ISSUE: February 2013

Simplified Analysis Of A DCM Boost Converter Driving An LED String

Part I: Theoretical Analysis

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The fixed-frequency boost converter lends itself very well to driving an LED string at a constant current. Working in discontinuous-conduction mode (DCM), the converter can be efficiently used for fast dimming operation. In this regard, the performance of a DCM-operated boost converter will be superior to that of its continuous-conduction mode (CCM)-operated counterpart because DCM operation offers better transient response. As a result of this fast transient performance, a boost converter operating in DCM will rapidly recharge the output capacitor when the LEDs are turned on, thus minimizing analog dimming of the LEDs.

However, the design of a DCM boost converter involves certain challenges such as properly stabilizing the design. Although small-signal models exist for performing this type of analysis, some difficulty arises in trying to apply these models to the LED driver applications. That's because the ac analysis of a boost converter driving LEDs differs from that using a standard resistive load. As the series diodes impose both dc and ac loading conditions, deriving the final transfer function is not a simple matter.

Here in part I of this two-part article, we present an easier method for analysis. Rather than using the classical small-signal model of the DCM boost converter, we develop a simplified model based on the output current expression for the converter and then use this model for analysis. In Part II of this article (Practical Considerations), we will delve into an implemented solution and verify measurement accuracy versus the theoretical derivation.

A Boost Converter Powering The LED String

Fig. 1 represents a simplified constant-frequency peak-current-operated boost converter driving an LED string. The output current is permanently monitored by a sense resistor R_{sense}. The voltage it develops goes to a control circuit that continually adjusts the power switch on-time to deliver a constant LED current I_{out}. This is the controlled output variable.

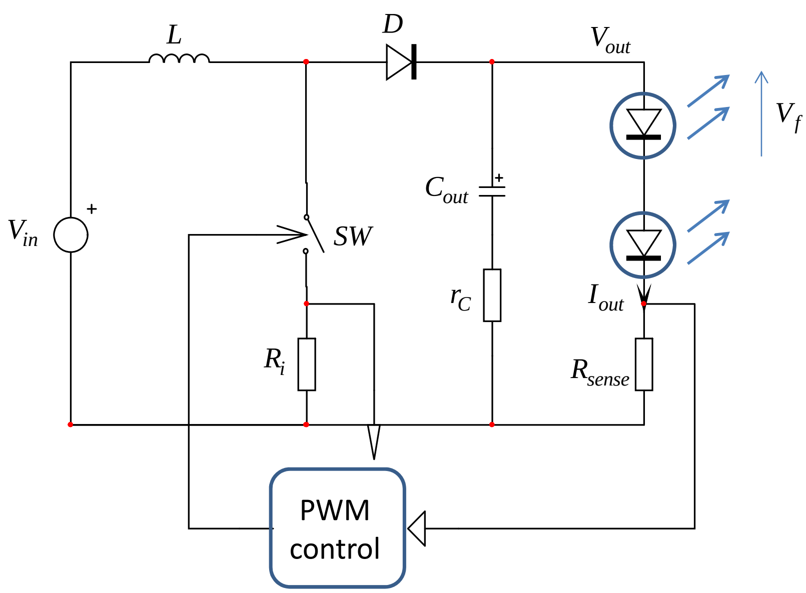

Fig. 1. A boost converter drives an LED string to deliver light. The output current is regulated to a setpoint value.

When lit, the LED string gives rise to a voltage across its connecting terminals. This voltage depends on the individual LED technology-related threshold voltage V_{T0} and its dynamic resistance r_d. The total drop across the LED string is thus the sum of the threshold voltages denoted as V_Z while the dynamic resistance r_{LEDs} represents the sum of the series dynamic resistances. Fig. 2 shows the adopted equivalent circuit.

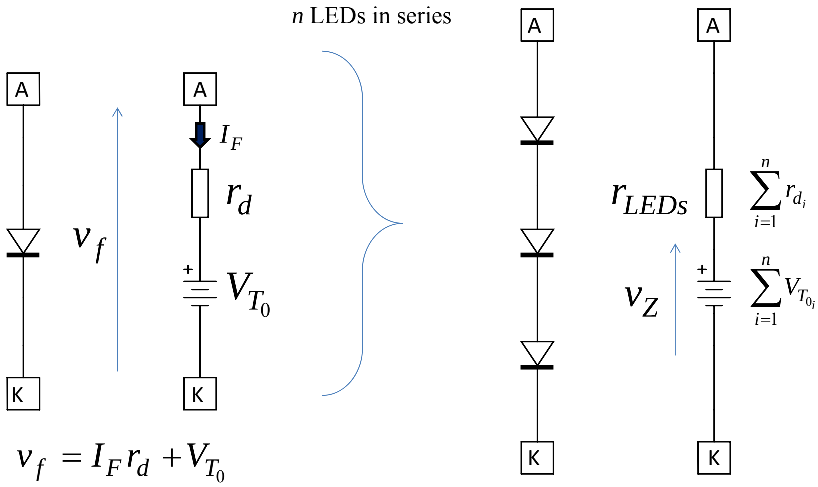

Fig. 2. The series connection of LEDs leads to a summing of their threshold voltages and the series connection of their individual dynamic resistances.

