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A Practical Primer On Motor Drives (Part 6): AC Line Power Calculations

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Having discussed measurement of voltages and currents in previous chapters, the discussion now turns to calculation of power as encountered in motor drives. Power is the rate at which energy transfers to a circuit. Units of power consumption are in watts, which equal one joule/second. For resistive loads with single-phase or three-phase applied ac voltage and current, power calculations are relatively straightforward, and power values, stated in watts, reflect "real" consumption.

For inductive or capacitive loads supplied by single-phase or three-phase ac voltages and currents, power calculations are much more complicated and require full understanding to make proper measurements. In this case, "power" is comprised of a "real" and a "reactive" component that are in a quadrature relationship. The quadrature sum of these two components is the "apparent" power.

The "apparent" power is what is required to run the device (e.g., a motor), even though the "real" power is all that is consumed during operation. There is no consumption of the "reactive" power, it is merely transferred within the circuit during each power cycle. High levels of reactive power in relation to real power are undesirable as it results in inefficient power delivery to the load.

This part reviews the relationships between voltage and current in linear and nonlinear loads, relationships between real and reactive power and how these parameters are calculated including the significance of phase angle and power factor. Requirements for sampling of voltage and current waveforms to enable accurate power calculations are also reviewed and measurement examples demonstrate how the power-related parameters are calculated by instruments such as the Motor Drive Analyzer. This section focuses specifically on power calculations in single-phase systems.

Power Consumed by Linear, Resistive Loads

Assume for a purely linear load that a sinewave applied to the load will not be distorted. Furthermore, also assume that the resistive load will consume all the power supplied to it. Here is the formula for calculating power in a resistive circuit for an ac (or a dc) system, with current or voltage expressed in RMS terms:

Power (P) = I^2R, or following Ohm's law, = V^2/R = $V*I$.

In this case, we may compute power consumption for a single cycle in real time on a digitally sampled signal (such as provided by an oscilloscope, power analyzer, or Motor Drive Analyzer) by multiplying the instantaneous voltage and current sampled values over a single sinewave period.

In vector terms, for a purely resistive load the voltage and current vectors for a single-phase system are in phase, and the corresponding line current (I) and line voltage (V) vector magnitudes are simply instantaneously multiplied, as shown in Fig. 1.

Fig. 1 Vector representation of instantaneous voltage and current for a purely linear (reistive) load.

The screen image in Fig. 2 shows the 120-Vac line-neutral waveform captured on channel 1 (C1, yellow) and the line current waveform captured on channel 8 (C8, orange) of the Motor Drive Analyzer when using our standard resistive load (the bread toaster.) Note that the voltage and current signals are in-phase with each other, as would be expected for a resistive load, and the signals are sinusoidal (with some small amount of distortion present in the ac line supply.)

As shown earlier, the voltage signal has the same distortion with or without a load, so it appears that this is a linear load. The RMS voltage and current, power (P, in watts) and power factor (PF) appear at the bottom of the screen image in the Numerics table.

