

Use Worst-Case Analysis Tool To Efficiently Validate Your Designs

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When it’s time to validate their electronic circuit designs, many engineers depend on testing of prototypes and running of Monte Carlo simulations, believing that these two methods are sufficient to ensure design reliability. However, bench measurements and Monte Carlo analysis will not catch all potential design problems and the potential field failures that they miss can prove to be very costly. To truly validate a design, worst-case analysis is required.

Design Master software allows engineers to efficiently and thoroughly validate their designs, using an advanced form of worst-case analysis called WCA+. This program provides a fully integrated set of analysis tools, including a worst-case solver, probability estimates for out-of-spec results, sensitivities, and optimum values (see the reference.)

This article demonstrates how to use Design Master by going step-by-step through a simple analysis example. The circuit being analyzed is the familiar voltage divider and the analysis is performed two ways. In the first example, the resistors are specified with fixed component values and tolerances. In the second example, the resistor values are assigned formulas that account for the effects of variables such as temperature or aging.

A Simple Voltage Divider

Consider the case where a circuit bias voltage, Vbias, of +2.5 V ±2% is desired. The circuit shown in Fig. 1 is used to generate Vbias, where R1 and R2 are 10-kΩ 1% resistors and V5 is +5 V ±2%. Design Master can help you answer the following questions:

- Will the circuit meet specification?
- If not, what percentage of circuits off the production line will not meet spec?
- If not, what parameters have the greatest effect on meeting spec, and what are their optimum values?

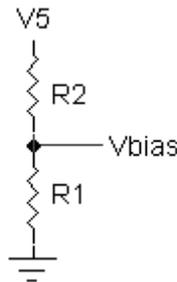


Fig. 1. Voltage divider used to generate a bias voltage, Vbias.

Step 1. Enter the design formula for Vbias in the worksheet.

Vbias is defined by the formula $V5 \cdot R1 / (R1 + R2)$. Its ±2% specification limits are entered under MinLimit and MaxLimit (Fig. 2.)

Rec	Status	Name	Description	Formula	MinLimit	Minimum	Average	Maximum	MaxLimit
1	Incomplete	Vbias	Bias voltage	$V5 \cdot R1 / (R1 + R2)$	2.45	0	0	0	2.55

Fig. 2. Inputting the formula for Vbias in the worksheet.

After entering the formula and pressing ENTER, Design Master detects the undefined terms in the formula (V5, R1, and R2) and prompts you to define them one at a time (Fig. 3.)

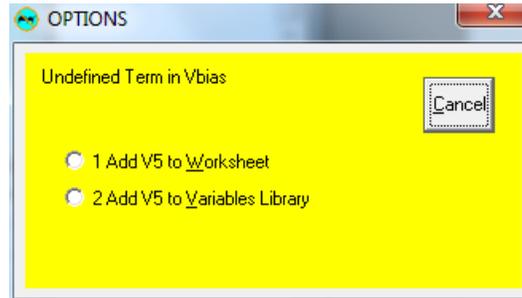


Fig. 3. Prompts to define the undefined term.

Since V5 is an independent variable (e.g. the output of a fixed regulator with a defined tolerance), Option 2 is clicked. This inserts the name "V5" into the Variables Library, where you can later enter its min/max data.

Note that an independent variable is defined as a term whose min/max values are not dependent on any other parameters. Examples of independent variables are initial tolerance, ambient temperature, source frequency, and time.

Next, Design Master detects undefined term R1 (Fig. 4.)

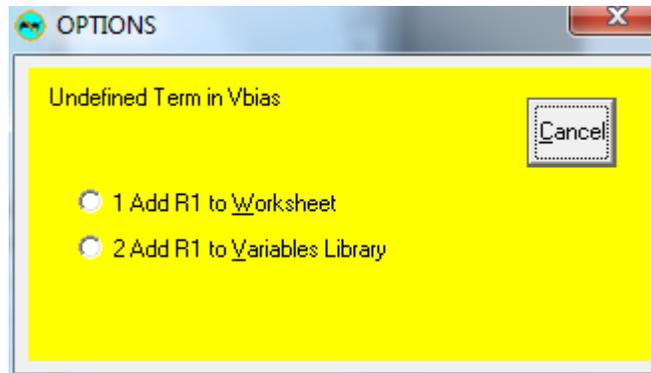


Fig. 4. Detection of an undefined term.

