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Current-Loop Control In Switching Converters

Part 6: Slope Compensation

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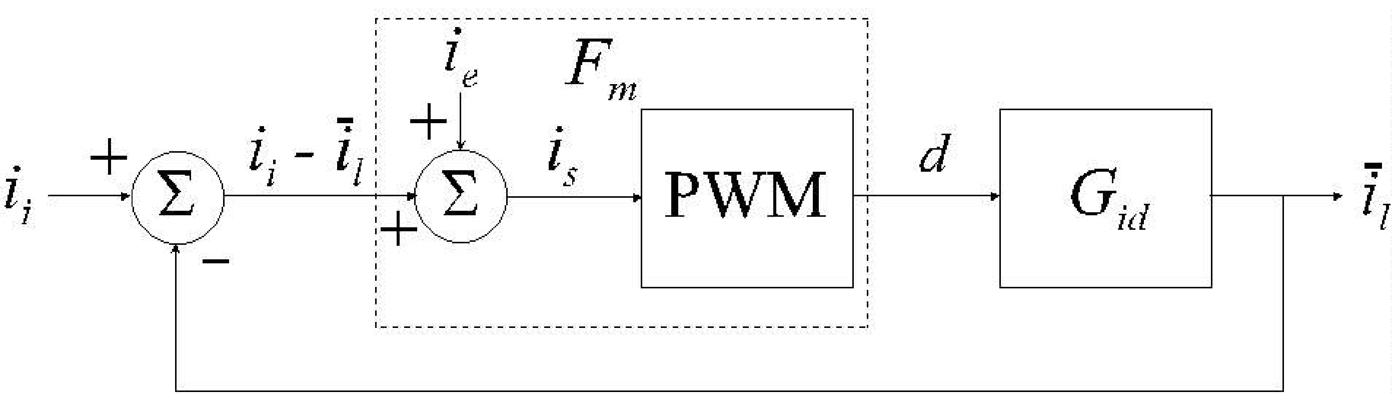
The last installment of this article presented a refined model of current-mode control that provides a deeper unification of the quasi-static or low-frequency current-loop behavior with the sampling aspects by deriving the dynamics equations for transfer functions from the average current variable rather than the valley current. Here in part 6 of this series, the effect of slope compensation is included in the refined model.

Specifically, we'll analyze the impact of three different slope-compensation schemes on the refined model, noting similarities and differences in the key waveform equations. We'll also note how the refined model with slope compensation compares to earlier models of current-model control. Finally, we'll examine the implications of this analysis in terms of establishing guidelines for converter design that ensure loop stability.

Slope Compensation

Slope compensation is a dynamic compensation technique by which the slope of the feedback inductor current is modified by the addition of a ramp to result in the sensed current. In this section, the incremental output variable $i_l(i_i)$ is expressed as $i_l(i_i\ ,\ i_e)$ to include the slope-compensation function, i_e. The slope-compensated waveform is input to the PWM circuit as the sensed current, i_s, as shown in the block diagram.



Slope compensation schemes differ in how i_e is added to i_i and the summation requires an additional summing block, as shown. In the scheme of the diagram, the summation occurs within the forward path with the error quantity i_{Ce} and the compensation function, i_e;

$$i_s = i_{Ce} + i_e = [i_i - i_l] + i_e\,.$$

The two other possible locations for the summation of i_e are in the input path and in the feedback path. For the input path, the input, i_i effectively becomes $i_i + i_e$:

$$i_s = [i_i + i_e] - i_l\,.$$

A summer in the feedback path subtracts i_e from i_l. Then the compensated error quantity, i_s, is the sensed feedback current subtracted from the input to form the error quantity:

$$i_s = i_i - [i_l - i_e]\,.$$

Because addition is associative and commutative, and with no additional processing between summations, the three means of combining the three quantities comprising i_s are equivalent. Consequently, we can consider the effect of i_e to be that of an addition to the commanded input, i_i. It should therefore be possible to substitute $i_i + i_e$ into the uncompensated closed-loop $i_l(i_i)$ for i_i and derive the compensated inductor-current waveform equations.

