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Current-Loop Control In Switching Converters

Part 2: A Waveform-Based Model

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In part 2 of this article series on current-loop control, we continue to lay the groundwork for the development of a waveform-based model by deriving time-domain expressions for inductor current that describe the closed-loop converter behavior without introduction of slope compensation into the PWM block. We then derive the equations relating inductor current slopes to converter parameters under steady-state operation. These slope equations allow us to analyze the effect of small changes that occur from cycle to cycle and are of interest in incremental (small-signal) and linear analysis.

The Waveform Equations

Waveform-based models of the current-loop converter begin with the time-domain waveforms of inductor current, i_L, and the peak- or valley-current-commanding input quantity, i_I, as shown in Fig. 1.

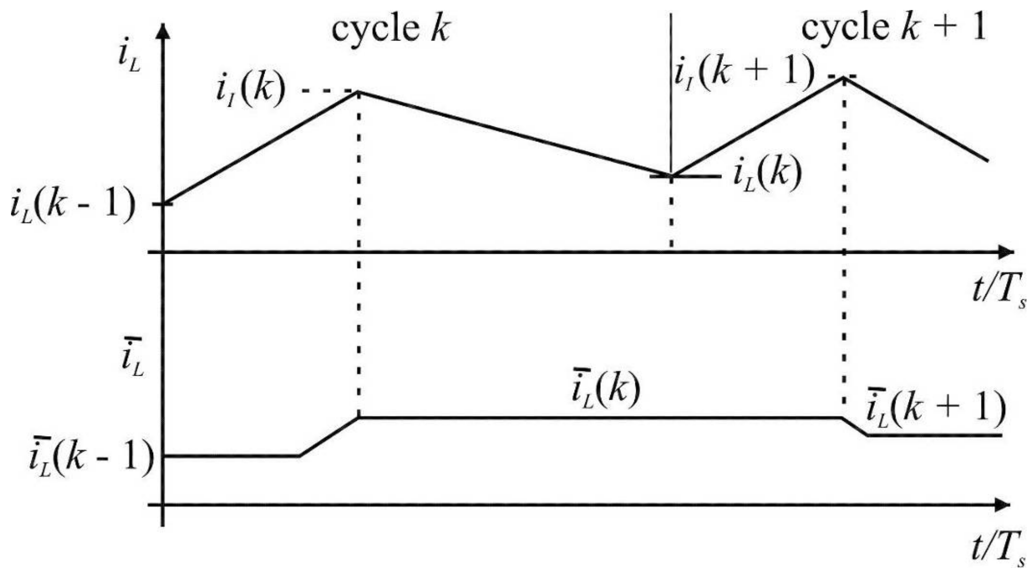

Fig. 1. Inductor current waveforms for a power converter operating under peak current control.

The total duty ratio (or duty cycle), $\delta = D + d$, is the sum of the steady-state or quasi-static D and incremental d for which $d = d\delta$ (the derivative, not the product of $d \cdot \delta$) and where the total current slopes are defined as $m_X = M_x + m_x$. A perturbation at the peak of i_L of cycle k will propagate to cycle $(k + 1)$ and affect $d(k + 1)$, not $d(k)$. The step change of $i_I(k)$ remains constant relative to the steady-state value of $i_L(t)$ until the end of the on-time of the next cycle, then begins or ends ramping with slope $\pm m_U$ for the difference that is i_I of that cycle.

We begin with the waveform equations for the inductor current, $i_L(t)$, and duty ratio, δ, using the notational shorthand that $x(k \cdot T_s) = x(k)$ and following standard electrical engineering notation for total- and small-signal variables♣;

♣ In the standard notation, static quantities are designated as having both upper-case letters with upper-case subscripts (M_E) while s-domain quantities have lower-case subscripts (M_e). Exception is taken here in allowing either case of subscripts for constants while making explicit the domain variable to designate s or other domains for the dependent variable of the function. The common practice of indicating incremental quantities with a circumflex (^) is not followed here; it is used to indicate peak value.

$$i_I(k) = i_L(k-1) + m_U \cdot \delta(k) \cdot T_s = m_D \cdot \delta'(k) \cdot T_s + i_L(k)$$

where up-slope is m_U during the on-time (Tan's, Tymerski's m_1 and Ridley's S_n), down-slope is $-m_D$ during off-time (Tan's, Tymerski's $-m_2$ and Ridley's $-S_f$), and i_I (i_c elsewhere) is the commanding input to the control loop in the form of a peak current, v_I/R_S. The term R_S (R_i elsewhere) is omitted in this analysis because it is easily reinserted into the loop equations. The index of i_I is that of the minimum or valley sample of i_L ending cycle k, $i_L(k)$. Input variable $i_I(k)$ changes where the peak of i_L occurs, and is the value of the cycle after it changes.