At this point, you could say: R1 is just a 10-kΩ resistor with a 1% tolerance. So why not just add it to the Variables Library with a min value of 9900 Ω and a max value of 10100 Ω?

That's one option, but in most cases it oversimplifies the situation because most resistors are significantly dependent on temperature, and also exhibit aging effects. For now, though, let's just add R1 and R2 to the Variables Library along with V5. (See the next section to learn how to enter R1 and R2 as formulas rather than variables.)

Step 2. Enter the min/max data in the Variables Library.

Now that all terms in Vbias are defined, we can enter the min/max data in the Variables Library.

R1 and R2 are 1% resistors, and V5 is a 2% supply voltage. Therefore, the following values are entered (Fig. 5.)

Rec	Name	Description	Minimum	Maximum	Scale	Samples	Peaking	Center	Type
1	R2	10K 1%	9900.0	10100.0	LIN	2	3.0	0.5	Stat
2	R1	10K 1%	9900.0	10100.0	LIN	2	3.0	0.5	Stat
3	V5	5V supply volts +/-2%	4.9	5.1	LIN	2	3.0	0.5	Stat

Fig. 5. Inputting the min/max data.

(Scale, Samples, Peaking, Center, and Type are left at their default values for this simple analysis.)

Step 3. Calculate.

All terms are fully defined, so now we select the Vbias row in the worksheet, and then click Calculate/This Record to get its results (Fig. 6.)

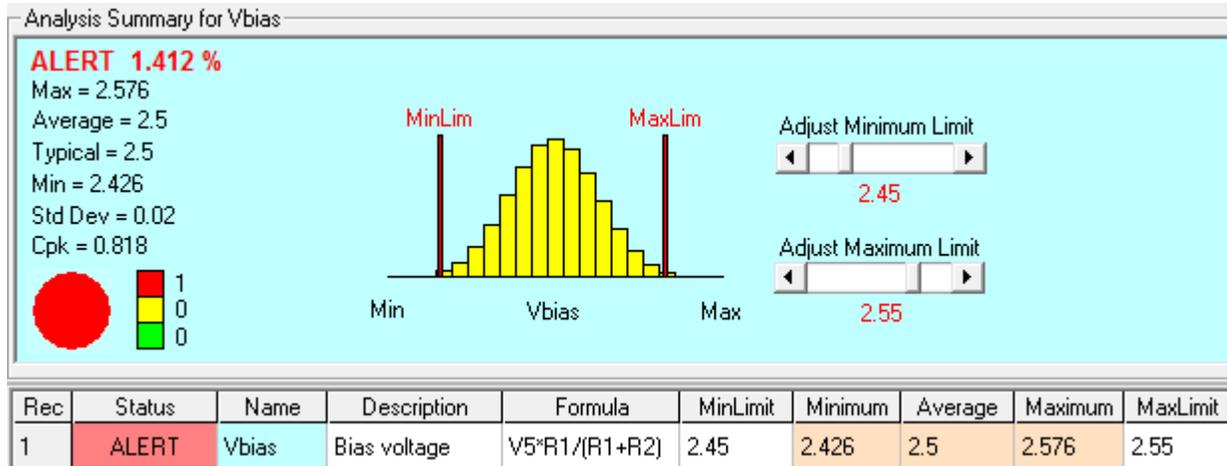


Fig. 6. Calculating Vbias values.

Step 4. Review results.

As can be seen, the design requirement is not achieved, with the worst-case minimum and maximum voltages both exceeding the 2% requirement. Design Master also advises us that the overall probability of exceeding the spec limits is roughly 1.4%.