Though eventually we want the incremental model as shown in the block diagram, the derivations begin with the total variables. By adding the total $i_E(t)$ as a compensating waveform to the loop error, this causes $\delta(k)$ to be altered, as its value each cycle is determined by the F_m block. F_m contains not only the comparator sampling function itself (labeled PWM) but also a summing block preceding it which outputs the sensed waveform, i_S. Slope compensation affects the sensed waveform as an addition to the error quantity, i_{CE}. The sensed current is

$$i_S(t) = [i_I(t) - i_L(t)] + i_E(t) = i_{CE}(t) + i_E(t)\,.$$

The resulting $i_L = G \cdot i_S \neq i_S$. The uncompensated closed-loop waveform equations of i_L and δ are affected by slope compensation in that δ is now determined by $F_m\,(i_S)$. However, the slopes of i_L remain the uncompensated slopes because they are determined by converter circuit parameters, not by F_m. The slopes and offset of i_S alone are modified by i_E. The forward-path transmittance, G, however, is an incremental transfer function and the total-variable i_L/i_S is nonlinear and can vary between on- and off-times of the cycle. This complicates the determination of F_m in that it must be linearized. (G_{id} is already linear.) The correct value for a linearized F_m is addressed in a later section.

By substituting $i_i + i_e$ for i_i into the uncompensated closed-loop equations, the compensated equation for i_l results;

$$i_l = \frac{G}{1+G}\cdot(i_i + i_e) = \frac{G}{1+G}\cdot i_i + \frac{G}{1+G}\cdot i_e\ .$$

where $G/(1 + G)$ is the unified-model uncompensated closed-loop transfer function of $i_l\,/i_i$. The compensated i_l is therefore the uncompensated i_l with the added response to slope compensation, i_e, by the same loop.

Returning to the total variables and substituting into $i_S(k)$ the uncompensated discrete-time $i_L(k)$ with $i_E(k)$ as its additional input,

$$\begin{aligned} i_S(k) &= i_I(k) - \left[-\left(\frac{m_D}{m_U}\right)\cdot i_L(k-1) + \left(1+\frac{m_D}{m_U}\right)\cdot[i_I(k)+i_E(k)] - m_D\cdot T_s\right] + i_E(k) \\ &= -\left(\frac{m_D}{m_U}\right)\cdot\big([i_I(k)+i_E(k)] - i_L(k-1)\big) + m_D\cdot T_s = -\left(\frac{m_D}{m_U}\right)\cdot\big([i_I(k)+i_E(k)] - i_L(k-1) - m_U\cdot T_s\big) \end{aligned}\ .$$

When an expression for $i_E(k)$ is substituted, it will (if it contains $\delta(k)$) include i_L and i_l terms, which modify the coefficient of i_S. The sensed current can be expressed more generally in steady-state by applying the slope equations;

$$i_S(k) = -\left(\frac{D}{D'}\right)\cdot\big([i_I(k)+i_E(k)] - i_L(k-1) - D'\cdot\Delta I_{L0}\big).$$

The sampling instant at $\delta\cdot T_s$ within the cycle (or $(k-1)\cdot T_s + \delta(k)\cdot T_s$ in total time) is determined by F_m according to the control law: $i_S(\delta\cdot T_s) = 0$. The general form of i_S describing the sampling event of cycle k is

$$i_S(\delta(k)\cdot T_s)=[i_I(k)-i_L(\delta(k)\cdot T_s)]+i_E(\delta(k)\cdot T_s)=0$$

where $i_I(k) = i_I(\delta(k)\cdot T_s)$. This equation describes the sensed current at the sampling time, $\delta(k)\cdot T_s$. The stability of i_S will, of course, affect loop stability because it determines δ. Loop stability analysis therefore can be an analysis of the stability of $i_S(t)$. The control law, $i_S(\delta\cdot T_s) = 0$, determines δ and can be solved for δ when i_E is substituted into i_S.

Returning to the general equation for $i_S(k)$, the expression $i_I(k) - i_L(k-1)$ in $i_S(k)$ has the form of i_{Ce} except that i_L is of the previous cycle. For steady-state operation, the waveform is stable and $i_L(k) = i_L(k-1)$. Substituting from the slope equation for m_D, then

$$\text{steady-state } i_S=\frac{m_D\cdot T_s}{1+\left(\dfrac{m_D}{m_U}\right)}=\left(\frac{m_U\cdot m_D}{m_U+m_D}\right)\cdot T_s=D\cdot D'\cdot\left(\frac{V_{off}\cdot T_s}{L}\right)=D\cdot D'\cdot\Delta I_{L0}\,.$$

The usual method of analysis derives the total closed-loop waveform equations from the waveform geometry under steady-state response, and then differentiates or perturbs them to produce the incremental equations. This approach is taken here with distinction maintained between i_L and i_S and with the additional demonstration that the waveform equations are equivalent to the uncompensated equations with the addition of i_E to i_I, thus localizing slope compensation in the model to F_m. The total-variable valley-current waveforms will be derived for completeness and used to show the localization of i_E to F_m. The total and incremental valley-current waveform equations will be shown to be equivalent for the given slope-compensation schemes while i_S differs with i_E. The average-current waveforms are also derived. Slopes will be held constant for these analyses.

