Current-Loop Control In Switching Converters

Part 6: Slope Compensation

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The last installment of this article presented a refined model of current-mode control that provides a deeper unification of the quasi-static or low-frequency current-loop behavior with the sampling aspects by deriving the dynamics equations for transfer functions from the average current variable rather than the valley current. Here in part 6 of this series, the effect of slope compensation is included in the refined model.

Specifically, we’ll analyze the impact of three different slope-compensation schemes on the refined model, noting similarities and differences in the key waveform equations. We’ll also note how the refined model with slope compensation compares to earlier models of current-model control. Finally, we’ll examine the implications of this analysis in terms of establishing guidelines for converter design that ensure loop stability.

Slope Compensation

Slope compensation is a dynamic compensation technique by which the slope of the feedback inductor current is modified by the addition of a ramp to result in the sensed current. In this section, the incremental output variable \( i_l \) is expressed as \( i_l + i_e \) to include the slope-compensation function, \( i_e \). The slope-compensated waveform is input to the PWM circuit as the sensed current, \( i_s \), as shown in the block diagram.

\[
i_s = i_{ce} + i_e = [i_l - i_l] + i_e.
\]

The two other possible locations for the summation of \( i_e \) are in the input path and in the feedback path. For the input path, the input, \( i_l \), effectively becomes \( i_l + i_e \):

\[
i_s = [i_l + i_e] - i_l.
\]

A summer in the feedback path subtracts \( i_e \) from \( i_l \). Then the compensated error quantity, \( i_{se} \), is the sensed feedback current subtracted from the input to form the error quantity:

\[
i_s = i_l - [i_l - i_e].
\]
Because addition is associative and commutative, and with no additional processing between summations, the three means of combining the three quantities comprising $i_L$ are equivalent. Consequently, we can consider the effect of $i_E$ to be that of an addition to the commanded input, $i_i$. It should therefore be possible to substitute $i_i + i_E$ into the uncompensated closed-loop $i(i_L)$ for $i_i$ and derive the compensated inductor-current waveform equations.

Though eventually we want the incremental model as shown in the block diagram, the derivations begin with the total variables. By adding the total $i_E(t)$ as a compensating waveform to the loop error, this causes $\delta(k)$ to be altered, as its value each cycle is determined by the $F_m$ block. $F_m$ contains not only the comparator sampling function itself (labeled PWM) but also a summing block preceding it which outputs the sensed waveform, $i_S$.

Slope compensation affects the sensed waveform as an addition to the error quantity, $i_{CE}$. The sensed current is

$$i_s(t) = [i_i(t) - i_L(t)] + i_E(t) = i_{CE}(t) + i_E(t).$$

The resulting $i_L = G_i i_S \neq i_S$. The uncompensated closed-loop waveform equations of $i_L$ and $\delta$ are affected by slope compensation in that $\delta$ is now determined by $F_m(i_S)$. However, the slopes of $i_L$ remain the uncompensated slopes because they are determined by converter circuit parameters, not by $F_m$. The slopes and offset of $i_S$ alone are modified by $i_E$. The forward-path transmittance, $G$, however, is an incremental transfer function and the total-variable $i_i/i_S$ is nonlinear and can vary between on- and off-times of the cycle. This complicates the determination of $F_m$ in that it must be linearized. ($G_{id}$ is already linear.) The correct value for a linearized $F_m$ is addressed in a later section.

By substituting $i_i + i_E$ for $i_i$ into the uncompensated closed-loop equations, the compensated equation for $i_i$ results;

$$i_i = \frac{G}{1+G}(i_i + i_E) = \frac{G}{1+G} i_i + \frac{G}{1+G} i_E.$$

where $G/(1 + G)$ is the unified-model uncompensated closed-loop transfer function of $i_i/i_i$. The compensated $i_i$ is therefore the uncompensated $i_i$ with the added response to slope compensation, $i_E$, by the same loop.