WCA+ is essential because it can detect small probabilities of circuit failure that can otherwise result in huge warranty repair costs and damage to a company's reputation. Low-probability failures are easily missed by prototype testing and Monte Carlo simulations.

Note that the Analysis Summary screen allows the min and max spec limits to be adjusted without having to rerun the analysis. This allows you to see what happens to the ALERT probability as the specification limits are changed.

Voltage Divider Example Using Part Formulas For R1 And R2

As mentioned in the previous section, to assume that R1 and R2 are simple variables is generally much too optimistic for reliable designs. R1 and R2 will usually be defined with formulas that capture their dependencies on temperature and aging.

Note: To ensure a highly reliable design, develop design equations from data sheet values, not from idealized approximations.

Step 1. Enter the design formula for Vbias in the worksheet.

As before, the formula for Vbias and its specification limits are entered in the worksheet (Fig. 7.)

Rec	Status	Name	Description	Formula	MinLimit	Minimum	Average	Maximum	MaxLimit
1	Incomplete	Vbias	Bias voltage	$V5 \cdot R1 / (R1 + R2)$	2.45	0	0	0	2.55

Fig. 7. Inputting the formula for Vbias in the worksheet.

Step 2. Assign the undefined terms.

After entering the formula and pressing ENTER, assign V5 to the Variables Library as before, but this time assign R1 and R2 to the worksheet (Fig. 8.)

Rec	Status	Name	Description	Formula	MinLimit	Minimum	Average	Maximum	MaxLimit
1	Incomplete	Vbias	Bias voltage	$V5 \cdot R1 / (R1 + R2)$	2.45	2.426	2.5	2.576	2.55
2		R2				0	0	0	
3		R1				0	0	0	

Fig. 8. Assigning R1 and R2 to the worksheet.

At this point, you can enter formulas for R1 and R2 that include temperature and aging dependencies. Alternatively, since Design Master comes with basic component models, you can simply import a resistor model by following the instructions below in Step 3.

Step 3. Import part models.

- a. Highlight the row where you want to import a model (Fig. 9.)

Rec	Status	Name	Description	Formula	MinLimit	Minimum	Average	Maximum	MaxLimit
1	Incomplete	Vbias	Bias voltage	$V5 \cdot R1 / (R1 + R2)$	2.45	2.426	2.5	2.576	2.55
2		R2				0	0	0	
3		R1				0	0	0	

Fig. 9. Selecting components requiring models.

- b. Select Parts/Import Part File from the worksheet menu (Fig. 10.)

