Current-Loop Control In Switching Converters

Part 7: F_{m0} In Models

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This article on modeling of current-mode control concludes with a discussion of the PWM factor, F_{m0}. The refined model of current-mode control presented in part 5 is shown to be compatible with the major existing models when their limiting assumptions are factored in. This discussion refers back to expressions for the three slope-compensation methods derived in part 6. Finally, this lengthy process of modeling the current-loop ends by discussing the advantages and limitations of waveform-based models versus circuit-based models, and why it would be desirable to merge these two approaches.

F_{m0} In Models

Perhaps the most difficult model parameter to determine is F_{m0}, which is the static form of the PWM transfer function F_m. Expressions for it differ by the method of slope compensation (which affects $i_S(k)$), whether the inductor current is the valley i_l or average $\bar{i}_l$, and the configuration of the model for feedback of v_O and v_G into the error quantity driving F_m.

Slope expressions can be converted to duty-ratio expressions using slope formulas modified to include m_E:

$$m_U + m_E = \delta' \cdot \frac{v_{OFF}}{L}\ ; \quad \begin{cases} m_D - m_E = \delta \cdot \dfrac{v_{OFF}}{L}, \text{slope-compensations } 1, 2 \\ m_D = \delta \cdot \dfrac{v_{OFF}}{L}, \text{slope-compensation } 3 \end{cases} .$$

From these it follows for slope-compensation methods 1 and 2 that for steady-state response,

$$(m_U + m_E) - (m_D - m_E) = m_U - m_D + 2 \cdot m_E = (\delta' - \delta) \cdot (m_U + m_D) = (\delta' - \delta) \cdot \frac{v_{OFF}}{L} .$$

Simple Unified Model

In their model development, Hong, Choi, and Ahn[1] begin with a steady-state expression (eq 14) for F_{mV0} (F_m') that is equivalent to

$$F_{mV0} = \frac{f_s}{\frac{1}{2} \cdot (m_U - m_D) + m_E} = \frac{2 \cdot f_s}{m_U - m_D + 2 \cdot m_E} = \frac{2 \cdot f_s}{(V_{off}/L) \cdot (D' - D)} = \frac{1}{\Delta I_{L0} \cdot \frac{1}{2} \cdot (D' - D)}$$

$$F_{mV0} = \frac{1}{\Delta I_{L0} \cdot (\frac{1}{2} - D)} .$$

F_{mV0} of the simple unified model is based on i_l and not $\bar{i}_l$. In their paper, Hong, Choi, and Ahn claim that the simplified derivation of the unified model, as given above for F_{mV0} is identical to the unified model of Tan. Tan gives both the $\bar{i}_l$-based unified (eqn 7, p. 400) and simple sampled-i_l unified (eqn 17, p. 401) expressions for F_{m0}.[2]

This F_{m0} goes to infinity at D = ½ as will the static loop gain. This of itself can make continuous feedback loops unstable. The additional (½ – D) factor in ω_p will cause ω_p to move to 0 s^{-1} as D approaches ½. As F_{m0}

approaches infinity by the same factor, the closed-loop 1 + $G \cdot H$ multiplication of ω_p keeps the closed-loop pole frequency at the Nyquist frequency.

LF-Avg Model And Unified Model Of Tan

F_{mV0} results from T_{CV} and G_{idV} and is not the F_{m0} of the $\bar{i}_l$-based lf-avg model of E&M[3]: $F_{m0} = 1/M_e \cdot T_s = 1/\Delta I_e$, where d is implicit in additional terms of $\bar{i}_l$ containing m_u and m_d. A more explicit $\bar{i}_l$-based F_{m0} used by Tan expands the terms in d in the $\bar{i}_l$ equation (repeated here):

$$\bar{i}_l = i_i - M_e \cdot T_s \cdot d - \frac{T_s}{2 \cdot L} \cdot [V_{off} \cdot (D' - D) \cdot d + (D \cdot D') \cdot v_{off}].$$

Then for $v_{off} = 0$,

$$\text{lf-avg } F_{m0} = \frac{d}{i_i - \bar{i}_l}(s = 0\,\text{s}^{-1}) = \frac{1}{M_e \cdot T_s + \Delta I_{L0} \cdot \frac{1}{2} \cdot (D' - D)} = \frac{1}{\Delta I_e + \Delta I_{L0} \cdot (\frac{1}{2} - D)}$$

which is effectively a lf-avg F_{m0} after the feedback loops for v_O and v_G have been reorganized according to the unified-model block diagram. Hong, Choi, and Ahn claim (p. 730, near bottom) that the resulting $F_m(s)$ "agrees precisely" with that of Tan. However, Tan's equation (eq 7, p. 400) for the full unified model is the lf-avg F_{m0}.

