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Current-Loop Control In Switching Converters

Part 4: Clarifications Of Existing Models

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The progression of models, from the low-frequency averaged model through Ridley's sampled-loop model and on to Tan and Middlebrook's effort to unify them in the *unified* model continues in the next part in the refinement of the unified model, the *refined* model.

Moving forward in this discussion of current-mode control, we digress in our derivation of a unified model by taking a closer look at the existing models. The goal here is to obtain a better understanding of the various current-loop models by uncovering the similarities among these models as well as some of their underlying assumptions. These concepts set the stage for explaining the refined model of current-mode control that will be presented in Part 5.

Equivalence Of Low-Frequency-Averaged And Unified i_L

An expression for the average current can be written from waveform geometry, noting that the average is half the Δi_L of the on- and off-time intervals and that the interval durations are included in the averaging;

$$\bar{i}_L = i_I - \tfrac{1}{2}\cdot[(m_U\cdot\delta\cdot T_s)\cdot\delta + (m_D\cdot\delta'\cdot T_s)\cdot\delta']\cdot T_s = i_I - \tfrac{1}{2}\cdot[(m_U\cdot\delta^2 + m_D\cdot\delta'^2)]\cdot T_s$$

The incremental lf-avg i_L is given in E&M (p. 461, eqn 12.64) as

$$\bar{i}_l = i_i - M_e\cdot T_s\cdot d - \frac{T_s}{2}\cdot(D^2\cdot m_u + D'^2\cdot m_d)$$

and includes an additional term for slope compensation.[1] The E&M derivation allows the slopes to vary with incremental values of m_u and m_d to include the effects of v_g and v_o. The i_l are averaged over one period of the switching cycle. Substituting the incremental slopes into the lf-avg i_l,

$$\begin{aligned}\bar{i}_l &= i_i - M_e\cdot T_s\cdot d - \frac{T_s}{2}\cdot\left[D^2\cdot(D'\cdot v_{off} - V_{off}\cdot d) + D'^2\cdot(D\cdot v_{off} + V_{off}\cdot d)\right]\\ &= i_i - M_e\cdot T_s\cdot d - \frac{V_{off}\cdot T_s}{2\cdot L}\cdot(D'^2 - D^2)\cdot d - \frac{T_s}{2\cdot L}\cdot(D^2\cdot D' + D'^2\cdot D)\cdot v_{off}\\ &= i_i - M_e\cdot T_s\cdot d - \frac{V_{off}\cdot T_s}{2\cdot L}\cdot(D' - D)\cdot d - \frac{T_s}{2\cdot L}\cdot(D\cdot D')\cdot v_{off}\end{aligned}$$

Using

$$(D'^2 - D^2) = (D' + D)\cdot(D' - D) = D' - D$$

and

$$(D^2\cdot D' + D'^2\cdot D) = D\cdot D'\cdot(D + D') = D\cdot D'$$

then the resulting

$$\bar{i}_l = i_i - M_e\cdot T_s\cdot d - \frac{T_s}{2\cdot L}\cdot[V_{off}\cdot(D' - D)\cdot d + (D\cdot D')\cdot v_{off}]$$

This is equivalent to the corresponding expression of the unified model of Tan (p. 400, eqn 4).[2] This equivalency depends on the steady-state assumption because the averaging intervals are not the same for the two models. The lf-avg model averages i_l over the switching cycle while the unified model of Tan averages between sampling-point peaks of i_L. Yet for the same per-cycle waveforms, the average i_l is the same in value.

For control of the current-waveform minimum extremum or the valley current, i_I,

$$\bar{i}_L = \tfrac{1}{2}\cdot[(i_I + m_U\cdot\delta\cdot T_s) + i_I] = i_I + \tfrac{1}{2}\cdot m_U\cdot\delta\cdot T_s = i_I + \tfrac{1}{2}\cdot\left(\frac{\delta'\cdot v_{OFF}}{L}\right)\cdot\delta\cdot T_s$$

$$= i_I + \tfrac{1}{2}\cdot\delta\cdot\delta'\cdot\left(\frac{T_s\cdot v_{OFF}}{L}\right) = i_I + \tfrac{1}{2}\cdot\delta\cdot\delta'\cdot\Delta i_{L0}$$

Moving He(s) Through The Summing Block

One scheme for attempting to unify Ridley's model[3] with the average current of the lf-avg model is to simply move the $H_e(s)$ block through the loop summer, distributing it in both input and output branches of the summer:

$$i_i - H_e\cdot i_l = i_{Ce} = H_e\cdot\left(\frac{1}{H_e}\cdot i_i - i_l\right)$$