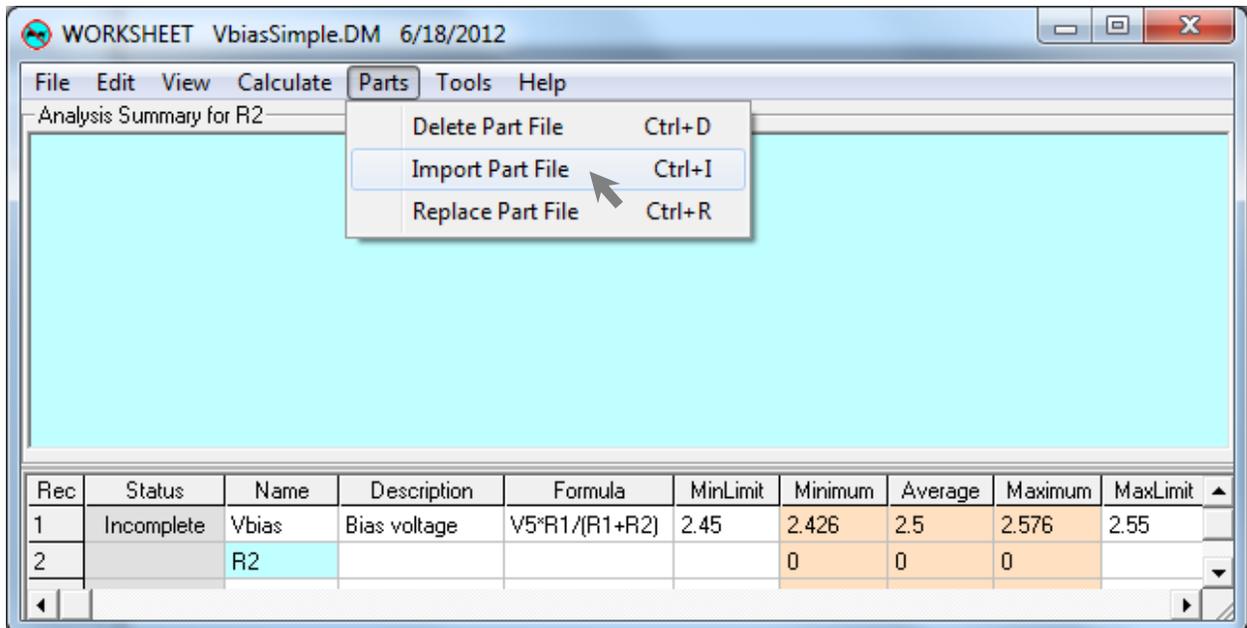


Fig. 10. Selecting Import Part File.

c. Select a Part File (Fig. 11.)

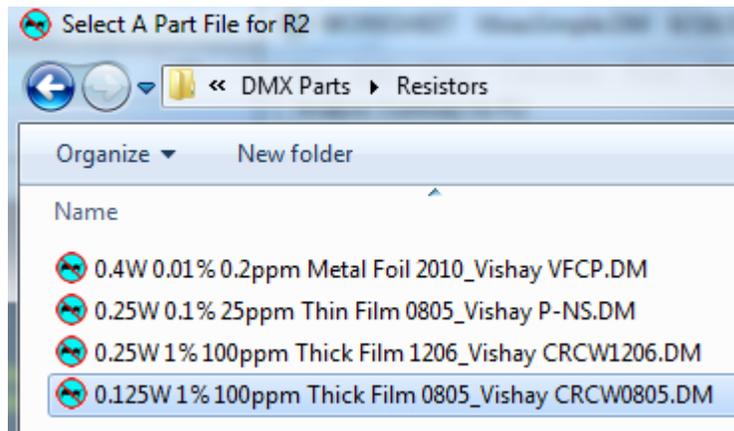


Fig. 11. Choosing a Part File.

d. Click the file and the model's formulas and variables are automatically inserted into the Worksheet and Variables Library (Fig. 12.)

Rec	Status	Name	Description	Formula	MinLimit	Minimum	Average	Maximum	MaxLimit
1	Incomplete	Vbias	Bias voltage	$V5 \cdot R1_R / (R1_R + R2_R)$	2.45	0	0	0	2.55
2			R2_Vishay CRCW0805 PROPERTIES			0	0	0	
3	Incomplete	R2_R	Resistance, ohms	$R2_Rnom \cdot (1 + R2_TOLiF) \cdot (1 + R2_TOLtcf \cdot (R2_Tc - 25)) \cdot (1 + R2_TOLage \cdot (R2_Life / 1000)^{0.5})$		0	0	0	
4	Incomplete	R2_Rnom	Nominal resistance, ohms			0	0	0	
5	Incomplete	R2_TOLage	Aging (life) tolerance @ 1000 hours (< 10Meg)	$R2_TOLagePOL \cdot 0.0006 \cdot 2^{((R2_Tc - 30) / 26.4)}$		0	0	0	
6			R2_Vishay CRCW0805 STRESS or OPERATING LIMITS			0	0	0	
7	Incomplete	R2_Tc	Case temperature			0	0	0	
8	Incomplete	R2_Life	Required operating hours			0	0	0	