Both the simple-unified and lf-avg (with slope compensation) models have the (½ – D) factor in F_{m0}.

Sampled-Loop Model

The sampled-loop F_{m0} of Ridley (p. 275, Table I)[4] is based on a PWM circuit that coincides with slope compensation method 3 waveform analysis (no m_E during off-time):

$$F_{m0} = \frac{1}{(m_U + m_E) \cdot T_s} = \frac{1}{\Delta I_{L0} \cdot D'}.$$

It is lower in value than the previous expressions in that 1 – D in the denominator is larger than (½ – D), and is the same as the refined model. Tymerski[5] uses Ridley's F_{m0}. It does not contain discontinuity at half D and does not become infinite there. The sampled-loop model accounts for the "half-D resonance" in the pole-pair linear-term coefficient and thereby places the dynamics of the loop in the dynamic factor of the loop transfer function.

Holloway and Eirea (and Middlebrook) Model

Holloway and Eirea[6] give an improved F_{m0} for the sampled-loop model, which can be derived from an average-current equation from Tymerski based on the first slope-compensation method. In steady-state,

$$\bar{i}_L + \tfrac{1}{2} \cdot m_U \cdot D \cdot T_s = i_I - m_E \cdot D \cdot T_s.$$

Solving,

$$F_{m0} = \frac{1}{(\frac{1}{2} \cdot m_U + m_E) \cdot T_s} = \frac{1}{m_E \cdot T_s + \frac{1}{2} \cdot \Delta I_{L0} \cdot D'}.$$

This F_{m0} is for the sampled-loop model of Ridley but with $\bar{i}_l$ instead of i_l and is thus more useful.

Refined Model

As derived for the uncompensated loop,

$$F_{m0} = \frac{4}{D'} \cdot \frac{L \cdot f_s}{V_{off}} = \frac{4}{\Delta I_{L0} \cdot D'}.$$

It is similar to the sampled-loop and Holloway and Eirea values except for the scale factor of 4 that gives it twice the gain. The total forward path, G, thus has a net static gain that is twice that of the sampled-loop and Holloway and Eirea models because G_{id} (for average i_l) is half that of G_{idV}. For all of the models, F_{m0} increases (nonlinearly) with D and is positive.

The cubic denominator in $F_{m0}(s)$ can be simplified to

$$F_m(s) \cong \frac{2}{\Delta I_{L0} \cdot \frac{1}{2} \cdot D'} \cdot \frac{1}{\left(\frac{s}{\omega_s/2}\right) \cdot \frac{1}{2} \cdot \pi \cdot D' + 1}, \; s << \frac{\omega_s}{2}.$$

Then if the ½ of G_{id} is brought into F_{m0},

$$F_{m0} = \frac{1}{\Delta I_{L0} \cdot \frac{1}{2} \cdot D'}.$$

and is the same as the Holloway and Eirea F_{m0}.

To conclude about F_{m0}, when the ½ factor that is the difference of the valley and average current variables in the models is taken into account, all dynamic models considered agree except the unified model of Tan. Even the earlier Middlebrook model is in agreement. The refined model, when reduced for comparison, is also in agreement.

F_m *With Localized Slope Compensation*

In the derivation of the simple unified model, the feedback formula was equated to T_{CV} and given G_{idV}, F_{mV} is free and can be solved. This implicit method of finding F_m entangles the slope-compensation scheme associated with F_m with the waveform equations so that it is hard to separate compensation effects from converter waveforms. In the direction taken here, no compensation methods were introduced when deriving current waveforms and slope formulas. They are based solely on converter parameters. In this section, i_s is derived at the input to the forward path of the loop.