Returning to the total variables and substituting into $i_S(k)$ the uncompensated discrete-time $i_L(k)$ with $i_E(k)$ as its additional input,

$$i_s(k) = i_i(k) \left[ -\left( \frac{m_D}{m_U} \right) i_L(k-1) + \left( 1 + \frac{m_D}{m_U} \right) [[i_i(k) + i_E(k)] - m_D \cdot T_s \right] + i_E(k)$$

$$= -\left( \frac{m_D}{m_U} \right) \left( [[i_i(k) + i_E(k)] - i_L(k-1)] + m_U \cdot T_s \right) = -\left( \frac{m_D}{m_U} \right) \left( [[i_i(k) + i_E(k)] - i_L(k-1)] - m_U \cdot T_s \right).$$

When an expression for $i_E(k)$ is substituted, it will (if it contains $\delta(k)$) include $i_L$ and $i_i$ terms, which modify the coefficient of $i_S$. The sensed current can be expressed more generally in steady-state by applying the slope equations;

$$i_s(k) = -\left( \frac{D}{D'} \right) \left( [[i_i(k) + i_E(k)] - i_L(k-1) - D' \cdot \Delta I_{L0} \right).$$

The sampling instant at $\delta \cdot T_s$ within the cycle (or $(k - 1) \cdot T_s + \delta(k) \cdot T_s$ in total time) is determined by $F_m$ according to the control law: $i_S(\delta \cdot T_s) = 0$. The general form of $i_S$ describing the sampling event of cycle $k$ is
where \( i_I(k) = i_L(\delta(k) \cdot T_s) \). This equation describes the sensed current at the sampling time, \( \delta(k) \cdot T_s \). The stability of \( i_S \) will, of course, affect loop stability because it determines \( \delta \). Loop stability analysis therefore can be an analysis of the stability of \( i_S(t) \). The control law, \( i_S(\delta \cdot T_s) = 0 \), determines \( \delta \) and can be solved for \( \delta \) when \( i_E \) is substituted into \( i_S \).

Returning to the general equation for \( i_S(k) \), the expression \( i_I(k) - i_L(k - 1) \) in \( i_S(k) \) has the form of \( i_{Ce} \) except that \( i_I \) is of the previous cycle. For steady-state operation, the waveform is stable and \( i_L(k) = i_L(k - 1) \).

Substituting from the slope equation for \( m_D \), then

\[
\text{steady-state } i_S = \frac{m_D \cdot T_s}{1 + \frac{m_D}{m_U}} = \left( \frac{m_U \cdot m_D}{m_U + m_D} \right) \cdot T_s = D \cdot D' \left( \frac{V_{off} \cdot T_s}{L} \right) = D \cdot D' \Delta I_{L0}.
\]

The usual method of analysis derives the total closed-loop waveform equations from the waveform geometry under steady-state response, and then differentiates or perturbs them to produce the incremental equations. This approach is taken here with distinction maintained between \( i_L \) and \( i_S \) and with the additional demonstration that the waveform equations are equivalent to the uncompensated equations with the addition of \( i_E \) to \( i_I \), thus localizing slope compensation in the model to \( F_m \). The total-variable valley-current waveforms will be derived for completeness and used to show the localization of \( i_E \) to \( F_m \). The total and incremental valley-current waveform equations will be shown to be equivalent for the given slope-compensation schemes while \( i_S \) differs with \( i_E \). The average-current waveforms are also derived. Slopes will be held constant for these analyses.

**First Slope-Compensation Scheme**

To the total-variable loop error \( i_{CE} \) is added a negative-slope compensation ramp of \( -m_E \) slope (elsewhere \( -m_c \)),

\[
i_E(t) = -m_E \cdot t
\]

and \( i_E(\delta \cdot T_s) \) at switching is

\[
i_E(\delta \cdot T_s) = -m_E \cdot \delta \cdot T_s.
\]

The \( i_L \) slope, \( m_D \), remains unchanged by slope compensation and at the end of the off-time,

\[
i_L(k) = i_L(\delta \cdot T_s) - m_D \cdot \delta \cdot T_s.
\]

For the up-slope segment, the slope of \( i_L \) remains the unaltered \( m_U \) and the control-law equation is

\[
i_S(\delta \cdot T_s) = [i_I(k) - i_L(\delta \cdot T_s)] + (-m_E \cdot \delta \cdot T_s) = [i_I(k) - (i_L(k - 1) + m_U \cdot \delta \cdot T_s)] - m_E \cdot \delta \cdot T_s = 0.
\]

At the sampling point, the inductor-current peak value is

\[
i_L(\delta \cdot T_s) = i_I(k) - m_E \cdot \delta \cdot T_s = i_L(k - 1) + m_U \cdot \delta \cdot T_s.
\]