Rec	Name	Description	Minimum	Maximum	Scale	Samples	Peaking	Center	Type
1	V5	5V supply volts +/-2%	4.9	5.1	LIN	2	3.0	0.5	Stat
2	R2_rCA	Vishay CRCW0805 thermal resistance, case to ambient, C/W [est]	352.0	440.0	LIN	2	3.0	0.5	Stat
3	R2_TOLagePi	Vishay CRCW0805 aging tolerance polarity	-1.0	1.0	LIN	2	1.0	0.5	Dyn
4	R2_TOLiF	Vishay CRCW0805 1% initial tolerance (1 ohm to 10 Meg)	-0.01	0.01	LIN	2	3.0	0.5	Stat
5	R2_TOLtcf	Vishay CRCW0805 temperature coefficient, 100ppm /C (10 ohm to 10 Meg)	-1.0E-4	1.0E-4	LIN	2	3.0	0.5	Stat

Fig. 12. Inserting variables into worksheet and Variables Library.

The imported model has locked (shaded) cells, and unlocked (white) cells for data entry. Design Master cells can be locked or unlocked to create fill-in-the-blank templates.

e. Repeat the above for R1.

Step 4. Fill in the blanks.

Enter any missing formulas or variables data (Fig. 13.)

Rec	Status	Name	Description	Formula	MinLimit	Minimum	Average	Maximum	MaxLimit
1	Incomplete	Vbias	Bias voltage	$V5 \cdot R1_R / (R1_R + R2_R)$	2.45	0	0	0	2.55
2			R2_Vishay CRCW0805 PROPERTIES			0	0	0	
3	Incomplete	R2_R	Resistance, ohms	$R2_Rnom \cdot (1 + R2_TOLIF) \cdot (1 + R2_TOLtcF \cdot (R2_Tc - 25)) \cdot (1 + R2_TOLage \cdot (R2_Life/1000)^{0.5})$		0	0	0	
4	Incomplete	R2_Rnom	Nominal resistance, ohms	10E3		0	0	0	
5	Incomplete	R2_TOLage	Aging (life) tolerance @ 1000 hours (< 10Meg)	$R2_TOLagePOL \cdot 0.0006 \cdot 2^{((R2_Tc - 30)/26.4)}$		0	0	0	
6			R2_Vishay CRCW0805 STRESS or OPERATING LIMITS			0	0	0	
7	Incomplete	R2_Tc	Case temperature	Ta		0	0	0	
8	Incomplete	R2_Life	Required operating hours	10000		0	0	0	
9			R1_Vishay CRCW0805 PROPERTIES			0	0	0	
10	Incomplete	R1_R	Resistance, ohms	$R1_Rnom \cdot (1 + R1_TOLIF) \cdot (1 + R1_TOLtcF \cdot (R1_Tc - 25)) \cdot (1 + R1_TOLage \cdot (R1_Life/1000)^{0.5})$		0	0	0	
11	Incomplete	R1_Rnom	Nominal resistance, ohms	10E3		0	0	0	
12	Incomplete	R1_TOLage	Aging (life) tolerance @ 1000 hours (< 10Meg)	$R1_TOLagePOL \cdot 0.0006 \cdot 2^{((R1_Tc - 30)/26.4)}$		0	0	0	
13			R1_Vishay CRCW0805 STRESS or OPERATING LIMITS			0	0	0	
14	Incomplete	R1_Tc	Case temperature	Ta		0	0	0	
15	Incomplete	R1_Life	Required operating hours	10000		0	0	0	

Fig. 13. Inputting missing formulas and variables.

The case temperature for R1 and R2 was defined as variable Ta. When entered, Ta is detected by Design Master as an undefined term. The term is added to the Variables Library and then its min/max values are entered (Fig. 14.)