First Slope-Compensation Scheme

To the total-variable loop error i_{CE} is added a negative-slope compensation ramp of $-m_E$ slope (elsewhere $-m_c$),

$$i_E(t)=-m_E\cdot t$$

and $i_E(\delta T_s)$ at switching is

$$i_E(\delta\cdot T_s)=-m_E\cdot\delta\cdot T_s\,.$$

The i_L slope, m_D, remains unchanged by slope compensation and at the end of the off-time,

$$i_L(k)=i_L(\delta\cdot T_s)-m_D\cdot\delta'\cdot T_s\,.$$

For the up-slope segment, the slope of i_L remains the unaltered m_U and the control-law equation is

$$i_S(\delta\cdot T_s)=[i_I(k)-i_L(\delta\cdot T_s)]+(-m_E\cdot\delta\cdot T_s)=[i_I(k)-(i_L(k-1)+m_U\cdot\delta\cdot T_s)]-m_E\cdot\delta\cdot T_s=0\,.$$

At the sampling point, the inductor-current peak value is

$$i_L(\delta\cdot T_s)=i_I(k)-m_E\cdot\delta\cdot T_s=i_L(k-1)+m_U\cdot\delta\cdot T_s\,.$$

Solving for $\delta\cdot T_s$,

$$\delta \cdot T_s = \frac{i_I(k) - i_L(k-1)}{m_U + m_E} = \frac{-\Delta i_L(k)}{m_U + m_E}.$$

Substituting this into $i_L(\delta \cdot T_s)$,

$$i_L(\delta \cdot T_s) = \left(\frac{m_E}{m_U + m_E}\right) \cdot i_L(k-1) + \left(\frac{m_U}{m_U + m_E}\right) \cdot i_I(k).$$

Substituting this peak value into the off-time equation, the resulting compensated total valley inductor current is

$$i_L(k) = -\left(\frac{m_D - m_E}{m_U + m_E}\right) \cdot i_L(k-1) + \left(\frac{m_U + m_D}{m_U + m_E}\right) \cdot i_I(k) - m_D \cdot T_s.$$

Comparing the slope-compensated $i_L(k)$ to the uncompensated $i_L(k)$, what has changed are the coefficients which consist of slope expressions. It would appear that i_E has caused i_L to have changed its slopes but this would be a misleading interpretation of the compensated i_L expression. All that compensation has affected is a change in δ as expressed in slopes. The converter circuit parameters have not changed and they determine i_L slopes. The discrete-time coefficients are ratios of slopes and are dependent only on δ. Consequently, it should be possible to express i_L in its observable circuit slopes, modified only by the introduction of i_E at the loop input.

To show that the total $i_L(k)$ can be written with localization of i_E (as added to i_I), start with the uncompensated i_L and add i_E to i_I:

$$i_L(k) = -\left(\frac{m_D}{m_U}\right) \cdot i_L(k-1) + \left(1 + \frac{m_D}{m_U}\right) \cdot (i_I(k) + i_E(k)) - m_D \cdot T_s.$$

Substituting for i_E, this becomes

$$i_L(k) = -\left(\frac{m_D}{m_U}\right) \cdot i_L(k-1) + \left(1 + \frac{m_D}{m_U}\right) \cdot (i_I(k) - m_E \cdot \delta(k) \cdot T_s) - m_D \cdot T_s.$$

Substituting the other of the two uncompensated waveform quantities,

$$\delta(k) = \frac{i_L(k) - i_L(k-1)}{(m_U + m_D) \cdot T_s} + \frac{m_D}{m_U + m_D}$$

then

$$i_L(k) = -\left(\frac{m_D}{m_U}\right) \cdot i_L(k-1) + \left(1 + \frac{m_D}{m_U}\right) \cdot \left[i_I(k) - m_E \cdot \left(\frac{i_L(k) - i_L(k-1)}{(m_U + m_D) \cdot T_s} + \frac{m_D}{m_U + m_D}\right) \cdot T_s \right] - m_D \cdot T_s.$$

After some algebra, this reduces to the same $i_L(k)$ as derived above. Consequently, the effect of compensation in the waveform equations can be localized to the input error summer as i_E added to i_I in the general uncompensated waveform equations.