Solving for \( \delta \cdot T_s \),
\[ \delta \cdot T_s = \frac{i_L(k) - i_L(k-1)}{m_U + m_E} = \frac{-\Delta i_L(k)}{m_U + m_E}. \]

Substituting this into \( i_L(\delta \cdot T_s) \),

\[ i_L(\delta \cdot T_s) = \left( \frac{m_E}{m_U + m_E} \right) i_L(k-1) + \left( \frac{m_U}{m_U + m_E} \right) i_I(k). \]

Substituting this peak value into the off-time equation, the resulting compensated total valley inductor current is

\[ i_L(k) = \left( \frac{m_D - m_E}{m_U + m_E} \right) i_L(k-1) + \left( \frac{m_U + m_D}{m_U + m_E} \right) i_I(k) - m_D \cdot T_s. \]

Comparing the slope-compensated \( i_L(k) \) to the uncompensated \( i_L(k) \), what has changed are the coefficients which consist of slope expressions. It would appear that \( i_E \) has caused \( i_L \) to have changed its slopes but this would be a misleading interpretation of the compensated \( i_L \) expression. All that compensation has affected is a change in \( \delta \) as expressed in slopes. The converter circuit parameters have not changed and they determine \( i_L \) slopes. The discrete-time coefficients are ratios of slopes and are dependent only on \( \delta \). Consequently, it should be possible to express \( i_L \) in its observable circuit slopes, modified only by the introduction of \( i_E \) at the loop input.

To show that the total \( i_L(k) \) can be written with localization of \( i_E \) (as added to \( i_I \), start with the uncompensated \( i_L \) and add \( i_E \) to it:

\[ i_L(k) = \left( \frac{m_D}{m_U} \right) i_L(k-1) + \left( 1 + \frac{m_D}{m_U} \right) (i_I(k) + i_E(k)) - m_D \cdot T_s. \]

Substituting for \( i_E \), this becomes

\[ i_L(k) = \left( \frac{m_D}{m_U} \right) i_L(k-1) + \left( 1 + \frac{m_D}{m_U} \right) (i_I(k) - m_E \cdot \delta(k) \cdot T_s) - m_D \cdot T_s. \]

Substituting the other of the two uncompensated waveform quantities,

\[ \delta(k) = \frac{i_L(k) - i_L(k-1)}{(m_U + m_D) \cdot T_s} + \frac{m_D}{m_U + m_D} \]

then

\[ i_L(k) = \left( \frac{m_D}{m_U} \right) i_L(k-1) + \left( 1 + \frac{m_D}{m_U} \right) \left[ i_I(k) - m_E \left( \frac{i_L(k) - i_L(k-1)}{(m_U + m_D) \cdot T_s} + \frac{m_D}{m_U + m_D} \right) \cdot T_s \right] - m_D \cdot T_s. \]
After some algebra, this reduces to the same $i_L(k)$ as derived above. Consequently, the effect of compensation in the waveform equations can be localized to the input error summer as $i_E$ added to $i_I$ in the general uncompensated waveform equations.

The slope expressions can be replaced by the simpler duty-ratio expressions for steady-state waveforms (constant $\delta$) from charge balance. Applying the waveform equation of $\delta$ for steady-state response,

$$\Delta i_L(\text{on}) = (m_U + m_E) \cdot D \cdot T_s = -\Delta i_L(\text{off}) = (m_D - m_E) \cdot D' \cdot T_s.$$ 

The following conversion formulas apply to $i_L(k)$ as all three schemes fall out of this constraint:

$$\left(\frac{m_D - m_E}{m_U + m_E}\right) = \frac{D}{D'} \quad \left(\frac{m_U + m_D}{m_U + m_E}\right) = \frac{1}{D'} \quad \left(\frac{m_D - m_E}{m_U + m_D}\right) = D \quad \left(\frac{m_D + m_E}{m_U + m_D}\right) = D'.$$

Substituting from these expressions, the total slope-compensated valley inductor current in $D$ is

$$i_L(k) = \frac{D}{D'} \cdot i_L(k-1) + \frac{1}{D'} \cdot i_I(k) - m_D \cdot T_s.$$

In $D$, the equation is the same as for the uncompensated waveform. Only when the coefficients are expressed in slopes is there a difference. Consequently, the effect of slope compensation is entirely contained within its effect on $D$. Slope compensation modifies the unmodified waveform by adding to it a compensation ramp so that the sensed addition results in the desired $D$. The modified slopes of $i_S$ affect how $D$ is determined while the inductor-current waveform continues to be described by the same equation in $D$.