Rec	Name	Description	Minimum	Maximum	Scale	Samples	Peaking	Center	Type
1	V5	5V supply volts +/-2%	4.9	5.1	LIN	2	3.0	0.5	Stat
2	Ta	Ambient temperature, C	0	70.0	LIN	3	3.0	0.5	Dyn
3	R2_rCA	Vishay CRCW0805 thermal resistance, case to ambient, C/W [est]	352.0	440.0	LIN	2	3.0	0.5	Stat
4	R2_TOLagePl	Vishay CRCW0805 aging tolerance polarity	-1.0	1.0	LIN	2	1.0	0.5	Dyn
5	R2_TOLiF	Vishay CRCW0805 1% initial tolerance (1 ohm to 10 Meg)	-0.01	0.01	LIN	2	3.0	0.5	Stat
6	R2_TOLtcF	Vishay CRCW0805 temperature coefficient, 100ppm /C (10 ohm to 10 Meg)	-1.0E-4	1.0E-4	LIN	2	3.0	0.5	Stat
7	R1_rCA	Vishay CRCW0805 thermal resistance, case to ambient, C/W [est]	352.0	440.0	LIN	2	3.0	0.5	Stat
8	R1_TOLagePl	Vishay CRCW0805 aging tolerance polarity	-1.0	1.0	LIN	2	1.0	0.5	Dyn
9	R1_TOLiF	Vishay CRCW0805 1% initial tolerance (1 ohm to 10 Meg)	-0.01	0.01	LIN	2	3.0	0.5	Stat
10	R1_TOLtcF	Vishay CRCW0805 temperature coefficient, 100ppm /C (10 ohm to 10 Meg)	-1.0E-4	1.0E-4	LIN	2	3.0	0.5	Stat

Fig. 14. Entering minimum and maximum values for Ta.

Note that Ta is defined as a dynamic variable, as are the aging tolerance variables. In assigning static or dynamic status to a variable consider the following: A static variable's value, after initial selection (like picking a part out of a bin), does not vary throughout the life of the circuit; e.g. initial tolerance or temperature coefficient. In contrast, a dynamic variable's value can continue to move or cycle over its range of values during the life of the circuit; e.g. temperature or unregulated supply voltage.

Step 5. Calculate (Fig. 15.)

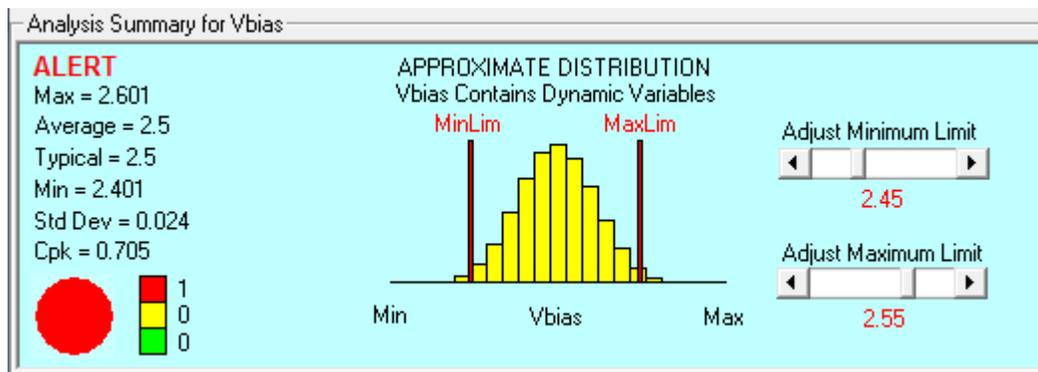


Fig. 15. Calculating Vbias values using formulas for R1 and R2.

Step 6. Review results.

As expected, by using full worst-case part parameters for R1 and R2, the worst case spread (2.401 V to 2.601 V) is wider than for the prior simpler analysis (2.426 V to 2.576V.) In this case, however, Design Master advises us that we have an "Approximate Distribution" and no ALERT value is provided for the out-of-spec results. This is because of the presence of the dynamic variables. When dynamic variables are present, select the Calculate/Make Min/Max Equations option (Fig. 16) ...