The slope expressions can be replaced by the simpler duty-ratio expressions for *steady-state* waveforms (constant δ) from charge balance. Applying the waveform equation of δ for steady-state response,

$$\Delta i_L(\text{on}) = (m_U + m_E)\cdot D\cdot T_s = -\Delta i_L(\text{off}) = (m_D - m_E)\cdot D'\cdot T_s\,.$$

The following conversion formulas apply to $i_L(k)$ as all three schemes fall out of this constraint:

$$\left(\frac{m_D - m_E}{m_U + m_E}\right) = \frac{D}{D'} \qquad \left(\frac{m_U + m_D}{m_U + m_E}\right) = \frac{1}{D'} \qquad \left(\frac{m_D - m_E}{m_U + m_D}\right) = D \qquad \left(\frac{m_D + m_E}{m_U + m_D}\right) = D'\,.$$

Substituting from these expressions, the total slope-compensated valley inductor current in D is

$$i_L(k) = -\frac{D}{D'}\cdot i_L(k-1) + \frac{1}{D'}\cdot i_I(k) - m_D\cdot T_s\ .$$

In D, the equation is the same as for the uncompensated waveform. Only when the coefficients are expressed in slopes is there a difference. Consequently, the effect of slope compensation is entirely contained within its effect on D. Slope compensation modifies the unmodified waveform by adding to it a compensation ramp so that the *sensed* addition results in the desired D. The modified slopes of i_S affect how D is determined while the inductor-current waveform continues to be described by the same equation in D.

Substituting $i_L(k)$ into i_S, the total equation for i_S at the end of the cycle ($k\cdot T_s$) is

$$i_S(k) = [i_I(k) - i_L(k)] + i_E(k) = i_I(k) - \left[-\left(\frac{m_D - m_E}{m_U + m_E}\right)\cdot i_L(k-1) + \left(\frac{m_U + m_D}{m_U + m_E}\right)\cdot i_I(k) - m_D\cdot T_s\right] - m_E\cdot T_s\,.$$

This simplifies to

$$i_S(k) = -\left(\frac{m_D - m_E}{m_U + m_E}\right)\cdot\left(i_I(k) - i_L(k-1) - (m_U + m_E)\cdot T_s\right).$$

This i_S has the same form as the general $i_S(k)$ but with different slope expressions. If written in D, however,

$$i_S(k) = -\left(\frac{D}{D'}\right)\cdot\left(i_I(k) - i_L(k-1) - (m_U + m_E)\cdot T_s\right).$$

When $i_E(k) = -m_E\cdot T_s$ is substituted in the general $i_S(k)$ written in D, the result is the same. Slope compensation changes the slope-ratio coefficients as it did for i_L, and they are the same in D as the uncompensated ratios. The constant term, which is related to ΔI_{L0}, remains unchanged.

Taking the differential of the total current, the compensated incremental valley inductor current is

$$i_l(k) = -\left(\frac{m_D - m_E}{m_U + m_E}\right) \cdot i_l(k-1) + \left(\frac{m_U + m_D}{m_U + m_E}\right) \cdot i_i(k).$$

The uncompensated incremental valley current,

$$i_l(k) = -\left(\frac{m_D}{m_U}\right) \cdot i_l(k-1) + \left(1 + \frac{m_D}{m_U}\right) \cdot i_i(k)$$

is affected only by an addition to i_i of i_e:

$$i_l(k) = -\left(\frac{m_D}{m_U}\right) \cdot i_l(k-1) + \left(1 + \frac{m_D}{m_U}\right) \cdot (i_i(k) + i_e(k)).$$

The incremental i_e is a function of the uncompensated $d(k)$,

$$d(k) = \frac{i_l(k) - i_l(k-1)}{(m_U + m_D) \cdot T_s}.$$

The uncompensated $d(k)$ is used in $i_e(k)$ because it is the d consistent with the uncompensated i_l. The effect of i_e is to modify the uncompensated waveform equations by its effect on the uncompensated loop. Substituting d,

$$i_e(k) = -m_E \cdot T_s \cdot d(k) = -m_E \cdot T_s \cdot \left(\frac{i_l(k) - i_l(k-1)}{(m_U + m_D) \cdot T_s}\right) = -\left(\frac{m_E}{m_U + m_D}\right) \cdot \left(i_l(k) - i_l(k-1)\right).$$

Substituting $i_e(k)$ into the uncompensated $i_l(k)$ and simplifying, the same incremental $i_l(k)$ results as derived above from the compensated total-variable expression for i_l.