Observing $i_I(k)$ in more detail, the plot of i_I shows $i_I(k)$ as the new value of i_I at the moment of switching (Fig. 2.) Both the sampled-loop model of Ridley and the unified model of Tan indexes i_I at the sampling point of the switching cycle (Ridley, p. 273, eqn. 4; Tan, p. 399, Fig. 4). This alignment of i_I with the sampling instant instead of the cycle appears at first to be insignificant because the value of i_I affects circuit behavior only at the sampling instant. However, in modeling the current loop using the average inductor current, i_I takes on an expanded role.

Consequently, over the off-time interval of cycle k,

$$-\Delta i_L(k) = i_I(k) - i_L(k) = i_{CE}(k)$$

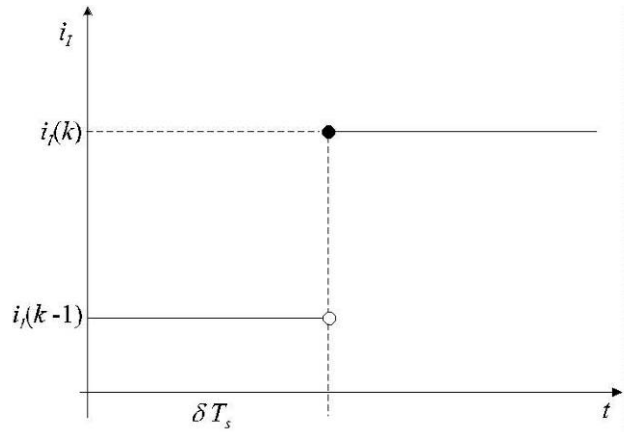

Fig. 2. A plot of i_I shows $i_I(k)$ as the new value of i_I at the moment of switching.

where $-\Delta i_L > 0$ and i_{CE} is the current-loop error. The previous sample interval ends at time $[\delta + (k-1)]\cdot T_s-$, as indicated by the open point of $i_I(k-1)$ at $[\delta + (k-1)]\cdot T_s$. Accordingly, Δi_L of cycle $k-1$ is the cycle k on-time Δi_L between the endpoint of cycle $k-1$ and the open point at $[\delta + (k-1)]\cdot T_s$, before i_I changes♣♣:

$$-\Delta i_L(k-1) = i_{CE}(k-1) = i_I(k-1) - i_L(k-1)$$

The equation for $-\Delta i_L(k)$ as derived from the off-time segment is

$$-\Delta i_L(k) = i_I(k) - i_L(k).$$

The change in inductor current for the up-slope is different than for the down-slope when not in steady-state. The down-slope Δi_L is that of the cycle:

$$\Delta i_L(k) = -(i_I(k) - i_L(k)) = i_L(k) - i_I(k).$$

♣♣ The total time of the waveform is the time within the cycle, where t is set to zero at the beginning of the cycle, plus the number of periods that have already gone by which is $(k-1)\cdot T_s$. Thus "cycle time" for cycle k is time offset by $(k-1)\cdot T_s$.

This implies (substituting $k - 1$ for k) that the up-slope Δi_L must be that of the previous ($k - 1$) cycle and that it ends just before i_l changes from $i_l(k - 1)$ to $i_l(k)$. Consequently, the instant i_l changes, $\bar{i}_L$ also either begins or ends its change, ending at the rate of m_U for a positive change and beginning $-m_U$ for a negative change. The time interval over which $\bar{i}_L$ changes is approximately zero for small changes and the change in $\bar{i}_L$ is nearly coincident with that of the moment of iteration of i_l.

The two rightmost expressions of the waveform equations can be solved for i_L;

$$i_L(k) = i_L(k-1) + (m_U + m_D)\cdot\delta(k)\cdot T_s - m_D\cdot T_s\,.$$

Then solving for δ,

$$\delta(k) = \frac{i_L(k) - i_L(k-1)}{(m_U + m_D)\cdot T_s} + \frac{m_D}{m_U + m_D}$$

and substituting into the first equation,

$$i_L(k) = -\left(\frac{m_D}{m_U}\right)\cdot i_L(k-1) + \left(1 + \frac{m_D}{m_U}\right)\cdot i_I(k) - m_D\cdot T_s\,.$$

The incremental waveform equations for i_L and δ are found by taking the differential of the above equations:

$$i_l(k) = -\left(\frac{m_D}{m_U}\right)\cdot i_l(k-1) + \left(1 + \frac{m_D}{m_U}\right)\cdot i_i(k)$$

$$d(k) = \frac{i_l(k) - i_l(k-1)}{(m_U + m_D)\cdot T_s}$$

where the slopes are assumed constant for now and are dependent on converter switch voltages. The incremental change in input, i_i, is coincident with the beginning of the change in inductor current, i_l. If the changes in i_l and i_i each cycle are the same for each variable ($i_l(k - 1) = i_l(k)$), then $i_l = i_i$.

The waveform equations describe the behavior of the discrete inductor-current samples at the end of each cycle. These are the minimum or valley points of the waveform cycle, not the average. The current of interest to us is the average current because it is the quantity of current that is most useful in converter performance specification and is the desired output current.