$$i_s(k) = [i_i(k) - \bar{i}_l(k)] + i_e(k) = i_{Ce}(k) + i_e(k)$$

The uncompensated incremental average-current waveform equation,

$$\bar{i}_l(k) = -\frac{D}{D'} \cdot \bar{i}_l(k-1) + \frac{1}{2} \cdot \left[\left(1 + \frac{1}{D'}\right) \cdot i_i(k) + \frac{D}{D'} \cdot i_i(k-1)\right]$$

can be expressed in i_{Ce} and i_i as

$$i_{Ce}(k) = i_i(k) - \bar{i}_l(k) = -\frac{D}{D'} \cdot \left(i_{Ce}(k-1) + \frac{1}{2} \cdot [i_i(k) - i_i(k-1)]\right).$$

Then in the z-domain,

$$T_{Ce}(z)=\frac{i_{Ce}(z)}{i_i(z)}=\frac{1}{2}\cdot\left(-\frac{D}{D'}\right)\cdot\frac{z-1}{z+\frac{D}{D'}}=\frac{1}{2}\cdot\frac{-D\cdot z+D}{D'\cdot z+D}\,.$$

Substituting $i_{Ce}(z)$ into i_s,

$$i_s(z)=-\frac{D}{D'}\cdot\left(z^{-1}\cdot i_{Ce}(z)+\tfrac{1}{2}\cdot(1-z^{-1})\cdot i_i(z)\right)+i_e(z)\,.$$

The term i_s is a time-dependent sum of i_{Ce} and i_e because it contains error and input quantities from cycles (k – 1) and k. In the sampled s-domain, it is

$$i_s^*(s)=-\frac{D}{D'}\cdot\left(e^{-s\cdot T_s}\cdot i_{Ce}^*(s)-\tfrac{1}{2}\cdot\left(1-e^{-s\cdot T_s}\right)\cdot i_i^*(s)\right)+i_e^*(s)\,.$$

This can be put in the form of a current loop with $H_e(s)$ moved through the summing block:

$$i_s^*(s)=H_e\cdot\left(\frac{1}{H_e}\cdot i_i^*-i_l^*\right)+i_e^*=-\frac{D}{D'}\cdot\left(H_0(s)\cdot[H_e(s)\cdot i_{Ce}^*(s)]-\tfrac{1}{2}\cdot e^{-s\cdot T_s}\cdot(s\cdot T_s)\cdot[(1/H_e(s))\cdot i_i^*(s)]\right)+i_e^*(s)\,.$$

The new input is $i_i^*(s)/H_e(s)$ and the new i_{Ce} is $H_e(s)\cdot i_{Ce}^*(s)$. Then $i_s*(s)$ has a cycle gain of $-D/D'$, the error is stepped (as are both input and feedback quantities), and the input is half the differentiated new input from the previous cycle. Compensation is added to this. A simpler expression for $i_s*(s)$ is

$$i_s^*(s)=-\frac{D}{D'}\cdot\left([i_{Ce}^*(s)+\tfrac{1}{2}\cdot i_i^*(s)]\cdot e^{-s\cdot T_s}-\tfrac{1}{2}\cdot i_i^*(s)\right)+i_e^*(s)\,.$$

The error plus half the input from the previous cycle has from it subtracted half the input of the present cycle (plus compensation), and this is amplified by the cycle gain. With no change in input (i_i = 0 A), the loop stability as expressed in i_s depends on the cycle gain and i_e. For $D \geq ½$ then $D/D' \geq 1$ and i_e is required to subtract from the i_{Ce} term to effectively reduce gain to less than one in magnitude.

Slope Variation From Input And Output Voltages

In the unified model, the effects of v_G and v_O are combined in the more general variable, v_{OFF}. The converter G_{id} is derived in steady-state with v_{OFF} = V_{off} and the effects of v_{off} are relegated to the F_{off} path in the model, which adds an additional loop error term to account for v_{OFF} variations:

$$F_{off}\cdot v_{off}=\frac{1}{2}\cdot\left(\frac{v_{off}\cdot T_s}{L}\right)\cdot D\cdot D'=\tfrac{1}{2}\cdot\Delta i_{l0}\cdot D\cdot D'\,.$$

One of the improvements of the unified model over the sampled-loop model was the relocation of sampling effects from the feedback path to the block, F_m , in which they occur. Placement of effects with their causes was not complete, however, in that the effects of v_{OFF} were not placed where they occur, in the converter block, G_{id}. By placing the effects of v_{OFF} back into G_{id}, and for generality adding A_{Ce} (which is 1 in most models), the resulting general block diagram for a voltage-regulated converter with a current loop is shown below, with R_i = $H_C \cdot R_S$ (see the figure.)