It will be your duty to characterize the string voltage drop and its total dynamic resistance. To measure it, bias the LED string to its nominal current, I_{F1}. Once LED thermal stability is reached, measure the total voltage drop across the LED string, V_{f1}. Change the current to a slightly lower value I_{F2} and measure the new voltage drop V_{f2}. From these values, you can calculate the total dynamic resistance as:

$$r_{LEDs} = \frac{V_{f_1} - V_{f_2}}{I_{F_1} - I_{F_2}}. \tag{1}$$

The "Zener" voltage is one of the string voltages V_{f1} minus r_{LEDs} times the current at this measurement point:

$$V_Z \approx V_{f_1} - R_{LEDs} I_{F_1}. \tag{2}$$

Let's assume we bias our LED string with a 100-mA current. The measured total drop is 27.5 V. We reduce the current to 80 mA and the new drop is 26.4 V. The total dynamic resistance is simply:

$$r_{LEDs} = \frac{27.5 - 26.4}{0.1 - 0.08} = 55\,\Omega. \tag{3}$$

From equation 2, the Zener voltage is simply:

$$V_Z = 27.5 - 0.1 \times 55 = 22\,\mathrm{V}. \tag{4}$$

Looking back to Fig. 1, the LED string is placed in series with a sense resistor R_{sense}. The total ac resistance is thus the combination of both elements:

$$R_{ac} = r_{LEDs} + R_{sense}. \tag{5}$$

The equivalent dc diagram simplifies quite a bit to that of Fig. 3. The dc output voltage V_{out} is made of the output current I_{out} circulating in the resistor R_{ac} plus the Zener voltage:

$$V_{out} = R_{ac} I_{out} + V_z . \tag{6}$$

In ac, as the Zener voltage is constant, the expression simplifies to:

$$V_{out}(s) = R_{ac} I_{out}(s) . \tag{7}$$

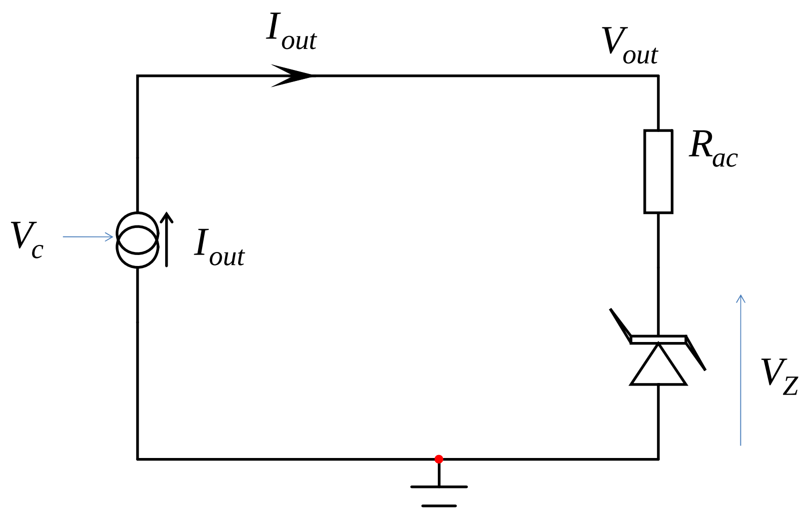

Fig. 3. The dc sketch shows the equivalent Zener diode and its dynamic resistance.

A Simplified Model

The current source actually represents the current taken from the input source and transmitted to the output without losses. This source is scaled up or down by the control voltage V_c, which sets the inductor peak current on a cycle-by-cycle basis. The controller works by observing the inductor peak current through the current-sense resistor R_i for the boost converter switch. When the voltage across R_i and the control voltage V_c match, the power switch is instructed to turn off.

If we now consider an ac diagram, the capacitor and its parasitic element come back as shown in Fig. 4. The Zener element alone does not play a role as its voltage remains constant during ac modulation: only its dynamic resistance r_{LEDs} stays in place, merged into R_{ac} as described by equation 5.