Substituting $i_L(k)$ into $i_S$, the total equation for $i_S$ at the end of the cycle ($kT_s$) is

$$i_S(k) = [i_L(k) - i_L(k)] + i_E(k) = i_I(k) - \left[-\left(\frac{m_D - m_E}{m_U + m_E}\right) \cdot i_L(k-1) + \left(\frac{m_U + m_D}{m_U + m_E}\right) \cdot i_I(k) - m_D \cdot T_s\right] - m_E \cdot T_s.$$ 

This simplifies to

$$i_S(k) = -\left(\frac{m_D - m_E}{m_U + m_E}\right) \cdot (i_I(k) - i_L(k-1) - (m_U + m_E) \cdot T_s).$$

This $i_S$ has the same form as the general $i_S(k)$ but with different slope expressions. If written in $D$, however,

$$i_S(k) = -\left(\frac{D}{D'}\right) \cdot (i_I(k) - i_L(k-1) - (m_U + m_E) \cdot T_s).$$

When $i_E(k) = -m_E T_s$ is substituted in the general $i_S(k)$ written in $D$, the result is the same. Slope compensation changes the slope-ratio coefficients as it did for $i_L$, and they are the same in $D$ as the uncompensated ratios. The constant term, which is related to $\Delta I_{L0}$, remains unchanged.

Taking the differential of the total current, the compensated incremental valley inductor current is
The uncompensated incremental valley current,

\[ i_s(k) = \left( \frac{m_D - m_E}{m_U + m_E} \right) i_i(k) - 1 + \left( \frac{m_U + m_D}{m_U + m_E} \right) i_i(k). \]

is affected only by an addition to \( i_i \) of \( i_e \):

\[ i_s(k) = \left( \frac{m_D}{m_U} \right) i_i(k) - 1 + \left( 1 + \frac{m_D}{m_U} \right) (i_i(k) + i_e(k)). \]

The incremental \( i_e \) is a function of the uncompensated \( d(k) \),

\[ d(k) = \frac{i_i(k) - i_i(k - 1)}{(m_U + m_D) \cdot T_s}. \]

The uncompensated \( d(k) \) is used in \( i_e(k) \) because it is the \( d \) consistent with the uncompensated \( i_i \). The effect of \( i_e \) is to modify the uncompensated waveform equations by its effect on the uncompensated loop. Substituting \( d \),

\[ i_e(k) = -m_E \cdot T_s \cdot d(k) = -m_E \cdot T_s \cdot \left( \frac{i_i(k) - i_i(k - 1)}{(m_U + m_D) \cdot T_s} \right) = -\left( \frac{m_E}{m_U + m_D} \right) \cdot (i_i(k) - i_i(k - 1)). \]

Substituting \( i_e(k) \) into the uncompensated \( i_i(k) \) and simplifying, the same incremental \( i_i(k) \) results as derived above from the compensated total-variable expression for \( i_i \).

The incremental \( i_s \) can be found by taking the differential of the total \( i_S(k) \);

\[ i_s(k) = -\left( \frac{m_D}{m_U} \right) \cdot [i_i(k) - i_i(k - 1)] + i_e(k). \]

This can be corroborated with the localization property by substituting \( i_e \) (having the uncompensated \( d \)) into \( i_s \). Then

\[ i_s(k) = [i_i(k) - i_i(k)] + i_e(k) = i_i(k) - \left[ -\left( \frac{m_D}{m_U} \right) i_i(k - 1) + \left( 1 + \frac{m_D}{m_U} \right) (i_i(k) + i_e(k)) \right] + i_e(k). \]

Collecting \( i_e \) terms, the result is equivalent to the previous equation:
\[ i_s(k) = \left( \frac{m_D}{m_U} \right) \cdot \left( i_i(k-1) - [i_i(k) + i_e(k)] \right). \]

For average total inductor current, the discrete-time \( \bar{i}_L \) is

\[ \bar{i}_L(k) = \frac{1}{2} \cdot (i_i(k) - m_E \cdot \delta(k) \cdot T_s) + i_L(k-1) \cdot \delta(k) + \frac{1}{2} \cdot (i_L(k) - m_E \cdot \delta(k) \cdot T_s) + i_L(k) \cdot \delta'(k) \]

or

\[ \bar{i}_L(k) = \frac{1}{2} \cdot (i_i(k) - m_E \cdot \delta(k) \cdot T_s) + \frac{1}{2} \cdot (i_L(k-1) \cdot \delta(k) + i_L(k) \cdot \delta'(k)). \]