The incremental i_s can be found by taking the differential of the total $i_S(k)$;

$$i_s(k) = -\left(\frac{m_D}{m_U}\right) \cdot \left([i_i(k) - i_l(k-1)] + i_e(k)\right).$$

This can be corroborated with the localization property by substituting i_e (having the uncompensated d) into i_s. Then

$$i_s(k) = [i_i(k) - i_l(k)] + i_e(k) = i_i(k) - \left[-\left(\frac{m_D}{m_U}\right) \cdot i_l(k-1) + \left(1 + \frac{m_D}{m_U}\right) \cdot [i_i(k) + i_e(k)]\right] + i_e(k).$$

Collecting i_e terms, the result is equivalent to the previous equation:

$$i_s(k) = \left(\frac{m_D}{m_U}\right) \cdot \left(i_l(k-1) - [i_i(k) + i_e(k)]\right).$$

For average total inductor current, the discrete-time $\bar{i}_L$ is

$$\bar{i}_L(k) = \tfrac{1}{2} \cdot ([i_I(k) - m_E \cdot \delta(k) \cdot T_s] + i_L(k-1)) \cdot \delta(k) + \tfrac{1}{2} \cdot ([i_I(k) - m_E \cdot \delta(k) \cdot T_s] + i_L(k)) \cdot \delta'(k)$$

or

$$\bar{i}_L(k) = \tfrac{1}{2} \cdot (i_I(k) - m_E \cdot \delta(k) \cdot T_s) + \tfrac{1}{2} \cdot (i_L(k-1) \cdot \delta(k) + i_L(k) \cdot \delta'(k)).$$

Substituting for $i_L(k)$ from the general waveform equation in δ coefficients and reducing,

$$\bar{i}_L(k) = \tfrac{1}{2} \cdot [i_L(k-1) + (i_I(k) - m_E \cdot T_s)].$$

Under steady-state operation and with constant slopes, the cycle valley current values are

$i_L(k-1) = i_L(k) = i_{LV}$. Then

$$\bar{i}_L = \tfrac{1}{2} \cdot ([i_I - m_E \cdot T_s] + i_{LV}) = \tfrac{1}{2} \cdot (i_I + i_{LV}) + \bar{i}_E.$$

where the average i_E over a cycle is $\bar{i}_E$. A negative i_E has the effect of reducing δ and causing $|\Delta i_L|$ to be less than that of the uncompensated loop.

The incremental $\bar{i}_L$ of the uncompensated steady-state waveform is

$$\bar{i}_l(k) = -\frac{D}{D'} \cdot \bar{i}_l(k-1) + \frac{1}{2} \cdot \left[\left(1 + \frac{1}{D'}\right) \cdot i_i(k) + \frac{D}{D'} \cdot i_i(k-1)\right].$$

By substituting i_l,

$$i_l(k) = 2 \cdot \bar{i}_l(k) - i_i(k)$$

into $d(k)$ with constant converter input and output voltages whereby $v_{OFF} = V_{off}$, then

$$d(k) = \frac{i_l(k) - i_l(k-1)}{(v_{OFF}/L) \cdot T_s} = \left(\frac{L}{V_{off} \cdot T_s}\right) \cdot \left([2 \cdot \bar{i}_l(k) - i_i(k)] - [2 \cdot \bar{i}_l(k-1) - i_i(k-1)]\right)$$
$$= -\left(\frac{1}{\Delta I_{L0}}\right) \cdot \left(2 \cdot [\bar{i}_l(k) - \bar{i}_l(k-1)] - [i_i(k) - i_i(k-1)]\right).$$

Substituting $d(k)$ into $i_e(k)$,

$$i_e(k) = -m_E \cdot d(k) \cdot T_s = -\left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot \left(2 \cdot [\bar{i}_l(k) - \bar{i}_l(k-1)] - [i_i(k) - i_i(k-1)]\right).$$

With i_e added to i_i, $\bar{i}_l$ becomes

$$\bar{i}_l(k) = -\frac{D}{D'} \cdot \bar{i}_l(k-1) + \frac{1}{2} \cdot \left(\frac{D'+1}{D'}\right) \cdot [i_i(k) + i_e(k)] + \frac{1}{2} \cdot \frac{D}{D'} \cdot [i_i(k-1) + i_e(k-1)].$$