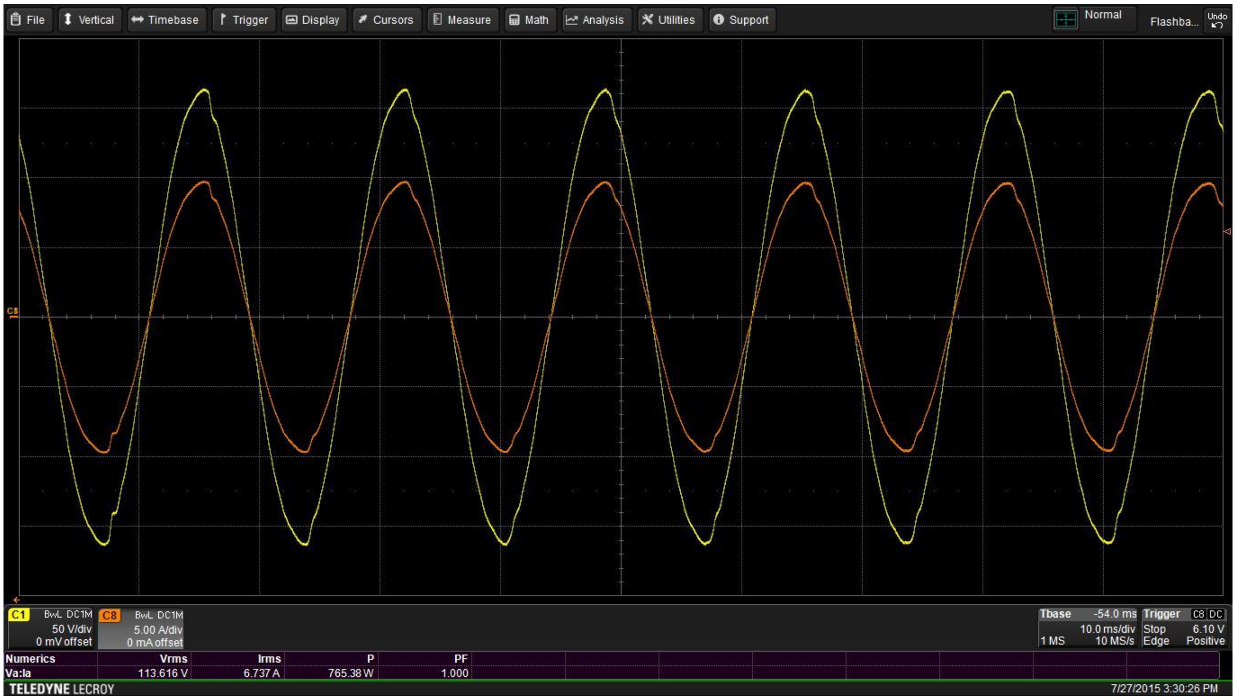

Fig. 2. Line-to-neutral voltage waveform and the associated current waveform for a linear load captured with a Teledyne LeCroy 8-channel, 12-bit Motor Drive Analyzer.

The Numerics table at the bottom of the display in Fig. 2 provides measurement parameters. A closeup of this table is shown in Fig. 3.

Numerics	Vrms	Irms	P	PF
Va:Ia	113.616 V	6.737 A	765.38 W	1.000

Fig. 3. The Numerics table lists key measurement values from the waveforms in Fig. 2.

Power Used (Supplied And Consumed) By Linear, Nonresistive Loads

If the load is inductive or capacitive, power is supplied but is not consumed, and the expression of power becomes more complex because the inductive or capacitive nature of the load causes the current and voltage vectors to be out of phase with respect to each other. Power therefore consists of a "real" (direct, or resistive) component in watts and referred to as "P", and a "reactive" (imaginary, or capacitive/inductive) component in volt-amperes reactive (VAr) and referred to as Q.

The reactive component of power is in quadrature with the real power. There is no consumption of the reactive power, Q, by the load, but rather it is simply transferred from one part of the circuit to another during a single ac period. It is also referred to as the "imaginary" power. The quadratic sum of these two components is the "apparent" power in volt-amperes (VA), referred to as "S". The apparent power S represents the total power required from the ac line for the circuit to work (Fig. 4.)

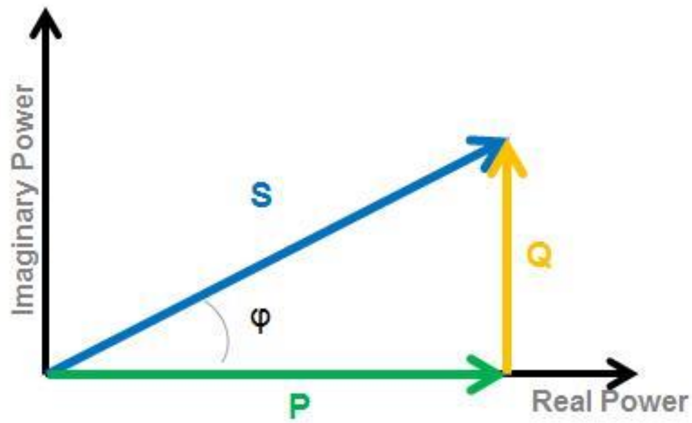

Fig. 4. Vector representation of real, imaginary and apparent power for a nonlinear (inductive or capacitive) load.