The converter is decomposed into G_{id} and G_{oi} blocks to allow i_L to be extracted from G_{id} as the feedback quantity in the current loop. Also, G_{oi} and Z_o are separated because G_{oi} is part of the converter while Z_o is its output-port load. For this modeling scheme, converter topologies must be solved for G_{id} and G_{oi}. This somewhat resembles the lf-avg model, which has two pairs of transfer functions, one for v_o and the other for i_l.

The effects of v_G and v_O on current slopes complicates waveform-based models immensely. By inputting v_{OFF} into G_{id}, no longer are the current and voltage loops neatly nested to allow for modular loop decomposition. Nested decomposition is retained by feeding back the effects of v_{OFF} to the current-loop summing block. This feedback path, F_{off}, is in parallel with the voltage-feedback path and can be treated as having an effect outside the current loop. This frees G_{id} from v_{OFF} variations. The static V_{off} is input to G_{id} and as a constant also keeps Δi_{L0} constant. Then the effects of v_{off} can be handled as an additional current-loop input, through F_{off}. The effects of v_{off} are thus placed in the outer voltage loop. All waveform-based models feed back v_{OFF} to the current-loop error summing block.

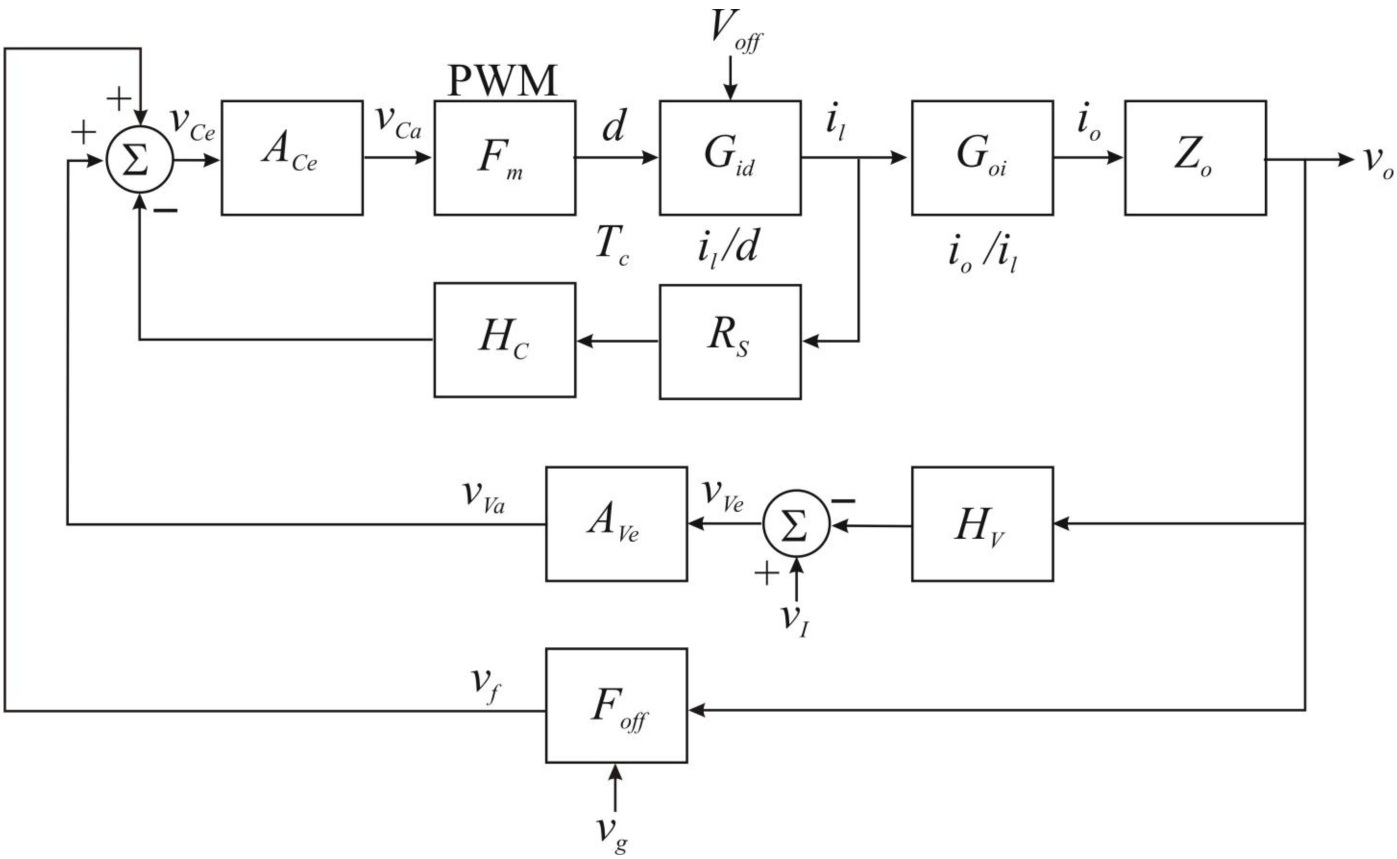

Figure. Generalized block diagram of the refined model for a voltage-regulated converter with a current loop.

The rationale for inclusion of the v_{OFF} effects in the error quantity is illustrated in the unified model. It uses the waveform-derived expression for the average inductor current,

$$\bar{i}_L = i_I - \tfrac{1}{2}\cdot[(m_U\cdot\delta\cdot T_s)\cdot\delta + (m_D\cdot\delta'\cdot T_s)\cdot\delta']\cdot T_s = i_I - \tfrac{1}{2}\cdot[(m_U\cdot\delta^2 + m_D\cdot\delta'^2)]\cdot T_s\,.$$

This can be written explicitly as the error,

$$i_{CE} = i_I - \bar{i}_L = \tfrac{1}{2}\cdot[(m_U\cdot\delta^2 + m_D\cdot\delta'^2)]\cdot T_s\,.$$

Substituting for the total slopes from the slope equations,

$$i_{CE} = i_I - \bar{i}_L = \tfrac{1}{2}\cdot[(\frac{\delta'\cdot v_{OFF}}{L}\cdot\delta^2 + \frac{\delta\cdot v_{OFF}}{L}\cdot\delta'^2)]\cdot T_s = \tfrac{1}{2}\cdot[(\delta'\cdot\delta^2 + \delta\cdot\delta'^2)]\cdot\Delta i_{L0}\,.$$