Substituting for i_e and after some algebra, this reduces to a second-order difference equation:

$$\left(\left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot (D'+1) + D'\right) \cdot \bar{i}_l(k) = \left(2 \cdot \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D' - D\right) \cdot \bar{i}_l(k-1) + \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D \cdot \bar{i}_l(k-2)$$
$$+ \frac{1}{2} \cdot (D'+1) \cdot i_i(k) - \frac{1}{2} \cdot \left(2 \cdot \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) - D\right) \cdot i_i(k-1) - \frac{1}{2} \cdot \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D \cdot i_i(k-2).$$

Second Slope-Compensation Scheme

For the second scheme, compensating slope, m_E, is added to both up-slope and down-slope segments of the loop error. This is accomplished by subtracting from i_L the same compensating ramp as in the first scheme,

$$i_E(t) = -m_E \cdot t.$$

The result is then subtracted from i_I, so that

$$i_S(k) = i_I(k) - [i_L(k) - i_E(k)] = [i_I(k) - i_L(k)] + i_E(k) = i_{Ce}(k) + i_E(k).$$

This is equivalent to the first scheme, though i_S peaks at i_I while the on-time slope of i_S is steeper because of its addition of m_E. The two effects cancel, resulting in the same δ. The waveform equations are derived from the waveform geometry. At the sample time,

$$i_S(\delta \cdot T_s) = i_I(k) - [i_L(k-1) + m_U \cdot \delta \cdot T_s] - m_E \cdot \delta \cdot T = i_I(k) - i_L(k-1) - (m_U + m_E) \cdot \delta \cdot T_s = 0.$$

Then

$$\delta \cdot T_s = \frac{i_I(k) - i_L(k-1)}{m_U + m_E}.$$

This is identical to the first scheme. During the off-time, i_S continues to have added to it the $-m_E$ slope, and at $k \cdot T_S$,

$$i_S(k) = i_I(k) - i_L(k) + i_E(k) = i_I(k) - [i_I(k) - m_D \cdot \delta' \cdot T_s] - m_E \cdot \delta' \cdot T_s = (m_D - m_E) \cdot \delta' \cdot T_s.$$

Substituting for δ and simplifying,

$$i_S(k) = -\left(\frac{m_D - m_E}{m_U + m_E}\right)\cdot\left(i_I(k) - i_L(k-1)\right) + (m_D - m_E)\cdot T_s\,.$$

Compared to the first scheme, the two expressions for i_S are equal. In the first scheme, $i_I + i_E$ is compared to i_L while in the second scheme, they are combined before being compared to zero. Yet it is the dynamics of the combination of all three waveforms that determines the stability of δ and the value of $i_S(k)$, whether they are added before or at the inputs of the comparator. The comparator responds to the difference between its inputs, and that constitutes the completion of the summation in i_S. In the first scheme, there is no single i_S circuit waveform as in the second, though it can be regarded as existing at the comparator input. It is this abstracted i_S and not either comparator input waveform that is relevant to the analysis of the first scheme and which makes it equivalent to the second scheme. The comparator output is the sign or polarity of i_S. The comparator operations of the PWM block are equivalent between schemes.

Solving for i_L by substituting the same δ as in the first scheme into the same uncompensated i_L equation produces the same result:

$$i_L(k) = -\left(\frac{m_D - m_E}{m_U + m_E}\right)\cdot i_L(k-1) + \left(\frac{m_U + m_D}{m_U + m_E}\right)\cdot i_I(k) - m_D\cdot T_s\,.$$

The average inductor current for the second compensation scheme is the same as the first; the waveform equations are identical and the average current must also be the same. Under steady-state operation, $i_L(k-1) = i_L(k) = i_{LV}$ and

$$i_{LV} = -\frac{D}{D'}\cdot i_{LV} + \frac{1}{D'}\cdot i_I - m_D\cdot T_s = i_I - D'\cdot m_D\cdot T_s = i_I - D\cdot D'\cdot(\Delta I_{L0})\,.$$

Then taking the steady-state average current from the first scheme and developing it further by substituting i_{LV},

$$\bar{i}_L = \tfrac{1}{2}\cdot([i_I - m_E\cdot T_s] + i_{LV}) = i_I - \tfrac{1}{2}\cdot(m_E\cdot T_s + D\cdot D'\cdot(\Delta I_{L0}))\,.$$

The equations for $i_I(k)$ and $\bar{i}_L(k)$ must be the same as those of the first scheme because the waveform and averaging equations from which they are derived are the same.