The waveform equations were also constructed without reference to slope-compensation schemes. They describe the actual behavior of $i_L(k)$ in slopes that can be measured from $i_L(t)$ at the converter output. The introduction of a compensating slope, i_E, occurs in the PWM function where it will be placed later, and not in the converter function (G_{id}). The waveform equations as derived above are the resulting closed-loop converter behavior without introduction of slope compensation into the PWM block.

The incremental variables are the small changes that occur from cycle to cycle and are of interest in incremental (small-signal) and linear analysis. Incremental equations describe the effect of infinitesimal perturbations from the steady-state operating parameter values of the quantities involved. These equations are not necessarily linear equations until they have been linearized by setting to zero the products of incremental quantities on the grounds that the product of already small changes is negligible. Instances of this occur in the following section.

The Slope Equations

The slope equations relate converter inductor current slopes to converter parameters under steady-state operation. On the general assumption that the converter PWM switch is operating in steady-state and inductor flux is balanced ($\Delta\lambda = L \cdot \Delta i_L = 0$ over T_s), then

$$m = \frac{\Delta i_L}{\Delta t} = \frac{v_L}{L} = \frac{V_L}{L} \ .$$

For steady-state response, $\Delta i_L = 0$ over a switching cycle of period T_s and

$$m_U \cdot (\delta \cdot T_s) - m_D \cdot (\delta' \cdot T_s) = \Delta i_L = 0 \ .$$

This balance must be maintained by control and if violated, these equations are not valid.

For flux balance of L for any one cycle,

$$\Delta\lambda = \int_0^{T_s} (v_C - v_L) \cdot dt = 0$$

where subscript C denotes the switch common terminal in series with the inductor, and v_L is the voltage on the other terminal of the inductor. Then it follows from the above equation that for steady-state operation, $v_C = v_L$. Flux balance can be expressed as

$$\Delta\lambda_{\mathrm{ON}} + \Delta\lambda_{\mathrm{OFF}} = 0.$$

Expressing the $\Delta\lambda$ terms in PWM-switch terminal voltages,

$$(v_A - v_L) \cdot \delta = (v_P - v_L) \cdot \delta'$$

where A and P are respectively the active and passive PWM-switch terminals. Then solving and substituting $v_C = v_L$,

$$\frac{v_A - v_C}{\delta'} = \frac{v_P - v_C}{\delta} \Rightarrow \frac{v_{AC}}{\delta'} = \frac{v_{PC}}{\delta} \ .$$

Applying the basic PWM-switch relationships,

$$\frac{v_{AC}}{\delta'} = \frac{v_{PC}}{\delta} = v_{AP} = v_{OFF} \ .$$

The off-time inductor voltage corresponds with Δi_L for the cycle. Then

$$v_{AC} = \delta' \cdot v_{AP} \ ; v_{PC} = \delta \cdot v_{AP} \ .$$

Returning to the slopes and relating them to PWM-switch voltages,

$$m_U = \frac{v_A - v_L}{L} = \frac{v_{AC}}{L} = \delta' \cdot \frac{v_{AP}}{L} = \delta' \cdot \frac{v_{OFF}}{L}$$

$$m_D = \frac{v_P - v_L}{L} = \frac{v_{PC}}{L} = \delta \cdot \frac{v_{AP}}{L} = \delta \cdot \frac{v_{OFF}}{L}$$

where total $v_{OFF} = V_{off} + v_{off}$.

These slopes are held constant as is $\bar{i}_L$ between switching times of successive cycles (but not over the cycle). For constant slopes, then $v_{OFF} = V_{off} = V_{ap}$, a constant inductor voltage.

The unified model of Tan uses the *total slope equations*:

$$m_U = \delta' \cdot \frac{v_{OFF}}{L}\,;\, m_D = \delta \cdot \frac{v_{OFF}}{L}\,.$$

These slopes relate to circuit quantities and describe what occurs in the converter circuit. What occurs in the PWM controller includes the compensating slope of magnitude m_E which contributes to the *sensed current*, i_S. The above relationships are not necessarily the slopes of i_S; they are derived for i_L instead.

The total slopes, when perturbed and linearized, result in the *incremental slope equations*:

$$m_u = (D' \cdot v_{off} - V_{off} \cdot d)/L$$

$$m_d = (D \cdot v_{off} + V_{off} \cdot d)/L$$

Closure

The waveform equations are discrete-time functions that can be also expressed in the frequency domain. The waveform and slope equations derived here lay the foundation for waveform-based current-loop modeling. In the next article, the dynamic equations for transfer functions of blocks in the current loop are derived.

About The Author

Dennis Feucht has been involved in power electronics for 25 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been doing current-loop converter modeling and converter optimization.

For more on current-mode control methods, see the How2Power Design Guide, select the Advanced Search option, go to Search by Design Guide Category, and select "Control Methods" in the Design Area category.