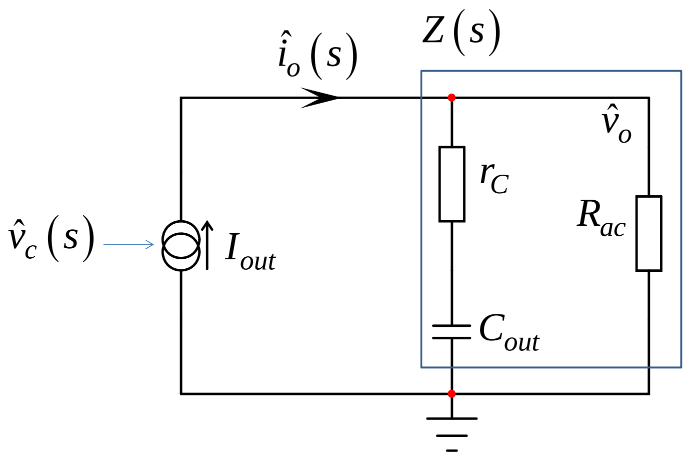

Fig. 4. The ac model uses the total resistance R_{ac} associated with the capacitor model.

From this drawing, it is possible to express the small-signal output voltage level when the control voltage is modulated:

$$\hat{v}_o(s) = \hat{i}_o(s) Z(s). \tag{8}$$

As previously mentioned, the current-source value depends on the control and output voltages. To derive a small-signal equivalent model, we extract the partial derivatives of I_{out} with respect to the control voltage V_c and the output voltage V_{out}:

$$\hat{i}_o(s) = \left.\frac{\partial I_{out}}{\partial V_c}\right|_{\hat{v}_{in}=0,\hat{v}_o=0} \hat{v}_c(s) + \left.\frac{\partial I_{out}}{\partial V_{out}}\right|_{\hat{v}_{in}=0,\hat{v}_c=0} \hat{v}_o(s). \tag{9}$$

Substituting equation 9 into equation 8, the latter can be rewritten:

$$\hat{v}_o(s) = \left[\left.\frac{\partial I_{out}}{\partial V_c}\right|_{\hat{v}_{in}=0,\hat{v}_{out}=0} \hat{v}_c(s) + \left.\frac{\partial I_{out}}{\partial V_{out}}\right|_{\hat{v}_{in}=0,\hat{v}_c=0} \hat{v}_o(s)\right] Z(s). \tag{10}$$

Reference [1] (equation 1-111, p. 49) has derived the dc transfer function for the DCM boost converter as:

$$M = \frac{V_{out}}{V_{in}} = \frac{1+\sqrt{1+\dfrac{2T_{sw}D^2R_{dc}}{L}}}{2}. \tag{11}$$

In this last expression, the resistance loading the converter must be replaced by V_{out}/I_{out} . The new expression then becomes:

$$\frac{V_{out}}{V_{in}} = \frac{1+\sqrt{1+\dfrac{2T_{sw}D^2\dfrac{V_{out}}{I_{out}}}{L}}}{2}. \tag{12}$$

From this equation, we need to derive the duty ratio expression and the control voltage V_c. In the presence of a compensation ramp, the control voltage is no longer a fixed dc voltage but a ramp whose slope affects the final peak current point. Fig. 5 shows the resulting waveform. The peak current value is reached sooner than in the absence of a ramp, as if we would artificially increase the current-control sense resistor R_i. It has the effect of decreasing the gain of the current-control loop and damping the double poles in continuous-conduction mode. When the converter transitions to DCM, the ramp is still present and must be accounted for.

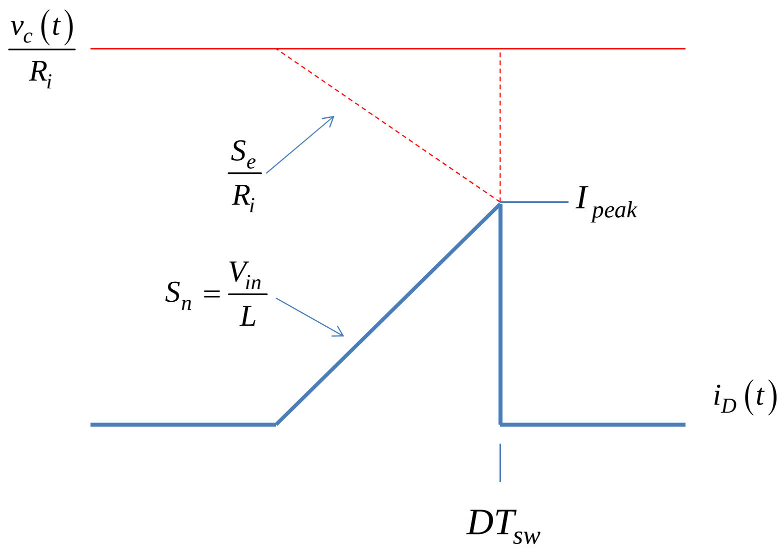

Fig. 5. The peak current is not equal to the control voltage divided by R_{sense} because of the compensation ramp.