This would put $1/H_e(s)$ in the input path. Not only is the input quantity then sampled and held, resulting in an iterative discrete-time $i_l\,(k)$, it is also advanced by being shifted to the $k - 1$ cycle:

$$\frac{1}{H_e(s)} = \frac{e^{s\cdot T_s} - 1}{s\cdot T_s} = \left(\frac{1 - e^{-s\cdot T_s}}{s\cdot T_s}\right)\cdot e^{s\cdot T_s} = H_0(s)\cdot e^{s\cdot T_s} = H_0(s)\cdot z$$

The additional forward-path $H_e(s)$ combines with the PWM block (F_{m0}) to form a new $F_m(s)$. Expressed using $H_0(s)$,

$$H_e(s) = \frac{s\cdot T_s}{e^{s\cdot T_s} - 1} = \frac{e^{-s\cdot T_s}}{H_0(s)} = \left(\frac{s\cdot T_s}{1 - e^{-s\cdot T_s}}\right)\cdot e^{-s\cdot T_s}$$

The $H_e(s)$ function can thus be interpreted as an inverse ZOH with a one-cycle time delay: a differentiator which repeats every T_s, making impulses out of the steps in the stepped incremental current waveform and shifting them in time to the next cycle. This one-cycle delay in the forward path is negated in the input path by $1/H_e$ but not without first combining the feedback with input i_i (k –1) from the previous cycle.

The movement of $H_e\,(s)$ forward in the feedback block diagram shows possibilities for a unified model while removing the obstacle to transfer-function formation of the time-variance of the loop. In the Ridley model, the sampling function is in the feedback loop. In contrast, sampling occurs in the forward path of the unified model. This difference is important because of time-variance. With the sampler in the feedback loop, the sensed inductor current,

$$v_l = i_l\cdot R_S$$

is sampled; $v_l \to v_l{}^*$ and is the input to $H_e(s)$. In sampling systems, *a sampled variable is a free variable*. Thus $v_l{}^*$ is the free variable; the error quantity, v_e, is not. This prevents the construction of a transfer function in the loop of the Ridley model.

To show this, write the feedback loop equations starting with the sampled quantity in the feedback path:

$$v_l^* = (R_S\cdot i_l)^*$$

$$v_e = v_i - H_e\cdot v_l^*$$

$$i_l = (F_m\cdot G_{id})\cdot v_e = G\cdot v_e$$

Then converting this equation to a sampled form and substituting,

$$i_l^* = (G\cdot v_i - G\cdot H_e \cdot v_l^*)* = (G\cdot v_i)* - (G\cdot H_e)*\cdot v_l^* = (G\cdot v_i)* - (G\cdot H_e)*\cdot(R_S\cdot i_l)*$$

The factors of the last term, $(R_S\cdot i_l)*$ are not separable;

$$(R_S\cdot i_l)* \neq R_S^*\cdot i_l^*$$

If the sampler is moved after H_e, the feedback quantity input to the summer is $(H_e\cdot R_S\cdot i_l)*$ and this expression is also not separable; i_l cannot be separated out. In either case, the transfer function, i_l / i_i, cannot be derived without ignoring Nyquist-band harmonics and the piecewise discontinuities in the waveform steps, as does the lf-avg model.

The unified model effectively places the sampler at the input to the forward path so that v_e is sampled: $v_e \to v_e*$. This makes v_e* and hence i_l* free variables (because they are related by addition over which the * operation is distributive). The loop equations for the unified model are, beginning with the sampled quantity,

$$v_e = v_i - R_S\cdot i_l \Rightarrow v_e^* = (v_i - R_S\cdot i_l)*$$

$$i_l = (F_m(s)\cdot G_{id})\cdot v_e^* = G\cdot v_e^*$$

where in this case G is for the unified forward path. Then sampling i_l and substituting,

$$v_e^* = v_i^* - (R_S\cdot G)*\cdot v_e^* = \frac{v_i^*}{1+(R_S\cdot G)*}$$

noting that in general

$$(T\cdot X*)* = T*\cdot X*$$

and the already-sampled $X*$ is free to be factored out. Then for the current,

$$i_l^* = (G\cdot v_e^*)* = G*\cdot v_e^* = G*\cdot\frac{v_i^*}{1+(R_S\cdot G)*}$$

Because v_i^* is a free variable, the transfer function exists and is

$$\frac{i_l^*}{v_i^*} = \frac{G*}{1+(R_S\cdot G)*}$$

Consequently, the unified model has a transfer function, though time-variant, whereas the sampled-loop model cannot be decomposed algebraically and constructed into a transfer function. This is a justification for preferring a model that samples v_e, with sampler at the input to the forward path, G, so that a transfer function can be derived.