Substituting for \( i_L(k) \) from the general waveform equation in \( \delta \) coefficients and reducing,

\[ \bar{i}_L(k) = \left. \frac{1}{2} \cdot (i_i(k) - m_E \cdot \delta(k) \cdot T_s) \right. + \left. \frac{1}{2} \cdot (i_L(k-1) + (i_i(k) - m_E \cdot T_s)). \right. \]

Under steady-state operation and with constant slopes, the cycle valley current values are

\[ i_L(k-1) = i_L(k) = i_{LV}. \]

Then

\[ \bar{i}_L = \frac{1}{2} \cdot (i_i - m_E \cdot T_s) + i_{LW} = \frac{1}{2} \cdot (i_L + i_{LV}) + \bar{i}_E. \]

where the average \( i_E \) over a cycle is \( \bar{i}_E \). A negative \( i_E \) has the effect of reducing \( \delta \) and causing \( \Delta i_L \) to be less than that of the uncompensated loop.

The incremental \( \bar{i}_L \) of the uncompensated steady-state waveform is

\[ \bar{i}_l(k) = -\frac{D}{D'} \cdot \bar{i}_l(k-1) + \frac{1}{2} \cdot \left[ \left( 1 + \frac{1}{D'} \right) \cdot i_i(k) + \frac{D}{D'} \cdot i_i(k-1) \right]. \]

By substituting \( i_i \),

\[ i_i(k) = 2 \cdot \bar{i}_l(k) - i_i(k) \]

into \( d(k) \) with constant converter input and output voltages whereby \( v_{OFF} = v_{off} \), then

\[ d(k) = \frac{i_i(k) - i_i(k-1)}{(v_{OFF} / L) \cdot T_s} = \left( \frac{L}{v_{off} \cdot T_s} \right) \cdot \left( [2 \cdot \bar{i}_l(k) - i_i(k)] - [2 \cdot \bar{i}_l(k-1) - i_i(k-1)] \right) \]

\[ = -\left( \frac{1}{\Delta I_{L0}} \right) \cdot \left( 2 \cdot [i_i(k) - \bar{i}_l(k-1)] - [i_i(k) - i_i(k-1)] \right). \]

Substituting \( d(k) \) into \( i_d(k) \),
\[ i_e(k) = -m_E \cdot d(k) \cdot T_s = \left( \frac{m_E \cdot T_s}{\Delta I_{L0}} \right) \cdot \left( 2 \cdot \left( \bar{i}_i(k) - \bar{i}_i(k-1) \right) - \left[ i_i(k) - i_i(k-1) \right] \right). \]

With \( i_e \) added to \( i_i \), \( \bar{i}_i \) becomes

\[ \bar{i}_i(k) = -\frac{D}{D'} \cdot i_i(k-1) + \frac{1}{2} \left( \frac{D'+1}{D'} \right) \cdot \left[ i_i(k) + i_e(k) \right] + \frac{1}{2} \cdot \frac{D}{D'} \cdot \left[ i_i(k-1) + i_e(k-1) \right]. \]

Substituting for \( i_e \) and after some algebra, this reduces to a second-order difference equation:

\[
\begin{align*}
\left( \frac{m_E \cdot T_s}{\Delta I_{L0}} \right) \cdot (D'+1 + D') \cdot \bar{i}_i(k) &= \left( 2 \cdot \left( \frac{m_E \cdot T_s}{\Delta I_{L0}} \right) \cdot D' - D \right) \cdot \bar{i}_i(k-1) + \left( \frac{m_E \cdot T_s}{\Delta I_{L0}} \right) \cdot D \cdot \bar{i}_i(k-2) \\
&+ \frac{1}{2} \cdot (D'+1) \cdot i_i(k) - \frac{1}{2} \cdot \left( 2 \cdot \left( \frac{m_E \cdot T_s}{\Delta I_{L0}} \right) - D \right) \cdot i_i(k-1) - \frac{1}{2} \cdot \left( \frac{m_E \cdot T_s}{\Delta I_{L0}} \right) \cdot D \cdot i_i(k-2).
\end{align*}
\]