The equations are the following ones, accounting for the scaling factor R_i as the external ramp S_e is a voltage ramp:

$$I_{peak} = \frac{V_c}{R_i} - \frac{S_e}{R_i} DT_{sw} \,. \tag{13}$$

A similar expression is derived involving the inductor current slope:

$$I_{peak} = \frac{DT_{sw} V_{in}}{L} \,. \tag{14}$$

Solving for D, we have:

$$D = \frac{V_c L}{S_e T_{sw} L + R_i T_{sw} V_{in}} \,. \tag{15}$$

This expression for D is now injected into equation 12 and we solve for I_{out}:

$$I_{out} = \frac{2V_{out} L V_c^{\,2}}{T_{sw}} \frac{1}{\left[\left(\frac{2V_{out}}{V_{in}} - 1\right)^2 - 1\right]\left(S_e L + R_i V_{in}\right)^2} \,. \tag{16}$$

To obtain the small-signal value, we will calculate I_{out} partial derivatives with respect to the control voltage, V_c and output voltage V_{out} as described in equation 10:

$$\left.\frac{\partial I_{out}}{\partial V_c}\right|_{\hat{v}_{in},\hat{v}_{out}} \hat{v}_c = \frac{d}{dV_c}\left(\frac{2V_{out}LV_c^{\,2}}{T_{sw}}\frac{1}{\left[\left(\frac{2V_{out}}{V_{in}}-1\right)^2-1\right]\left(S_eL+R_iV_{in}\right)^2}\right)\hat{v}_c \qquad (17)$$

$$\left.\frac{\partial I_{out}}{\partial V_c}\right|_{\hat{v}_{in},\hat{v}_{out}} \hat{v}_c = \frac{V_{in}^{\,2}V_cL}{T_{sw}\left(V_{out}-V_{in}\right)\left(S_eL+R_iV_{in}\right)^2}\hat{v}_c\,. \qquad (18)$$

The expression in equation 18 characterizes the impact of the small-signal modulation of v_c on the output current. Then, calculating the I_{out} partial derivative with respect to V_{out}:

$$\left.\frac{\partial I_{out}}{\partial V_{out}}\right|_{\hat{v}_{in},\hat{v}_c} \hat{v}_o = \frac{d}{dV_{out}}\left(\frac{2V_{out}LV_c^{\,2}}{T_{sw}}\frac{1}{\left[\left(\frac{2V_{out}}{V_{in}}-1\right)^2-1\right]\left(S_eL+R_iV_{in}\right)^2}\right)\hat{v}_o \qquad (19)$$

$$\left.\frac{\partial I_{out}}{\partial V_{out}}\right|_{\hat{v}_{in}=0,\hat{v}_c=0} \hat{v}_o = -\frac{V_{in}^{\,2}V_c^{\,2}L}{2T_{sw}\left(V_{in}-V_{out}\right)^2\left(S_eL+R_iV_{in}\right)^2}\hat{v}_o\,. \qquad (20)$$

This last equation expresses a current depending on a voltage multiplied by a coefficient having the dimension of a conductance *g*. It is a voltage-controlled current-source as drawn in Fig. 6.

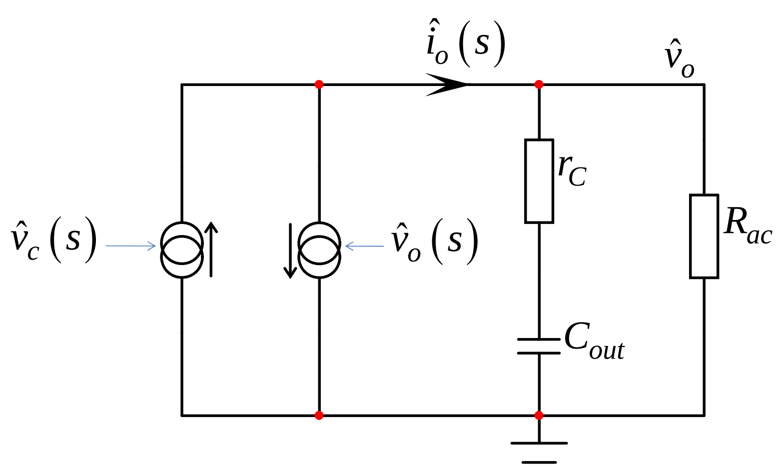

Fig. 6 The equation 20 coefficient is a voltage-controlled current-source, effectively a resistance.