The angle between the real power, P, and the apparent power, S, is the phase angle, ϕ, expressed in degrees (-90° to +90°) or radians (-π/2 to + π/2 radians), with a zero value representing a purely resistive load.

We also express phase angle as the power factor (λ, or PF). Power factor is simply the cos(ϕ) for sinusoidal waveforms, and is a unitless value from 0 to 1, specified as leading PF (the current vector leads the voltage vector, as in the case of a capacitive load) or lagging PF (current vector lags the voltage vector, as in the case of an inductive load). See Fig. 5.

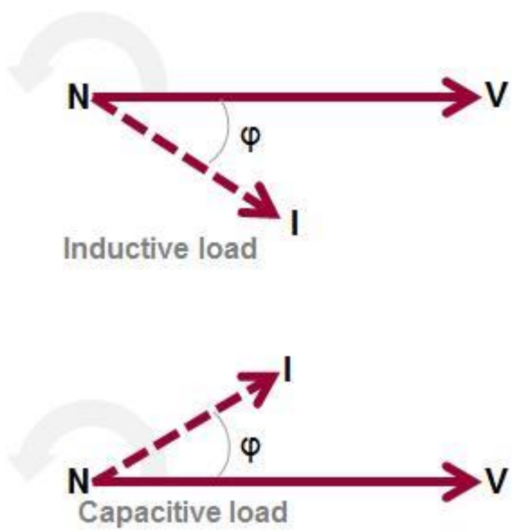

Fig. 5. The phase angle (φ) between voltage and current for a reactive load is the same as the phase angle between the load's real and apparent power and is used to calculate the power factor.

For a pure, single-frequency, sinusoidal waveform (e.g., a utility-supplied ac line voltage waveform with zero distortion), the phase angle is the phase difference between the applied line-to-neutral voltage sinusoid and the line current sinusoid, as shown in Fig. 6.

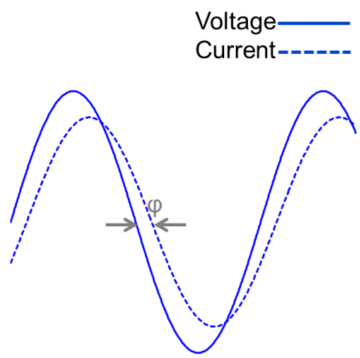

Fig. 6. Current leads voltage by a phase angle φ, indicating a capacitive load.

The phase angle ϕ shown above leads the voltage signal, indicating a capacitive load. By convention, the phase angle is positive if the voltage is leading the current (it would be negative for the example shown above). If we can accurately measure the phase angle, then we may calculate the real, reactive, and apparent power values as follows for a single power frequency cycle:

- Measure Vrms and Irms
- Measure phase angle
- Calculate apparent power (S) using $|S| = Vrms * Irms$
- Calculate real power (P) using $P = S * \cos\varphi$
- Calculate reactive power (Q) using $Q = \sqrt{S^2 - P^2}$.

However, the above is only true for a pure, single-frequency sinusoidal waveform, which is unlikely to be present. However, if the waveform distortion is within normal electric utility guidelines (<5%), it is a reasonable approximation.

Power Used (Supplied And Consumed) By Nonlinear, Nonresistive Loads

More often, loads are not purely resistive and may be nonlinear. Therefore, the voltage or current waveforms are out-of-phase and distorted. Therefore, the simple calculation technique of measuring phase angle between a single-frequency voltage and current sinewave pair does not apply because the distorted waveform is composed of a complex Fourier series of multiple frequencies, each with a unique phase angle for a given harmonic.

Therefore, we employ a different approach based on digital sampling techniques. This technique is quite common given the pervasiveness of high-resolution analog-to-digital converters with high bandwidth (e.g., digital storage oscilloscopes and power analyzers).

After digitally sampling the waveforms, the mathematical calculations are easy. This technique is mathematically correct whether the waveforms are pure sinusoids, distorted sinusoids, pulse-width modulated, or something else. This technique is described in the following steps.