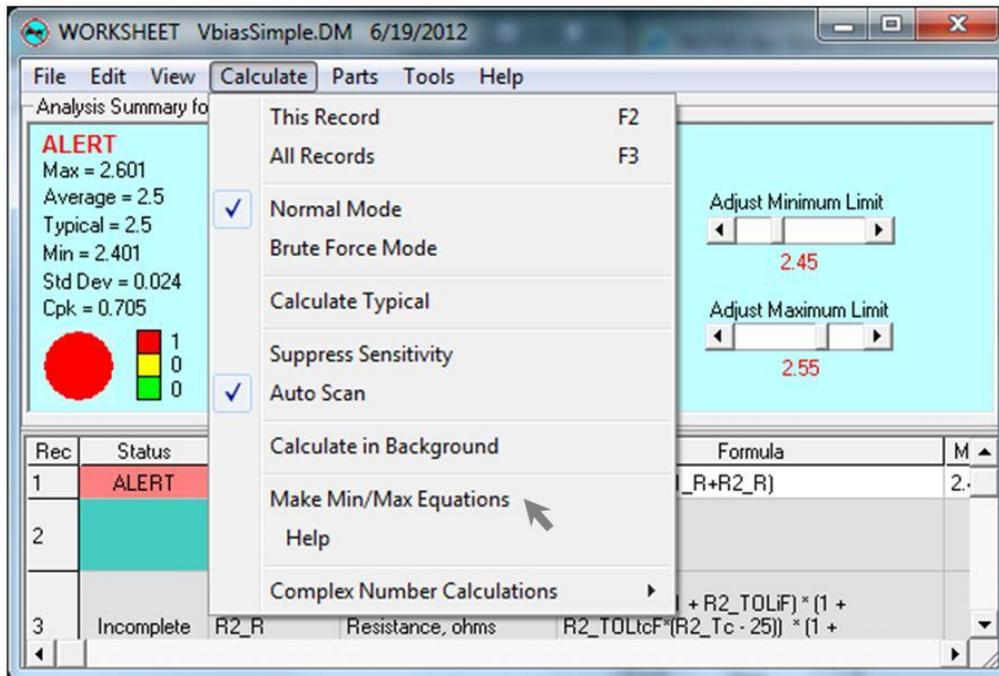


Fig. 16. Selecting the Calculate/Make Min/Max Equations option.

...and then highlight Vbias and click Calculate/This Record. This will generate two records, worst-case minimum and worst-case maximum (Fig. 17.)