The current direction of the $\hat{v}_o(s)$ controlled current source is reversed because of the negative sign in equation 20. As such, since we have a current-source driven by the voltage across it, it is simply a resistor whose definition is:

$$R_1 = \frac{2T_{sw}\left(V_{in} - V_{out}\right)^2 \left(S_e L + R_i V_{in}\right)^2}{V_c^{\,2} V_{in}^{\,2} L}. \tag{21}$$

In this simplified derivation, the current source illustrates the energy that is absorbed from the input source and transmitted to the output. The current source expression does not carry information related to the converter operating mode. For instance, looking at equation 16, we do not know if the part operates at a fixed frequency, transmits energy to the output load during the on-time or during the off-time, and so on.

Lacking such information, the model will mask second-order contributions such as a right-half plane zero (RHPZ), for instance. However, we know from previous analysis that the RHPZ still exists in DCM operation but since it is relegated to high frequencies, we can omit its presence in this case.

The benefit of this simplified approach is that it enables you to quickly derive an approximate model that gives you the low-frequency behavior of the considered structure: dc gain and pole/zero combination. An alternative would be to use the small-signal model of the DCM current-mode boost converter and carry the complete analysis with a load made of the elements in Fig. 4. This model would give an exact result but would require more iterations and complex equations.

The Complete AC Model

Now that we have derived all of our coefficients, we can update the model originally presented in Fig. 4. The updated schematic appears in Fig. 7. R_1 corresponds to the equation 20 coefficient and induces a current directly proportional to the output voltage modulation.

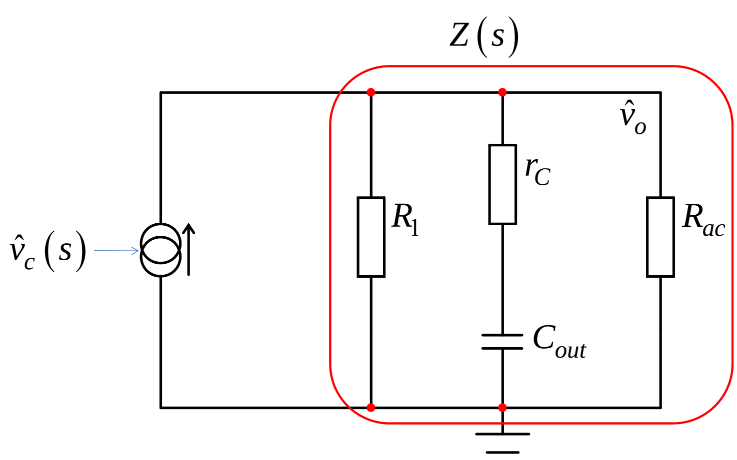

Fig. 7. This is the updated ac model from which we will calculate the complete transfer function.

To derive the transfer function of interest, $\hat{v}_o/\hat{v}_c$, we will simplify the circuit by looking at the impedance Z loading the current source. It is defined as:

$$Z(s) = \frac{\left(r_C + \frac{1}{sC_{out}}\right)R_{eq}}{\left(r_C + \frac{1}{sC_{out}}\right) + R_{eq}} = R_{eq}\frac{1 + sr_c C_{out}}{1 + sC_{out}\left(R_{eq} + r_C\right)}. \tag{22}$$

In the above equation, R_{eq} is the parallel combination of R_{ac} and R_1:

$$R_{eq} = \frac{R_1 R_{ac}}{R_1 + R_{ac}}. \qquad (23)$$

The complete transfer equation is thus the coefficient given by equation 18 multiplied by the resistance in equation 23 and followed by the pole/zero combination from equation 22:

$$H(s) = H_0 \frac{1+\frac{s}{\omega_z}}{1+\frac{s}{\omega_p}}. \qquad (24)$$

where

$$H_0 = \frac{{V_{in}}^2 V_c L}{T_{sw}\left(V_{out} - V_{in}\right)\left(S_e L + R_i V_{in}\right)^2} R_{eq}\ , \qquad (25)$$

$$\omega_z = \frac{1}{r_C C_{out}}, \text{ and} \qquad (26)$$

$$\omega_p = \frac{1}{\left(r_C + R_{eq}\right) C_{out}}. \qquad (27)$$

Deriving The Operating Points

Before plotting the ac function, we need to express the operating points and the output current dependence on control voltage V_c. We know that the output voltage is equal to:

$$V_{out} = I_{out} R_{ac} + V_Z\ . \qquad (28)$$

We can substitute this definition into equation 12:

$$\frac{V_Z + I_{out} R_{ac}}{V_{in}} = \frac{\sqrt{\frac{I_{out} L + 2D^2 T_{sw} V_Z + 2D^2 I_{out} R_{ac} T_{sw}}{I_{out} L}}}{2} + \frac{1}{2}. \qquad (29)$$