Step 1: Digitally Sample The Waveform

A digital sampling system should oversample the waveform by 10x (or more) to capture enough detail to be able to re-create the analog waveform from the digital samples. The requisite oversampling depends on the harmonic content of the signal—pure, lower-frequency sinewaves (e.g., an ideal utility supply voltage) require less oversampling than a complex PWM waveform.

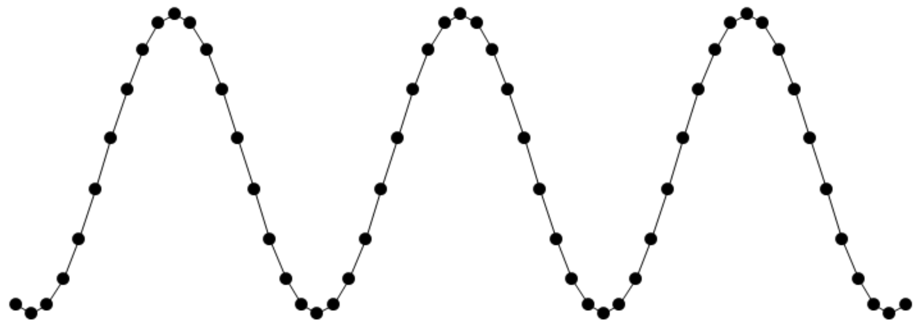

Fig 7. Accurate measurement of voltage and current waveforms using digital techniques, particularly those waveforms associated with nonlinear loads, requires oversampling.

Step 2: Determine The Cyclic Period

Using the digitally sampled data, a complex software algorithm determines a 50% amplitude value for the waveform (the 50% amplitude value equals approximately 0 V for a voltage signal probed line-to-line, or a line current signal). The algorithm then determines a 50% (or zero) crossing point for each individual cycle, and a time measurement for the start and end of each full cycle present in the acquisition, thereby identifying the exact location of the beginning and end of each cyclic period. A software algorithm determines the 50% (zero) crossing point determination with high precision, typically by combining the following measurement techniques:

- User-settable high-frequency filtering via low pass filter cutoff setting
- Localized interpolation/oversampling at the 50% (zero) crossing point
- Elimination or minimization of the effects of perturbations or non-monotonicities at the 50% (zero) crossing point with a user-defined hysteresis band control.

In the case of a pure, single-frequency sinewave (as shown in Fig. 8), this is a relatively straightforward process and default settings are likely sufficient. For a more complex, distorted PWM waveform, advanced settings may be required to avoid detection of false periods. In both cases, one should visually review the accuracy of the zero-crossing point detection (on the measurement instrument) because the power calculation accuracy depends on it.

The image in Fig. 8 depicts the detection of two full cyclic periods in this short acquisition:

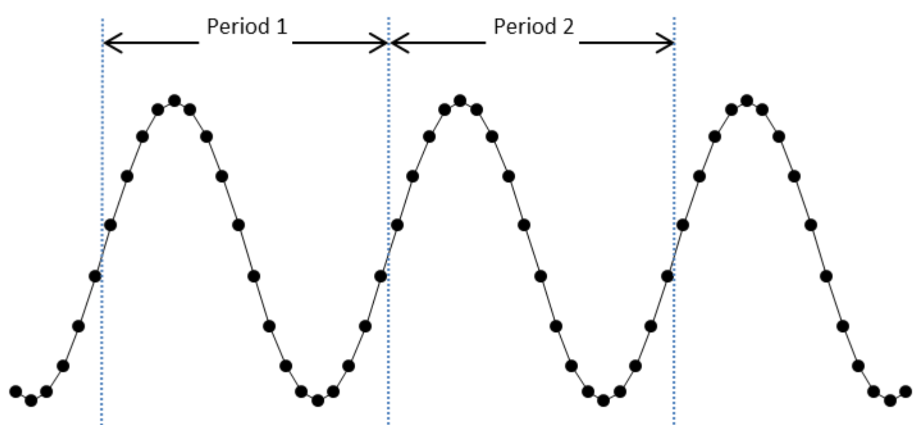

Fig 8. Determining 50% or zero-crossing points is easiest in the case of a pure sinewave.