This simplifies to

$$i_{CE} = i_I - \bar{i}_L = \tfrac{1}{2} \cdot \delta \cdot \delta' (\delta + \delta') \cdot \Delta i_{L0} = \tfrac{1}{2} \cdot \delta \cdot \delta' \cdot \Delta i_{L0} \, .$$

Then the incremental i_{CE} can be determined by perturbing both δ and v_{OFF} of i_{CE} and linearizing to result in

$$i_{Ce} = i_i - \bar{i}_l = \frac{T_s}{2 \cdot L} \cdot [V_{off} \cdot (D' - D) \cdot d + (D \cdot D') \cdot v_{off}] \, .$$

$F_{off} \cdot v_{off}$ is the second term of the expression. Consequently, it is appealing to simply account for the effects of v_{OFF} as an additional feedback-path error term. The F_{off} block can be summed by the current-loop summer as an addition to v_{Va}. The slope compensation waveform v_E can be summed either at the summing block or with v_{Ca}, which can be treated for analysis as the error quantity in transfer function formation with loop sampling. Expressed explicitly,

$$F_{off} = \frac{v_f}{v_{off}} = \frac{T_s}{2 \cdot L} \cdot (D \cdot D')$$

where v_{off} is the PWM switch voltage v_{ap} during off-time.

Another reason for separating the effects of v_{off} from the current loop is the dependence of v_{off} on the circuit structure. For the three PWM-switch configurations, v_{off} differs among configurations. With v_{off} we face circuit dependencies for the model and are forced to abandon a purely waveform-based modeling scheme.

The first term in the i_{Ce} equation is the effect on G_{id} of the static V_{off} component of v_{OFF}. For i_i = 0 A, then the term for V_{off} adds to the previously-derived G_{id} as

$$\frac{T_s}{2 \cdot L} \cdot (D' - D) \cdot V_{off}$$

The current-loop model is thus essentially finished with the determination of F_{off} and $G_{id}(V_{off})$. Inclusion of F_{off} requires the conversion of loop input and error currents, i_I and i_{Ce} to voltages v_{Va} and v_{Ce} respectively:

$$v_I = R_S \cdot i_I \, ; v_{Ce} = R_S \cdot i_{Ce}$$

This is easily accomplished in the complete converter block diagram by moving R_S forward from its place in the feedback path, through the summing block to input and error quantities, where it is multiplied by them.