Third Slope-Compensation Scheme

The third scheme (found in Ridley and Sheehan) differs from the previous schemes in that the compensating ramp is non-zero only during the on-time. Within a cycle,

$$i_E(t) = -m_E\cdot t\,,\ 0 \le t \le \delta\cdot T_s\,.$$

The sensed current at the sample point is

$$i_S(\delta\cdot T_s) = i_I(k) - [i_L(k-1) + m_U\cdot\delta\cdot T_s] - m_E\cdot\delta\cdot T_s = i_I(k) - i_L(k-1) - (m_U + m_E)\cdot\delta\cdot T_s\,.$$

Then set $i_S(\delta\cdot T_s) = 0$ and solve for δ;

$$\delta\cdot T_s = \frac{i_I(k) - i_L(k-1)}{m_U + m_E}\,.$$

The δ equation is the same as for the first two schemes as it must be; the conditions during on-time are the same. The inductor current will therefore have the same waveform equation as the previous schemes:

$$i_L(k) = -\left(\frac{m_D - m_E}{m_U + m_E}\right)\cdot i_L(k-1) + \left(\frac{m_U + m_D}{m_U + m_E}\right)\cdot i_I(k) - m_D \cdot T_s.$$

Because the slope compensation is different, i_S differs from the previous schemes. For the off-time segment, substituting $\delta \cdot T_s$ into the sensed current, its value at the end of the cycle is

$$i_S(k) = i_I(k) - [i_I(k) - m_D \cdot \delta' \cdot T_s] = m_D \cdot T_s - m_D \cdot \left(\frac{i_I(k) - i_L(k-1)}{m_U + m_E}\right).$$

This becomes the sensed valley current;

$$i_S(k) = -\left(\frac{m_D}{m_U + m_E}\right)\cdot (i_I(k) - i_L(k-1) + (m_U + m_E)\cdot T_s).$$

The sensed valley current has the same form as i_S of the previous schemes but the slope-ratio coefficients are not the same. In steady-state,

$$-\Delta i_S = (m_U + m_E)\cdot D \cdot T_s = m_D \cdot D' \cdot T_s.$$

and the slope ratios are related to D by the following formulas:

$$\frac{m_D}{m_U + m_E} = \frac{D}{D'} \qquad 1 + \frac{m_D}{m_U + m_E} = \frac{1}{D'}.$$

Then in D, the steady-state $i_S(k)$ is the same as the previous schemes.

The average inductor current equation is the same as for the previous schemes, though $\bar{i}_L(k)$ can alternatively be derived for the third scheme from

$$\bar{i}_L(k) = \tfrac{1}{2}\cdot (m_U + m_E)\cdot \delta(k)\cdot T_s + i_L(k-1)$$
$$\bar{i}_L(k) = \tfrac{1}{2}\cdot m_D \cdot \delta'(k)\cdot T_s + i_L(k)$$

after $\delta(k)$ is eliminated. These equations do not include i_I though the effects of i_I on $\bar{i}_L$ are implicit in $\delta(i_I)$.

Incremental Average Inductor Current With Slope Compensation

The total and incremental waveform equations for the three slope-compensation schemes considered are equivalent whether the coefficients are expressed as slope ratios or duty-ratio. Thus the inductor current equation is independent of slope compensation scheme, and is

$$i_L(k) = -\left(\frac{m_D - m_E}{m_U + m_E}\right)\cdot i_L(k-1) + \left(\frac{m_U + m_D}{m_U + m_E}\right)\cdot i_I(k) - m_D \cdot T_s = -\frac{D}{D'}\cdot i_L(k-1) + \frac{1}{D'}\cdot i_I(k) - D\cdot \Delta I_{L0}.$$

Similarly, the incremental $i_I(k)$ is independent of slope-compensation scheme and is

$$i_l(k) = -\frac{D}{D'} \cdot i_l(k-1) + \frac{1}{D'} \cdot i_i(k).$$

The z-transform of i_l is

$$i_l(z) = -\frac{D}{D'} \cdot i_l(z) \cdot z^{-1} + \frac{1}{D'} \cdot i_i(z).$$