Step 3: Determine The Digital Sample Point Set In Each Detected Cyclic Period

Given the locations of the detected cyclic periods, the sample point set can easily be determined for each cycle (Fig. 9.) In this figure, there are ***N*** periods and ***N*** = 2. For a given cyclic period index ***i***, the digitally sampled waveform is represented as having a set of sample points ***j*** in cyclic period index ***i***, with $\boldsymbol{M_i}$ sample points beginning at $\boldsymbol{m_i}$ and continuing through $\boldsymbol{m_i + M_i - 1}$. For example, period 1 is cyclic period index ***i = 1***. There is a set of sample points ***j*** in cyclic period 1, beginning with point 7 and ending with point 24.

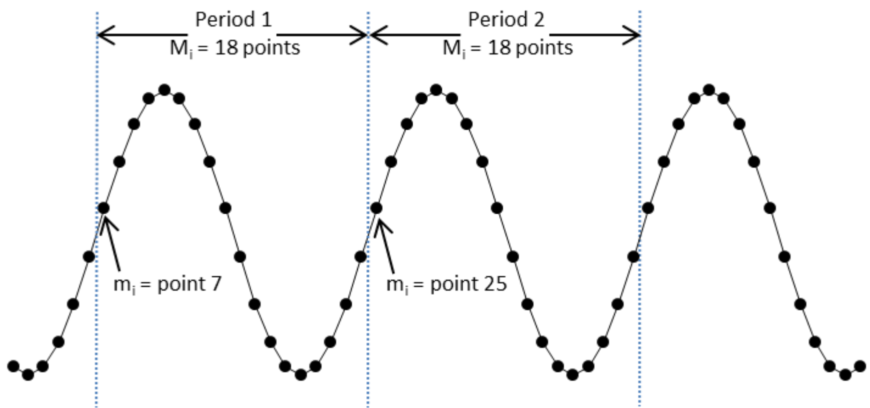

*Fig 9. For a given cyclic period index **i**, the digitally sampled waveform is represented as having a set of sample points **j** in cyclic period index **i**, with $\boldsymbol{M_i}$ sample points.*

Step 4: Calculations

Once the sample point sets are determined, we make all cyclic period voltage, current and power calculations with the appropriate set of points using the equations in the table.

Table. Equations for calculating voltage and power of sampled waveforms.

	Per-cycle Value Calculations	Mean Value Calculations
V_{RMS}	$Vrms_i = \sqrt{\frac{1}{M_i}\sum_{j=m_i}^{m_i+M_i-1} V_j^2}$	$Vrms = \frac{1}{N}\sum_{i=1}^{N} Vrms_i$
I_{RMS}	$Irms_i = \sqrt{\frac{1}{M_i}\sum_{j=m_i}^{m_i+M_i-1} I_j^2}$	$Irms = \frac{1}{N}\sum_{i=1}^{N} Irms_i$
Real power (P, in Watts)	$P_i = \frac{1}{M_i}\sum_{j=m_i}^{m_i+M_i-1} V_j * I_j$	$P = \frac{1}{N}\sum_{i=1}^{N} P_i$
Apparent power (S, in VA)	$S_i = Vrms_i * Irms_i$	$S = \frac{1}{N}\sum_{i=1}^{N} S_i$
Reactive power (Q, in VAr)	$magnitude\ Q_i = \sqrt{S_i^2 - P_i^2}$ *sign of Q_i is positive if the fundamental voltage vector leads the fundamental current vector*	$Q = \frac{1}{N}\sum_{i=1}^{N} Q_i$
Power factor (PF or λ)	$\lambda_i = \frac{P_i}{S_i}$	$\lambda = \frac{1}{N}\sum_{i=1}^{N} \lambda_i$
Phase angle (ϕ or φ)	$magnitude\ \phi_i = \cos^{-1}\lambda_i$ *sign of ϕ_i is positive if the fundamental voltage vector leads the fundamental current vector*	$\phi = \frac{1}{N}\sum_{i=1}^{N} \phi_i$

Example

Let's revisit the previous example of the single-phase standard resistive load (the bread toaster.)