**Second Slope-Compensation Scheme**

For the second scheme, compensating slope, \( m_E \), is added to both up-slope and down-slope segments of the loop error. This is accomplished by subtracting from \( i_L \) the same compensating ramp as in the first scheme,

\[ i_E(t) = -m_E \cdot t. \]

The result is then subtracted from \( i_i \), so that

\[ i_S(k) = i_i(k) - [i_L(k) - i_E(k)] = [i_i(k) - i_L(k)] + i_E(k) = i_{Ce}(k) + i_E(k). \]

This is equivalent to the first scheme, though \( i_S \) peaks at \( i_i \) while the on-time slope of \( i_S \) is steeper because of its addition of \( m_E \). The two effects cancel, resulting in the same \( \delta \). The waveform equations are derived from the waveform geometry. At the sample time,

\[ i_S(\delta \cdot T_s) = i_i(k) - [i_L(k-1) + m_U \cdot \delta \cdot T_s] - m_E \cdot \delta \cdot T_s = i_i(k) - i_L(k-1) - (m_U + m_E) \cdot \delta \cdot T_s = 0. \]

Then

\[ \delta \cdot T_s = \frac{i_i(k) - i_i(k-1)}{m_U + m_E}. \]

This is identical to the first scheme. During the off-time, \( i_S \) continues to have added to it the \(-m_E\) slope, and at \( k \cdot T_s \),

\[ i_S(k) = i_i(k) - i_L(k) + i_E(k) = i_i(k) - [i_i(k) - m_D \cdot \delta' \cdot T_s] - m_E \cdot \delta' \cdot T_s = (m_D - m_E) \cdot \delta' \cdot T_s. \]

Substituting for \( \delta \) and simplifying,
\[ i_s(k) = \left( \frac{m_D - m_E}{m_U + m_E} \right) \cdot (i_i(k) - i_L(k-1)) + (m_D - m_E) \cdot T_s. \]

Compared to the first scheme, the two expressions for \( i_s \) are equal. In the first scheme, \( i_1 + i_E \) is compared to \( i_L \) while in the second scheme, they are combined before being compared to zero. Yet it is the dynamics of the combination of all three waveforms that determines the stability of \( \delta \) and the value of \( i_s(k) \), whether they are added before or at the inputs of the comparator. The comparator responds to the difference between its inputs, and that constitutes the completion of the summation in \( i_s \). In the first scheme, there is no single \( i_s \) circuit waveform as in the second, though it can be regarded as existing at the comparator input. It is this abstracted \( i_s \) and not either comparator input waveform that is relevant to the analysis of the first scheme and which makes it equivalent to the second scheme. The comparator output is the sign or polarity of \( i_s \). The comparator operations of the PWM block are equivalent between schemes.

Solving for \( i_L \) by substituting the same \( \delta \) as in the first scheme into the same uncompensated \( i_L \) equation produces the same result:

\[ i_L(k) = \left( \frac{m_D - m_E}{m_U + m_E} \right) \cdot i_L(k-1) + \left( \frac{m_U + m_D}{m_U + m_E} \right) \cdot i_i(k) - m_D \cdot T_s. \]

The average inductor current for the second compensation scheme is the same as the first; the waveform equations are identical and the average current must also be the same. Under steady-state operation, \( i_i(k-1) = i_i(k) = i_{LV} \) and

\[ i_{LV} = -\frac{D}{D'} \cdot i_{LV} + \frac{1}{D'} \cdot i_L - m_D \cdot T_s = i_i - D \cdot m_D \cdot T_s = i_i - D \cdot D' \cdot (\Delta I_{L0}). \]

Then taking the steady-state average current from the first scheme and developing it further by substituting \( i_{LV} \),

\[ \bar{i}_L = \frac{1}{2} \cdot (i_i - m_E \cdot T_s) = i_i - \frac{1}{2} \cdot (m_E \cdot T_s + D \cdot D' \cdot (\Delta I_{L0})). \]

The equations for \( i_L(k) \) and \( \bar{i}_L(k) \) must be the same as those of the first scheme because the waveform and averaging equations from which they are derived are the same.

