a current-mode controlled power converter.

A *simple unified* model is found in Hong, Choi and Ahn's paper "The Unified Model for Current-Mode Control: an Alternative Derivation."[5] In this paper, the authors give a straightforward derivation of $Fm(s)$ using a modified slope based on average inductor current. It is simpler and easier to follow than the multiple derivations in the paper by Middlebrook and Tan.[4]

Refinement of the unified model, using a modified equation for the quasi-static F_m having an up-slope that's half that of Ridley's model, is found in an article by Holloway and Eirea in Power Electronics Technology.[6] The modification was used earlier by Middlebrook and is in Tymerski's paper. [3] It accounts for the difference between valley and average inductor current.

Circuit-Based Modeling

A different kind of refinement of the unified model is presented by Robert Sheehan in his paper, "Current-Mode Modeling for Peak, Valley, and Emulated Control Methods."[7] Instead of beginning with generalized converter behavior (waveform and slope equations) from which to derive general circuit structure, Sheehan begins with circuit structure and derives the circuit equations that describe circuit behavior. The variation of the inductor current slope caused by input and output voltage is implicit in the circuit equations because they are expressed in terms of converter input voltage, v_G and output voltage, v_O.

Sheehan analyzes multiple control schemes and gives the detailed derivations of reasonably complete converter circuit models—a compendium of modeling equations. He describes his discovery that if the external compensating ramp for the buck configuration is allowed to vary with the output voltage, an improved control scheme results.

At this point, it's important to note that Sheehan's current-loop modeling differs in a basic way from previous methods in that it is a *circuit-based* model rather than a *waveform-based* model. It begins with *circuit structure*, not *behavior*.

As we will see in the next part of this article series, the historic model development focused on the inductor current waveform and how it behaved. This waveform was simplified as a triangle-wave, and therefore could be applied to any converter for which inductor current, i_L could be modeled as a triangle-wave. Circuit-based modeling loses that generality for the sake of accuracy. The circuit is analyzed for its own behavior rather than having to make simplifying assumptions about the circuit to fit the waveform model. Sheehan has analyzed multiple converter circuits and has begun generalization of circuit-based modeling.

Subtleties In Waveform-Based Models

In waveform-based modeling, the quest to formulate a unified model led to different adaptations of the older low-frequency-average (lf-avg) and sampled-loop models. This series includes various clarifications of these models, points out discrepancies among them, and then presents a basis for a truly unified or *refined* model.

The groundwork for a refined model is laid by first showing relationships between previous models. The average inductor current of the lf-avg model of Erickson and Macsimović is the same as the average inductor current of the unified model of Tan and Middlebrook, though derived differently. The inductor current quantity used by Ridley is not the same as that of Tymerski and Tan. The lf-avg and unified models use average current, while the sampled-loop model uses discrete-time minimum or valley values of inductor current. This results in a

© 2011 How2Power. All rights reserved.

difference in dynamics between average and valley currents and is a key aspect of a fully-unified or refined model.

Some subtleties of previous models need to be made more explicit, thereby clarifying some of the complications in them. The time-domain waveforms of the sampled-loop and unified models are not continuous but piecewise-continuous. This results in a difference between continuous inductor current and quantized or stepped current. The actual inductor current waveform is stepped and correctly represents circuit behavior. When the validity of a model is limited to the first Nyquist band of frequencies, the higher frequencies in the steps can be overlooked.

From sampled-loop control theory, not every sampled loop has a transfer function. It depends on where the sampling occurs. Ridley's sampled-loop model does not have a transfer function because, as a sampled loop with sampling in the feedback path, the input to the loop is time-variant. It can be converted to a unified model by moving the block that accounts for sampling—$H_e(s)$ in the feedback loop—forward through the summing block and into the input and forward paths. Then, both input and error quantities are sampled.

A "simple unified model" is derived more simply in the work by Hong, Choi and Ahn than by Middlebrook and Tan, using Ridley's transfer function for the power stage of the converter, $i_L/d = G_{id}$. Their quasi-static F_m is different from either the lf-avg or sampled-loop model of Ridley, and differs from the lf-avg model only because the input and output voltage feedback is factored differently in the unified model.