In Fig. 10, the 120-Vac line-neutral waveform is captured on channel 1 (C1, yellow, top grid) and the line current waveform is captured on channel 8 (C8, orange, top grid) of the Motor Drive Analyzer. The bottom grid shows a 20,000:1 zoom waveform on channel 1 (Z1, yellow, bottom grid) and a 20,000:1 zoom waveform on channel 8 (Z8, orange, bottom grid). These two zoom traces are time-synchronized with each other.

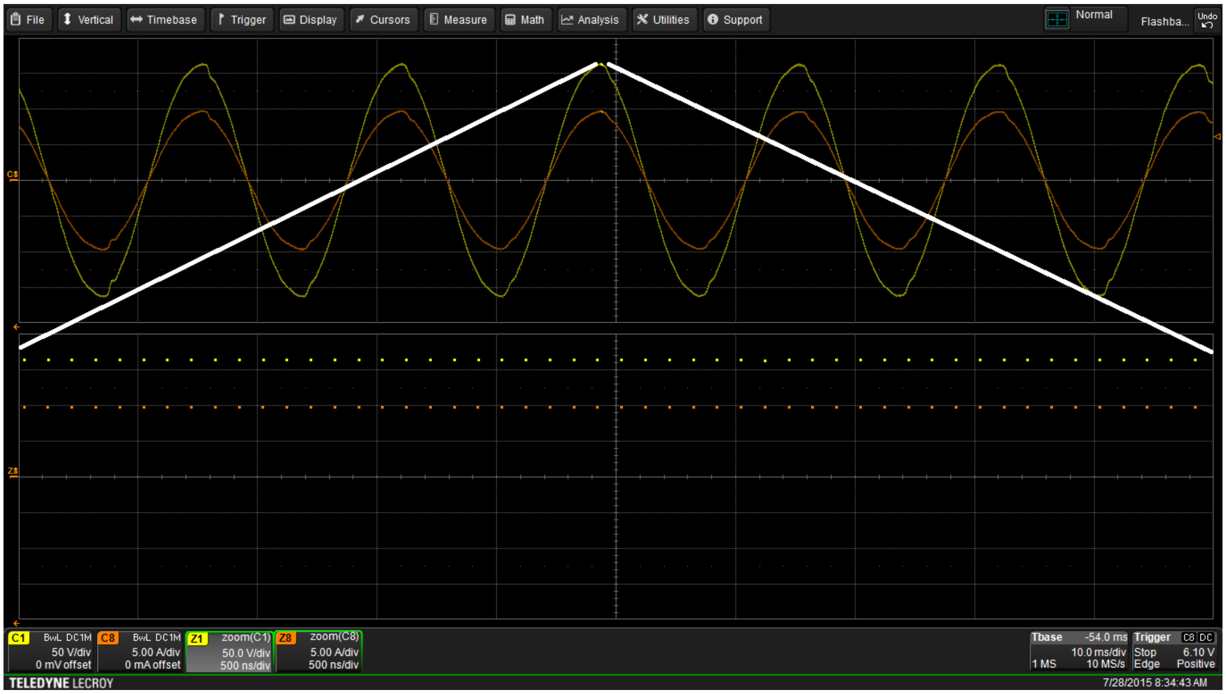

Fig. 10. Line-to-neutral voltage waveform (yellow) and the associated current waveform (orange) for a linear load are shown on channels 1 and 8 at the top of the display. Then, in the bottom of the display, small segments of these waveforms are magnified in the time base to show the sampled points that comprise the waveforms.

We add the white lines to the image to draw attention to the small part of the original waveform shown in the zoom traces. Note that in the Motor Drive Analyzer display settings, we have turned "off" the normal line connection between points on the waveforms to make it easier to see the distinct sample points. Also, note that the sample points are spaced 100 ns apart—the time scale of the Zx traces is 500 ns/div, and there are five points per division.

This is to be expected given that the sample rate for the full acquisition is 10 MS/s, and therefore the distance between points is 1/(10 MS/s) = 100 ns. This sample rate setting can be changed by the user to be lower (into the kS/s range) or higher (e.g., 2.5 GS/s), depending on the frequency content of the signal and the desired acquisition duration.