From this expression, we can solve for I_{out}:

$$I_{out} = \frac{\sqrt{2R_{ac} T_{sw} D^2 L {V_{in}}^2 + \left(V_Z L\right)^2 - 2L^2 V_Z V_{in} + \left(V_{in} L\right)^2} - V_Z L + V_{in} L}{2 R_{ac} L}. \qquad (30)$$

We can also replace the duty ratio *D* by its expression in equation 15. In this case, the output current expression becomes an ugly but useful equation:

$$I_{out} = \frac{\sqrt{V_Z{}^2 + V_{in}{}^2 - 2V_z V_{in} + \dfrac{2LR_{ac}T_{sw}V_c{}^2 V_{in}{}^2}{\left(S_e T_{sw} L + R_i T_{sw} V_{in}\right)^2}} - V_z + V_{in}}{2R_{ac}}. \quad (31)$$

Knowing the LED string voltage V_Z and its dynamic resistance r_{LEDs}, this expression allows us to predict the current delivered by the boost converter. Let's verify these formulas with a practical example.

Practical Application

In this section, we present the example of a DCM boost converter delivering a constant current to an LED string with a voltage of 22 V. We'll use the following circuit values in our calculations.

$L = 3.3\,\mu\text{H}$

$T_{sw} = 1\,\mu\text{s}$

$\text{R}_i = 250\,\text{m}\Omega$

$C_{out} = 2.2\,\mu\text{F}$

$r_C = 4\,\text{m}\Omega$

$R_{sense} = 11\,\Omega$

$r_{LEDs} = 55\,\Omega$

$V_{in} = 12\,\text{V}$

$\text{S}_e = 100\,\text{kV/s}$

$V_Z = 22\,\text{V}$

To impose the constant current, we assume the control voltage V_c to be 400 mV. We can calculate the duty ratio from equation 15:

$$D = \frac{V_c L}{S_e T_{sw} L + R_i T_{sw} V_{in}} = 39.6\%. \quad (32)$$

The output current is obtained from equation 31:

$$I_{out} = \frac{\sqrt{\left(V_Z L\right)^2 + \left(V_{in} L\right)^2 - 2V_z V_{in} L^2 + \dfrac{2L^3 R_{ac} T_{sw} V_c{}^2 V_{in}{}^2}{\left(S_e T_{sw} L + R_i T_{sw} V_{in}\right)^2}} - V_z L + V_{in} L}{2R_{ac} L} = 164\,\text{mA}. \quad (33)$$

The output voltage quickly follows:

$$V_{out} = I_{out}\left(r_d + R_{sense}\right) + V_Z = 32.85\,\text{V}. \quad (34)$$

The extra resistor R_1 calculated in equation 21 is found to be:

$$R_1 = \frac{2T_{sw}\left(V_{in} - V_{out}\right)^2\left(S_e L + R_i V_{in}\right)^2}{V_c^{\,2} V_{in}^{\,2} L} = 126.8\,\Omega\,. \tag{35}$$

When paralleled with R_{ac}, it becomes R_{eq} according to equation 23:

$$R_{eq} = \frac{R_1 R_{ac}}{R_1 + R_{ac}} = \frac{126.8 \times (11+55)}{126.8 + 11 + 55} = 43.4\,\Omega\,. \tag{36}$$

We can now evaluate the static gain, H_0:

$$20\log\left(G_0\right) = 20\log\left(\frac{V_{in}^{\,2} V_c L}{T_{sw}\left(V_{out} - V_{in}\right)\left(S_e L + R_i V_{in}\right)^2} R_{eq}\right) = 20\log\left(35.68\right) = 31\,\text{dB} \tag{37}$$

The pole and zero are derived:

$$f_z = \frac{1}{2\pi r_C C_{out}} = 18\,\text{MHz} \quad \textit{and} \tag{38}$$

$$f_p = \frac{1}{2\pi\left(r_C + R_{eq}\right)C_{out}} = 1.6\,\text{kHz}\,. \tag{39}$$

A SPICE simulation can be run to check the validity of the bias points. A large-signal auto-toggling current-mode model derived in Ref. [1], p. 161 was used. The schematic and the reflected bias points appear in Fig. 8. In this schematic, to obtain the right dynamic resistance and operating voltage, we used a simple shunt regulator mimicking the operation of a perfect Zener diode. This perfect diode exhibits a breakdown voltage V_Z of 22 V and its dynamic resistance is 55 Ω.