Expressed as a valley-current-gain transfer function in the z-domain,

$$T_{CV}(z) = \frac{i_l(z)}{i_i(z)} = \left(\frac{1}{D'}\right) \cdot \frac{z}{z + \left(\frac{D}{D'}\right)}.$$

and is stable when $D/D' < 1$ or $D < ½$. This is the closed-loop equation of the sampled-loop model of Ridley. It is equivalent to that of the unified model of Tan which samples $i_l(k)$ coincident with $\bar{i}_L(k)$. This current-loop transfer function is effectively the same as that used in the unified model of Tan (based on the same waveform difference equations) and consequently there is a discrepancy in the model between valley-current sampling and average-current derivation of F_{m0}. This results in a model that does not fully unify the average current with the sampling effects of the sampled-loop model. The refined model attempts to make a full unification by deriving F_{m0} as a consequence of using average i_l in the waveform equations of the current-loop transfer function.

The sensed current, i_S, is the same for the first and second schemes and differs from the third scheme only in how slope ratios are related to D. Expressed in D, they are equivalent:

$$i_S(k) = -\left(\frac{D}{D'}\right) \cdot (i_I(k) - i_L(k-1) + (m_U + m_E) \cdot T_s).$$

The $\bar{i}_l(k)$ difference equation is dependent on $i_L(k)$ and is independent of compensation scheme;

$$\left(\left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot (D'+1) + D'\right) \cdot \bar{i}_l(k) = \left(2 \cdot \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D' - D\right) \cdot \bar{i}_l(k-1) + \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D \cdot \bar{i}_l(k-2)$$
$$+ \frac{1}{2} \cdot (D'+1) \cdot i_i(k) - \frac{1}{2} \cdot \left(2 \cdot \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) - D\right) \cdot i_i(k-1) - \frac{1}{2} \cdot \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D \cdot i_i(k-2).$$

This second-order equation in the z-domain can be written as the closed-loop transfer function,

$$\frac{\bar{i}_l(z)}{i_i(z)} = \frac{1}{2} \cdot \frac{(D'+1) \cdot z^2 - \left(\left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) - D\right) \cdot z - \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D}{\left(\left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot (D'+1) + D'\right) \cdot z^2 - \left(2 \cdot \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D' - D\right) \cdot z - \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D}.$$

Setting m_E = 0 A/s, this reduces to $T_C(z)$:

$$T_C(z) = \frac{\bar{i}_l(z)}{i_i(z)} = \frac{1}{2} \cdot \frac{(D'+1) \cdot z + D}{D' \cdot z + D}.$$

In normalized form,

$$\frac{\bar{i}_l(z)}{i_i(z)} = \frac{1}{2} \cdot \frac{-\dfrac{D'+1}{\left(\dfrac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D} \cdot z^2 + \dfrac{\left(\dfrac{m_E \cdot T_s}{\Delta I_{L0}}\right) - D}{\left(\dfrac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D} \cdot z + 1}{-\left(\dfrac{D'+1}{D} + \dfrac{D'}{\left(\dfrac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D}\right) \cdot z^2 + \left(2 \cdot \dfrac{D'}{D} - \dfrac{1}{\left(\dfrac{m_E \cdot T_s}{\Delta I_{L0}}\right)}\right) \cdot z + 1}.$$

The frequently-occurring

$$\left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) = \frac{m_E}{m_D} \cdot D$$

and $m_E \cdot T_s$ is the peak-to-peak PWM current ramp. The condition often chosen for optimal dynamic response is

$$m_E = m_D$$

which results in the above expression reduced to *D*.

In the literature, the impression is sometimes given that *D* can exceed 0.5 in a stable loop because of m_E. However, the effect of m_E is to reduce the value of *D* so that it is within the stable interval. Consequently, the direct method of loop compensation is to design the loop to operate within an acceptable response range of *D* by selecting turns ratios and other relevant parameters to keep *D* within acceptable dynamic-response bounds. However, m_E also has an effect on F_{m0} and thus frequency response, but this affects continuous-feedback stability involving loop gain and not the stability related to discrete-time effects.

In the next and final section of this article, part 7, we will return to a comparison of the important PWM factor, F_{m0} in the current loop and show that the refined model is compatible with the major existing models when reduced to accommodate their limiting assumptions. Finally, we'll complete this lengthy process of modeling the current-loop with the merging of waveform-based and circuit-based modeling.

About The Author

Dennis Feucht has been involved in power electronics for 25 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been doing current-loop converter modeling and converter optimization.

For more on current-mode control methods, see the How2Power Design Guide, select the Advanced Search option, go to Search by Design Guide Category, and select “Control Methods” in the Design Area category.