The Refined Model

The refined model that will be derived in this article series is based on average discrete-time current in a sampled loop. The average inductor-current discrete-time equations are used as a basis for constructing the model. The PWM transfer function, $F_m(s)$, is derived by localizing PWM effects in the block diagram to the PWM block. The sampling effects in the loop are considered in equating the closed-loop feedback formula, $T_C(s)$, to the one derived from the closed-loop waveform equations.

Slope compensation schemes are analyzed as modifications of the uncompensated response. The average inductor current, discrete-time, slope-compensated waveform equations will be derived in the next article in this series. The uncompensated converter waveform equations are shown to be unaffected by slope compensation, though the sensed current in the PWM block is changed by it.

Finally, the effects of input and output voltage on the loop dynamics are included. In his paper, Sheehan does this implicitly, by starting with circuit analysis instead of generalized waveform analysis. However, the refined model developed in this series assumes steady-state converter port voltages and a linear inductor-current waveform from which is derived a simple expression for the converter $G_{id} = i_l/d$ transfer function. The model includes a total-variable v_{OFF} to account for input and output voltage in G_{id} , though the linear inductor-current waveform is no longer valid when a more complete G_{id} from circuit analysis is considered.

The unified model places sampling where it belongs—in the PWM block—but does not place v_{OFF} in the G_{id} block. The effects are instead included in the loop error quantity. In the refined model, they are moved to where they occur in the G_{id} block. However, there is some advantage in retaining their effects at the error-summing block.

Summary

This opening article has presented an overview of the current-loop modeling quagmire. Ideally, one might want to simply bypass the vicissitudes of historical development and concentrate on a refined model. However, it is hard to read much of the literature without some historical understanding because new developments refer to existing concepts, however lacking they may be in the finality of their development. The rest of this article series attempts to develop the "ultimate" waveform-based model, but does so late in the series. In the intervening parts, we will wade through refinement and clarification of existing concepts, for they are needed for an understanding of current-loop control in its present state of development.

References

1. Low-frequency cycle-averaged (lf-avg) model. *Fundamentals of Power Electronics, Second Edition*, Erickson and Macsimović, Kluwer, 2001.

2. Sampled-loop model: "A New, Continuous-Time Model For Current-Mode Control," Raymond B. Ridley, *IEEE Trans. on Power Electronics*, Vol. 6, No. 2, April 1991.

© 2011 How2Power. All rights reserved.

3. Early Unified Model: "State-space models for current-programmed pulsewidth-modulated converters," R. Tymerski and D. Li, *IEEE Trans. on Power Electronics*, Vol. 8, No. 3, July 1993, pp. 271- 278.

4. "A Unified Model for Current-Programmed Converters," F. Dong Tan and R.D. Middlebrook, *IEEE Trans. on Power Electronics*, Vol. 10, No. 4, July 1995, pp. 397 - 408.

5. "The Unified Model for Current-Mode Control: an Alternative Derivation," S. Hong, B. Choi, and H. Ahn, *Journal of Circuits, Systems, and Computers*, Vol. 13, No. 4 (2004), pp. 725-736, at www.worldscientific.com.

6. "Model Current-Mode Control With Ease and Accuracy," Rendon Holloway and Gabriel Eirea, *Power Electronics Technology*, November 2008, pp. 22 – 25, (www.powerelectronics.com.)

7. "Current-Mode Modeling for Peak, Valley, and Emulated Control Methods," Robert Sheehan, National Semiconductor website www.nsc.com under "papers."

About The Author

Dennis Feucht has been involved in power electronics for 25 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been doing current-loop converter modeling and converter optimization.

For more on current-mode control methods, see the How2Power Design Guide, select the Advanced Search option, go to Search by Design Guide Category, and select "Control Methods" in the Design Area category.

© 2011 How2Power. All rights reserved.