The screen image in Fig. 11 shows the zoom traces turned off and replaced by the 120-Vac line-to-neutral waveform (C1) with application of a 500-Hz low-pass filter— this is known as the "sync" signal. The zero-crossing algorithm has determined the cyclic periods and displayed them with a color-coded annotation on top of the waveform. The annotation indicates a good determination of the cyclic period as well as detection of five full periods. If necessary (as would often be the case with PWM waveforms), the hysteresis setting could be adjusted to aid in the zero-crossing determination.

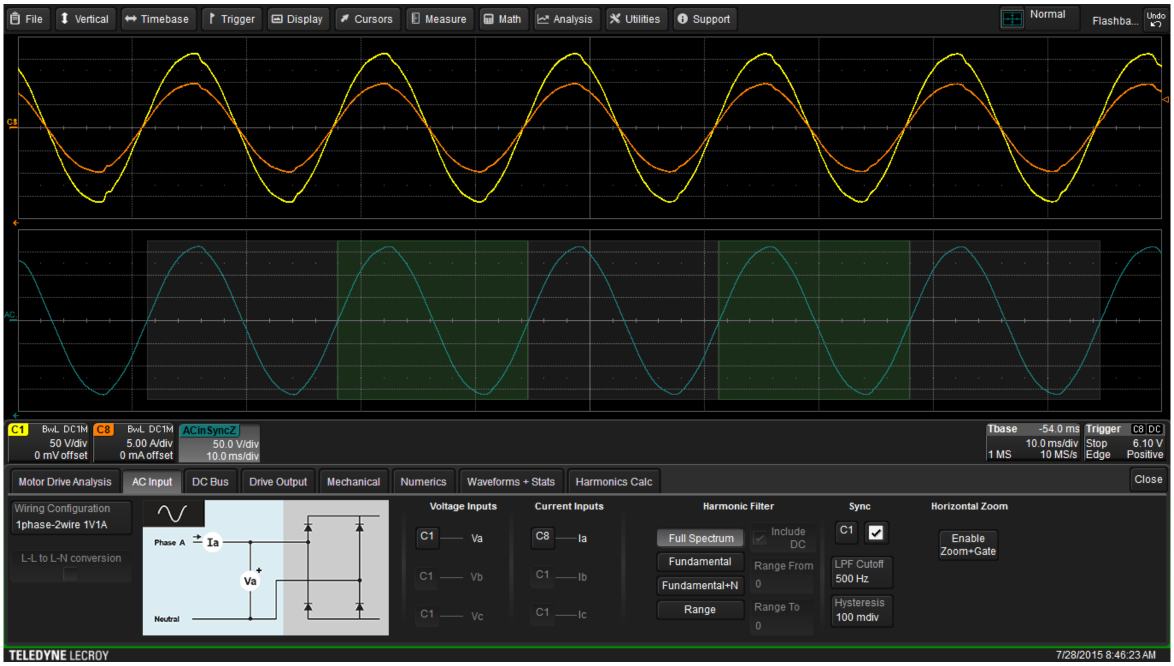

Fig 11. Line-to-neutral voltage waveform and the associated current waveform for a linear load are again shown on channels 1 and 8 at the top of the display. But the lower waveform is a low-pass filtered version of the voltage waveform, which serves as the sync signal.

Now, in the image below (Fig. 12), we have turned off the sync signal and we again display the Numerics calculation table, this time showing apparent and reactive power and phase angle in addition to real power and power factor. An enlarged version of the Numerics table appears in Fig. 13.

Fig 12. Line-to-neutral voltage waveform and the associated current waveform for a linear load are shown with the Numerics calculation table, which displays various power-related parameters for these waveforms.

Numerics	Vrms	Irms	P	S	Q	PF	Φ
Va:Ia	113.616 V	6.737 A	765.38 W	765.39 VA	-2.38 VAR	1.000	-177.9 m°

Fig 13. A close-up of the Numerics table shows the values calculated by the MDA for apparent and reactive power, real power, phase angle and power factor.