**Third Slope-Compensation Scheme**

The third scheme (found in Ridley and Sheehan) differs from the previous schemes in that the compensating ramp is non-zero only during the on-time. Within a cycle,

\[ i_E(t) = -m_E \cdot t, \quad 0 \leq t \leq \Delta T_s. \]

The sensed current at the sample point is

\[ i_s(\Delta T_s) = i_i(k) - [i_L(k-1) + m_U \cdot \Delta T_s] - m_E \cdot \Delta T_s = i_i(k) - i_L(k-1) - (m_U + m_E) \cdot \Delta T_s. \]

Then set \( i_s(\Delta T_s) = 0 \) and solve for \( \delta \),

\[ \delta \cdot T_s = \frac{i_i(k) - i_L(k-1)}{m_U + m_E}. \]

The \( \delta \) equation is the same as for the first two schemes as it must be; the conditions during on-time are the same. The inductor current will therefore have the same waveform equation as the previous schemes.
Because the slope compensation is different, \( i_s \) differs from the previous schemes. For the off-time segment, substituting \( \delta \cdot T_s \) into the sensed current, its value at the end of the cycle is

\[ i_s(k) = i_t(k) - [i_t(k) - m_D \cdot \delta \cdot T_s] = m_D \cdot T_s - m_D \cdot \left( \frac{i_t(k) - i_L(k-1)}{m_U + m_E} \right). \]

This becomes the sensed valley current;

\[ i_s(k) = -\left( \frac{m_D}{m_U + m_E} \right) \cdot (i_t(k) - i_L(k-1) + (m_U + m_E) \cdot T_s). \]

The sensed valley current has the same form as \( i_s \) of the previous schemes but the slope-ratio coefficients are not the same. In steady-state,

\[ -\Delta i_s = (m_U + m_E) \cdot D \cdot T_s = m_D \cdot D' \cdot T_s. \]

and the slope ratios are related to \( D \) by the following formulas:

\[ \frac{m_D}{m_U + m_E} = \frac{D}{D'} \quad 1 + \frac{m_D}{m_u + m_E} = \frac{1}{D'}. \]

Then in \( D \), the steady-state \( i_s(k) \) is the same as the previous schemes.

The average inductor current equation is the same as for the previous schemes, though \( \bar{i}_L(k) \) can alternatively be derived for the third scheme from

\[ \bar{i}_L(k) = \frac{1}{2} \cdot (m_u + m_E) \cdot \delta(k) \cdot T_s + i_L(k-1) \]

\[ \bar{i}_L(k) = \frac{1}{2} \cdot m_D \cdot \delta'(k) \cdot T_s + i_L(k) \]

after \( \delta(k) \) is eliminated. These equations do not include \( i_I \) though the effects of \( i_I \) on \( \bar{i}_L \) are implicit in \( \delta(i_I) \).

**Incremental Average Inductor Current With Slope Compensation**

The total and incremental waveform equations for the three slope-compensation schemes considered are equivalent whether the coefficients are expressed as slope ratios or duty-ratio. Thus the inductor current equation is independent of slope compensation scheme, and is

\[ i_L(k) = -\left( \frac{m_D - m_E}{m_U + m_E} \right) \cdot i_L(k-1) + \left( \frac{m_U + m_D}{m_U + m_E} \right) \cdot i_t(k) - m_D \cdot T_s = \frac{-D}{D'} \cdot i_L(k-1) + \frac{1}{D'} \cdot i_t(k) - D \cdot \Delta I_{L_0}. \]

Similarly, the incremental \( i_I(k) \) is independent of slope-compensation scheme and is
\[
i_j(k) = -\frac{D}{D'} \cdot i_j(k-1) + \frac{1}{D'} \cdot i_j(k).
\]

The \(z\)-transform of \(i_j\) is

\[
i_j(z) = -\frac{D}{D'} \cdot i_j(z) \cdot z^{-1} + \frac{1}{D'} \cdot i_j(z).
\]

Expressed as a valley-current-gain transfer function in the \(z\)-domain,

\[
T_{CV}(z) = \frac{i_j(z)}{i_j(z)} = \left(\frac{1}{D'}\right) \cdot \frac{z}{z + \left(\frac{D}{D'}\right)}.
\]

and is stable when \(D/D' < 1\) or \(D < \frac{1}{2}\). This is the closed-loop equation of the sampled-loop model of Ridley. It is equivalent to that of the unified model of Tan which samples \(i_j(k)\) coincident with \(\tilde{i}_l(k)\). This current-loop transfer function is effectively the same as that used in the unified model of Tan (based on the same waveform difference equations) and consequently there is a discrepancy in the model between valley-current sampling and average-current derivation of \(F_{m0}\). This results in a model that does not fully unify the average current with the sampling effects of the sampled-loop model. The refined model attempts to make a full unification by deriving \(F_{m0}\) as a consequence of using average \(i_l\) in the waveform equations of the current-loop transfer function.