Waveform- And Circuit-Based Model Convergence

The direction of model development taken here and in the lf-avg, sampled-loop, and unified models is to begin with circuit behavior instead of structure, as the inductor-current waveform from which to construct a general behavioral model. This has appeal in that the results are circuit-independent and apply to any topology of converter with the assumed waveform. The limitation of the waveform model that is avoided by circuit-based modeling is that it applies only to converters with the assumed linear inductor-current waveform.

The inability to achieve a purely waveform-based model because of the circuit dependence of v_{OFF} suggests that there is yet one further refinement needed in current-loop modeling, that of a convergence of a generalized circuit model with the refined waveform model. Circuit-based modeling has been advanced by Robert Sheehan[7] by beginning with particular circuits, analyzing them, and then seeking to generalize to a less circuit-dependent model. Sheehan's circuit-driven approach to converter modeling lumps G_{id}, G_{oi}, and Z_o together in the circuit equations to produce v_o/d as the transfer function of interest, with the effects of v_{OFF} included in the circuit equations. The circuit variable common to the three PWM-switch configurations is v_{SW}, the voltage applied at the PWM-switch common terminal to the inductor. This method of analysis should be appealing to electronics engineers because of its concrete focus on the circuits and their equations. Sheehan derives equations for the three PWM-switch configurations and then proceeds to generalize from them.

To illustrate the more accurate result of circuit-based modeling, G_{id} of the sampled-loop, unified, and refined models is derived from waveform-based loop analysis and is an integrator of constant inductor voltage, V_{off}. The result for G_{id} is a pole at the origin. Converter circuit analysis quickly reveals that quadratic pole-pairs and an RHP zero appear in G_{id} when the effects of Z_o, and consequently v_o, are included. The steady-state G_{id} relegates the effects of v_{off} to a different feedback path and block in the waveform models—in the refined model as the path through F_{off} (elsewhere F_v and F_g, or K). While this choice of functional decomposition is entirely valid, placement of the effects of v_{OFF} back into G_{id} is what makes Sheehan's approach appealing; it makes the relationship between model and circuit more explicit.

To illustrate the greater complexity of actual circuits, analysis of the common-inductor (buck-boost) configuration results in

$$G_{id} = \frac{i_l}{d} = \frac{I_L}{D'} \cdot \left(\frac{R_o / D + Z_o}{Z_o} \right) \cdot \frac{1}{s \cdot \dfrac{L / D'^2}{Z_o} + 1}$$

where R_o is the real part of Z_o. For a simple case, let the load $Z_o = R_o$. Then

$$G_{id} = \frac{I_L}{D'} \cdot \left(\frac{R_o / D + R_o}{R_o} \right) \cdot \frac{1}{s \cdot \dfrac{L / D'^2}{R_o} + 1} = \frac{I_L}{D'} \cdot \frac{D+1}{D} \cdot \frac{1}{s \cdot \dfrac{L / D'^2}{R_o} + 1}$$

Even in this simple case, the pole is not at the origin but is finite. Consequently, the inductor waveform is only approximately a triangle-wave.

Closure

As for now, both waveform and circuit modeling complement each other, though they also motivate a search for a convergence. Waveform-based models begin with general waveform behavior and apply to any circuit structure for which the behavior is an approximation. Waveform models lose their generality when attempting completion by including v_{OFF} because the F_{off} and G_{id} blocks are dependent upon the circuit configuration. Circuit-based modeling begins with particular circuits and suffers from a lack of behavioral generality which is then sought by discovering common features in the different circuit behaviors such as the common variable v_{SW}. A final step in this decades-long modeling effort is to seek the convergence of waveform-based modeling with circuit-based modeling.

Sheehan's circuit simulation results show that the unified model with delay in the forward path matches simulation results most closely. The refined model is essentially the same kind of model, though derived from first principles without the *ad hoc* combination of a choice of F_{m0} based on independent considerations. The refined model makes no such arbitrary choices yet the early results appear to match Sheehan's simulation results. More research on the extent of this matching is desired and the early returns leave me hopeful.

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For more on current-mode control methods, see the How2Power Design Guide, select the Advanced Search option, go to Search by Design Guide Category, and select "Control Methods" in the Design Area category.