Simple Unified Model Derivation Of $F_m(s)$

In their work, Hong, Choi, and Ahn state that it is more conceptually appealing than in Ridley's model to put the sampling function where circuit sampling occurs, in or adjacent to the PWM function of the forward path.[4] This is a major feature of the unified model, where what is *unified* are the lf-avg inductor current and the sampled loop dynamics with $F_m(s)$ in the forward path, allowing T_C formation for a time-variant feedback loop.

If the effects of slope compensation are included in the derivation of T_{CV}, then the PWM transfer function, F_m, can be found. Hong, Choi, and Ahn present a derivation that is rather straightforward based on instantaneous sampling (stepped i_l) and Ridley's sampled-loop model. The derivation given here parallels theirs. Given the existence of a transfer function, we can equate the closed-loop feedback equation,

$$T_{CV}(s) = \frac{i_l}{i_i} = \frac{F_{mV}(s)\cdot G_{idV}}{1+F_{mV}(s)\cdot G_{idV}} = \frac{1}{1+\dfrac{1}{F_{mV}(s)\cdot G_{idV}}}$$

with the incremental valley-current sampled-loop expression as derived in Ridley's model,

$$T_{CV}(s) \cong \frac{1}{\left(\dfrac{s}{\omega_s/2}\right)^2 + \pi\cdot(\tfrac{1}{2}-D)\cdot\left(\dfrac{s}{\omega_s/2}\right)+1}$$

Then

$$\left(\frac{s}{\omega_s/2}\right)^2 + \pi\cdot(\tfrac{1}{2}-D)\cdot\left(\frac{s}{\omega_s/2}\right) = \frac{1}{F_{mV}(s)\cdot G_{idV}(s)}$$

$G_{idV}(s)$ is the converter transfer function i_l/d and was derived from the valley current independent of converter topology;

$$G_{idV}(s) = \frac{i_l(s)}{d(s)} = \frac{V_{off}}{s\cdot L} = \frac{\Delta I_{L0}}{s\cdot T_s}$$

Having an expression for G_{idV}, we return to the derivation of $F_{mV}(s)$, make the substitution for G_{idV} and solve;

$$F_{mV}(s) = \frac{s\cdot L}{V_{off}}\cdot\frac{1}{s\cdot\left(\dfrac{s}{(\omega_s/2)^2}+\dfrac{\pi\cdot(\frac{1}{2}-D)}{\omega_s/2}\right)} = \frac{1}{\Delta I_{L0}\cdot(\frac{1}{2}-D)}\cdot\frac{1}{\dfrac{s}{\left(\dfrac{\omega_s}{2}\right)\cdot\pi\cdot(\frac{1}{2}-D)}+1}$$

The static F_{mV0} is

$$F_{mV0} = \frac{1}{\Delta I_{L0}\cdot(\frac{1}{2}-D)}$$

F_{mV} has a single pole at

$$\omega_P = \left(\frac{\omega_s}{2}\right)\cdot\pi\cdot(\tfrac{1}{2}-D)$$

which is the same pole as in the unified model of Tan.

The T_{CV} derived from the sampled loop was equated with the general closed-loop feedback expression which in itself does not include the effects of loop sampling. If i_{Ce}, the error current of the loop (at the output of the summer) of the unified model is sampled, then i_{Ce} is a free sampled variable, $i_{Ce}^*(s)$, and a transfer function can be constructed. The error quantity, however, is not the input to the PWM block when slope compensation is included.

Closure

The three sections of this part offered intimations of improvement in current-loop modeling by identifying what is the same in different models, though expressed differently; hidden assumptions underlying error in the sampled-loop model and how these assumptions might be circumvented; and a simplified development of the unified model as a prelude to a refinement of it. The various aspects of existing models expounded in this part prepare the way to examine the refined model, presented in the next part.

References

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About The Author

Dennis Feucht has been involved in power electronics for 25 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been doing current-loop converter modeling and converter optimization.

For more on current-mode control methods, see the How2Power Design Guide, select the Advanced Search option, go to Search by Design Guide Category, and select "Control Methods" in the Design Area category.