The sensed current, \(i_S\), is the same for the first and second schemes and differs from the third scheme only in how slope ratios are related to \(D\). Expressed in \(D\), they are equivalent:

\[
i_S(k) = -\left(\frac{D}{D'}\right) \cdot (i_l(k) - i_l(k-1) + (m_U + m_E) \cdot T_s).
\]

The \(\tilde{i}_l(k)\) difference equation is dependent on \(i_l(k)\) and is independent of compensation scheme;

\[
\left(\frac{m_E \cdot T_s}{\Delta I_{L0}} \right) \cdot (D'+1) + D' \cdot \tilde{i}_l(k) = 2 \left( \frac{m_E \cdot T_s}{\Delta I_{L0}} \right) \cdot D' - D \cdot \tilde{i}_l(k-1) + \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D \cdot \tilde{i}_l(k-2)
+ \frac{1}{2} \cdot (D'+1) \cdot i_l(k) - \frac{1}{2} \left(2 \left( \frac{m_E \cdot T_s}{\Delta I_{L0}} \right) - D \right) \cdot i_l(k-1) - \frac{1}{2} \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D \cdot i_l(k-2).
\]

This second-order equation in the \(z\)-domain can be written as the closed-loop transfer function,

\[
\frac{\tilde{i}_l(z)}{i_l(z)} = \frac{1}{2} \cdot \frac{(D'+1) \cdot z^2 - \left(\frac{m_E \cdot T_s}{\Delta I_{L0}} - D\right) \cdot z - \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D}{\left(\frac{m_E \cdot T_s}{\Delta I_{L0}} \right) \cdot (D'+1) + D' \cdot z^2 - 2 \left( \frac{m_E \cdot T_s}{\Delta I_{L0}} \right) \cdot D' - D \cdot z - \left(\frac{m_E \cdot T_s}{\Delta I_{L0}}\right) \cdot D}.
\]
Setting \( m_E = 0 \text{ A/s} \), this reduces to \( T_C(z) \):

\[
T_C(z) = \frac{\tilde{i}_L(z)}{i_L(z)} = \frac{1}{2} \cdot \left( \frac{D' + 1}{D'} \cdot z + D \right)
\]

In normalized form,

\[
\tilde{i}_L(z) = \frac{1}{2} \cdot \left( \frac{D' + 1}{D} \right) \cdot z^2 + \left( \frac{m_E \cdot T_s}{\Delta I_{L0}} \right) \cdot D \cdot z + \left( \frac{m_E \cdot T_s}{\Delta I_{L0}} \right)
\]

The frequently-occurring

\[
\left( \frac{m_E \cdot T_s}{\Delta I_{L0}} \right) = \frac{m_E}{m_D} \cdot D
\]

and \( m_E T_s \) is the peak-to-peak PWM current ramp. The condition often chosen for optimal dynamic response is

\[
m_E = m_D
\]

which results in the above expression reduced to \( D \).

In the literature, the impression is sometimes given that \( D \) can exceed 0.5 in a stable loop because of \( m_E \).
However, the effect of \( m_E \) is to reduce the value of \( D \) so that it is within the stable interval. Consequently, the direct method of loop compensation is to design the loop to operate within an acceptable response range of \( D \) by selecting turns ratios and other relevant parameters to keep \( D \) within acceptable dynamic-response bounds.

In the next and final section of this article, part 7, we will return to a comparison of the important PWM factor, \( F_{m0} \) in the current loop and show that the refined model is compatible with the major existing models when reduced to accommodate their limiting assumptions. Finally, we’ll complete this lengthy process of modeling the current-loop with the merging of waveform-based and circuit-based modeling.

**About The Author**

Dennis Feucht has been involved in power electronics for 25 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been doing current-loop converter modeling and converter optimization.
For more on current-mode control methods, see the [How2Power Design Guide](#), select the Advanced Search option, go to Search by Design Guide Category, and select "Control Methods" in the Design Area category.