It should be noted that a simple 22-V dc source would work for an ac analysis, but would not work for any transient simulations such as start-up. When an ac sweep is run, SPICE linearizes the circuit around its operating point and generates a small-signal model. The results displayed in the schematic are not far from what we have obtained through the analytical analysis. The current in the sense resistor with a 0.4-V control voltage reaches $1.77/11 \approx 161\,\text{mA}$, close to that calculated in equation 33.

The plant Bode plot is shown in Fig. 9. The dc gain is close to that calculated in equation 37 and the pole is located at the correct location (1.6 kHz). The phase that continues to drop is due to the high-frequency RHPZ located at high frequencies.

Our simplified approach cannot predict the presence of this RHPZ. Its existence relates to the topology arrangement: a boost converter first stores the source energy in the inductor during the on-time and dumps it into the load during the off-time. Any load condition changes, i.e. an increase in the output current, must first ramp through the inductor current before it is delivered to the output. This delay inherent to the operating mode is modeled through an RHPZ. This energy transfer delay does not explicitly appear in equation 16, which simply defines a current in relationship to control voltage, V_c. In DCM, however, the left-half plane zero defined in equation 38 occurs at a frequency significantly above the operation frequency F_{sw}.

It should be noted that we analyzed the output voltage even though we actually regulate the LED current. As we observe the voltage across the sense resistance R_{sense}, the feedback signal is that of V_{out} scaled down by the division ratio brought by r_{LEDs} and R_{sense}. The scaling adjustment becomes:

$$20Log_{10}\left(\frac{R_{sense}}{R_{sense}+r_{LEDs}}\right)=20Log_{10}\left(\frac{11}{11+55}\right)\approx -15.6\text{ dB}\,. \qquad (40)$$

This curve is also represented in Fig. 8.

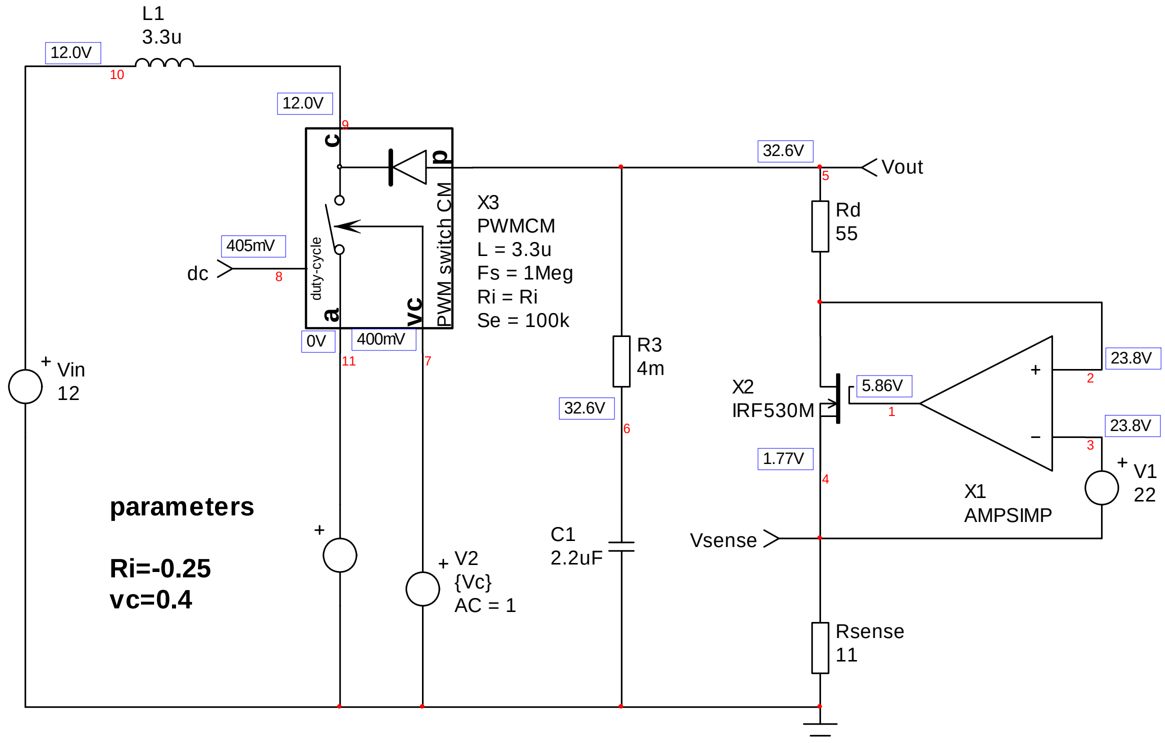

Fig. 8. The averaged model helps to verify the operating bias points but also the ac response.