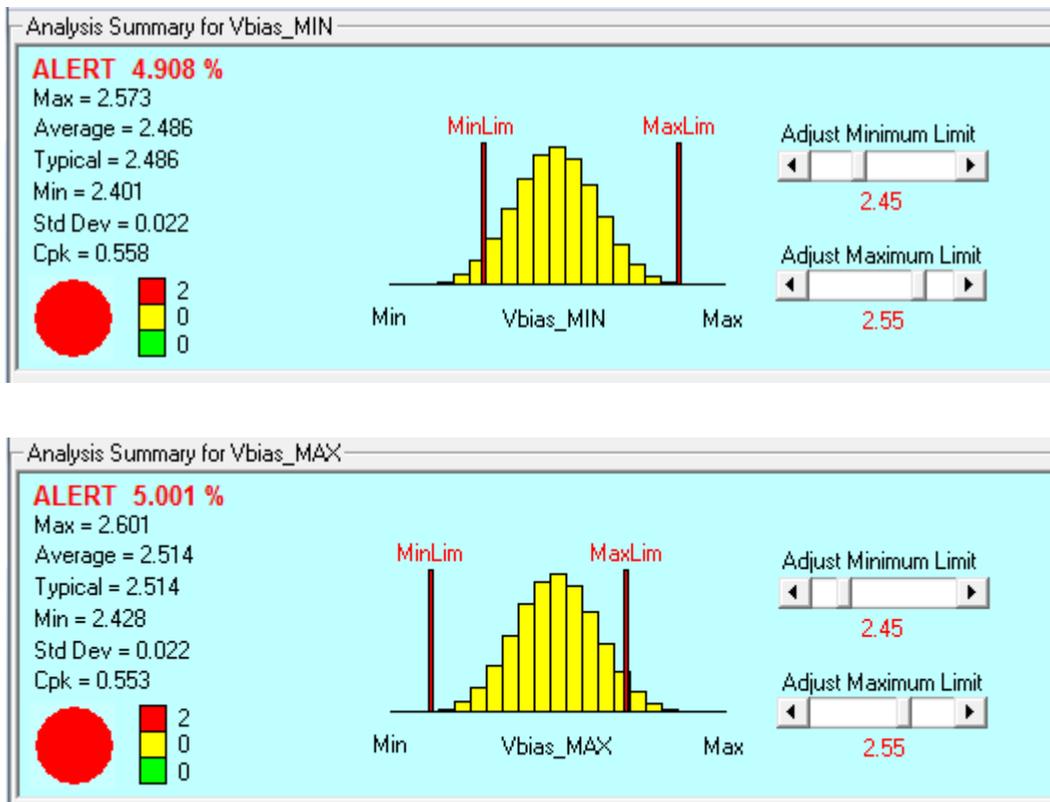


Fig. 17. Checking Alert Values when dynamic variables are used.

Design Master advises us that Vbias has an approximate 4.9% probability of being too low, and an approximate 5.0% probability of being too high.

Because the design is properly centered, we can't shift values to improve reliability, and must either widen the spec limits, use lower-tolerance or matched resistors, or use a more-accurate reference voltage V5. Which approach would be best? Design Master provides a set of Normalized Sensitivities along with Optimum Values to help you decide (Fig. 18.)

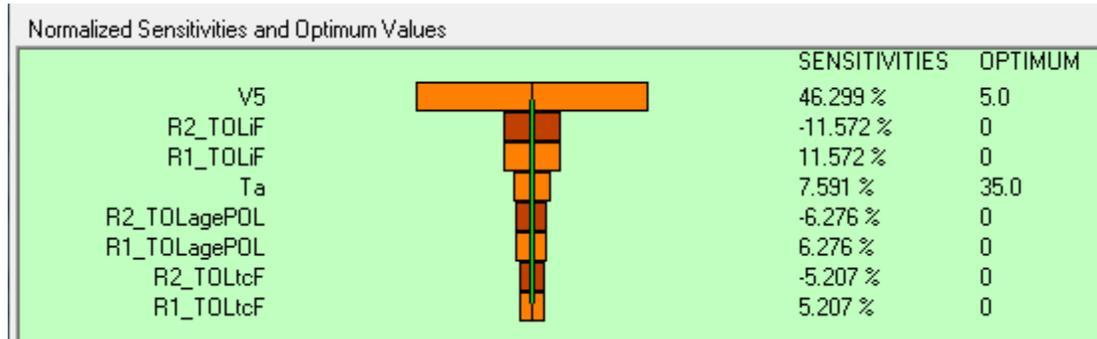


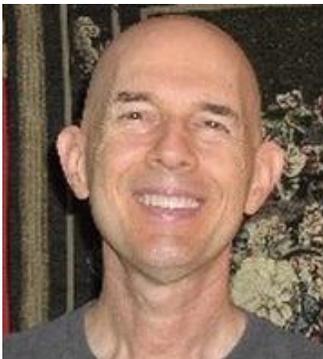
Fig. 18. Checking design sensitivities.

V5 has the greatest effect on Vbias, followed by the initial tolerances of R1 and R2, and then ambient temperature Ta. The remaining aging and temperature-coefficient parameters have diminishing effect.

Reference

For an overview of Design Master's capabilities, see "[Are Your Design Validation Tools Tough Enough?](#)"

About The Author



Ed Walker is president of [Design/Analysis Consultants, Inc. \(DACI\)](#), a consulting firm specializing in design validation, risk assessment, and optimization of electronics and electromechanical technologies. Walker is the author of *The Design Analysis Handbook, A Practical Guide to Design Validation* (Elsevier). He is also the creator of Design Master, a probability-based analysis software tool, which complements the Handbook. Numerous technology firms, including automotive, power supply, medical, telecommunications, computer, and integrated circuit developers, have adopted the design/analysis methodology developed by Walker and described in the Handbook. You can follow Ed through the [DACI Newsletter](#) and the [Engineering Thinking blog](#). Ed also writes edgy [psychological fiction](#).