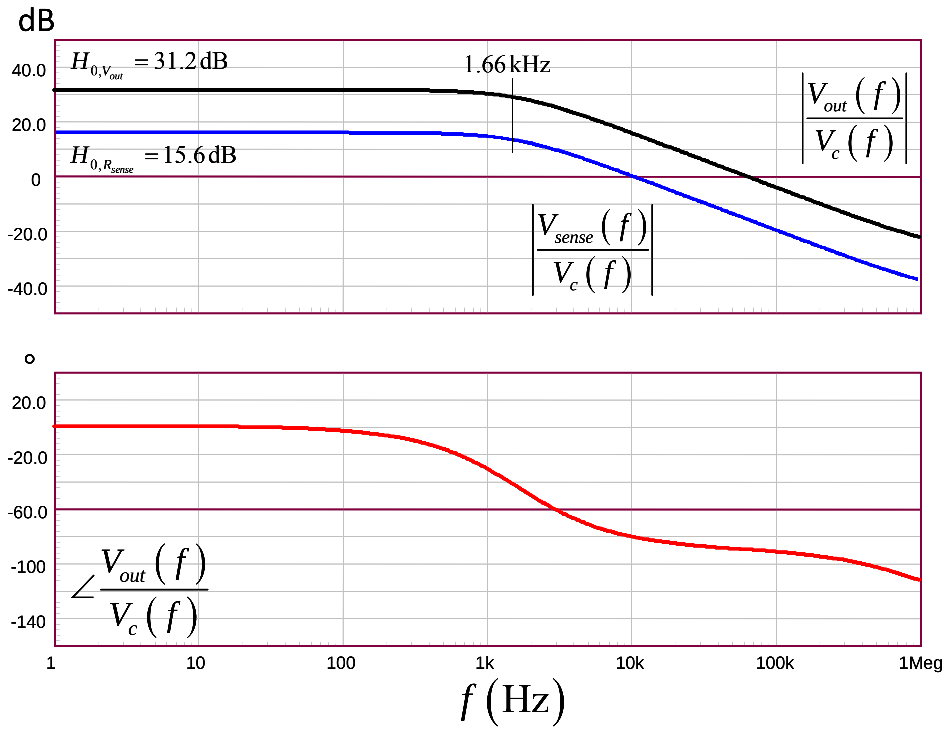

Fig. 9. The Bode plot confirms the dc gain and the pole location.

Conclusion

Here in part I of this article, we have described the derivation of the small-signal response of the boost converter driving an LED string. Rather than implementing the comprehensive small-signal model of the DCM boost converter, a simple equation was derived that describes a first-order response of the LED boost converter operating in discontinuous conduction mode. Despite its inherent limitation to first order, the answer obtained in a few lines is sufficient to stabilize the control loop. In Part II (Practical Considerations), we will delve into an implemented solution and verify empirical results versus the theoretical derivation.

Reference

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About The Authors

Christophe Basso is an application engineering director at ON Semiconductor in Toulouse, France. He has originated numerous integrated circuits among which the NCP120X series has set new standards for low standby power converters. SPICE simulation is also one of his favorite subjects and he has authored two books on the subject. Christophe's latest work is "Designing Control Loops for Linear and Switching Power Supplies: A Tutorial Guide."

Christophe received a BSEE-equivalent from the Montpellier University, France and an MSEE from the Institut National Polytechnique de Toulouse, France. He holds 17 patents on power conversion and often publishes papers in conferences and trade magazines.

Alain Laprade received an MEng in electrical engineering from McGill University in Montréal in 1984. Since then he has worked in power supply development for commercial, telecommunication, automotive, and aerospace applications as an applications engineer. In 2010, he relocated to Rhode Island to join ON Semiconductor's Analog Power Group New Product Development team.

For further reading on the design of LED drivers, see the How2Power Design Guide, select the Advanced Search option, go to Search by Design Guide Category and select "LED Lighting" in the Popular Topics category.