Note that the phase angle is not zero—it is a very small number that is a round-off error in terms of calculating the power factor, but it would seem to indicate that the load is slightly capacitive. We can test this assumption by calculating the power measurements on the fundamental only instead of the full spectrum of the waveform (as set in the Harmonic Filter dialog box in the screen image in Fig. 14.)

With the Harmonic Filter set to measure power based on the 60-Hz fundamental waveform only, the phase angle is essentially zero and the real power and the apparent power are equal (Fig. 15.) The visible distortion in the waveforms does have some small measurement impact in this case. The data shows that the bread toaster coils are not purely resistive, but have some complex impedance that behaves capacitively at the higher frequencies present in the slightly distorted voltage waveform.

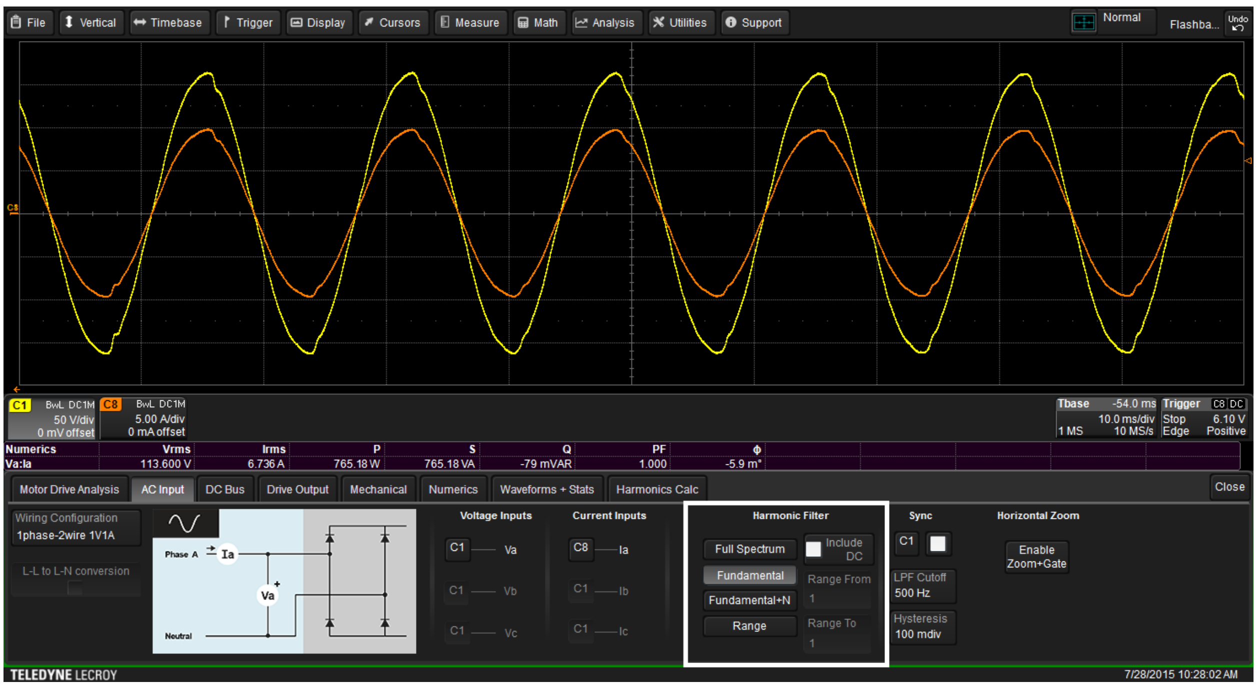

Fig 14. Voltage and current waveforms with power measured for just the 60-Hz fundamental. Apparent and real power are equal and phase angle is zero. This suggests that the visible distortion in the waveforms is due to some complex impedance in the load at higher frequencies.

Numerics	Vrms	Irms	P	S	Q	PF	Φ
Va:Ia	113.600 V	6.736 A	765.18 W	765.18 VA	-79 mVAR	1.000	-5.9 m°

Fig 15. A closeup of the Numerics table in Fig 14.

Further harmonic order analysis reveals significant odd harmonic distortion in the voltage and current signals. Fig. 16 below provides harmonic order calculations in percentages and spectral displays to the right (because it is always 100%, the fundamental is excluded from the spectral display).

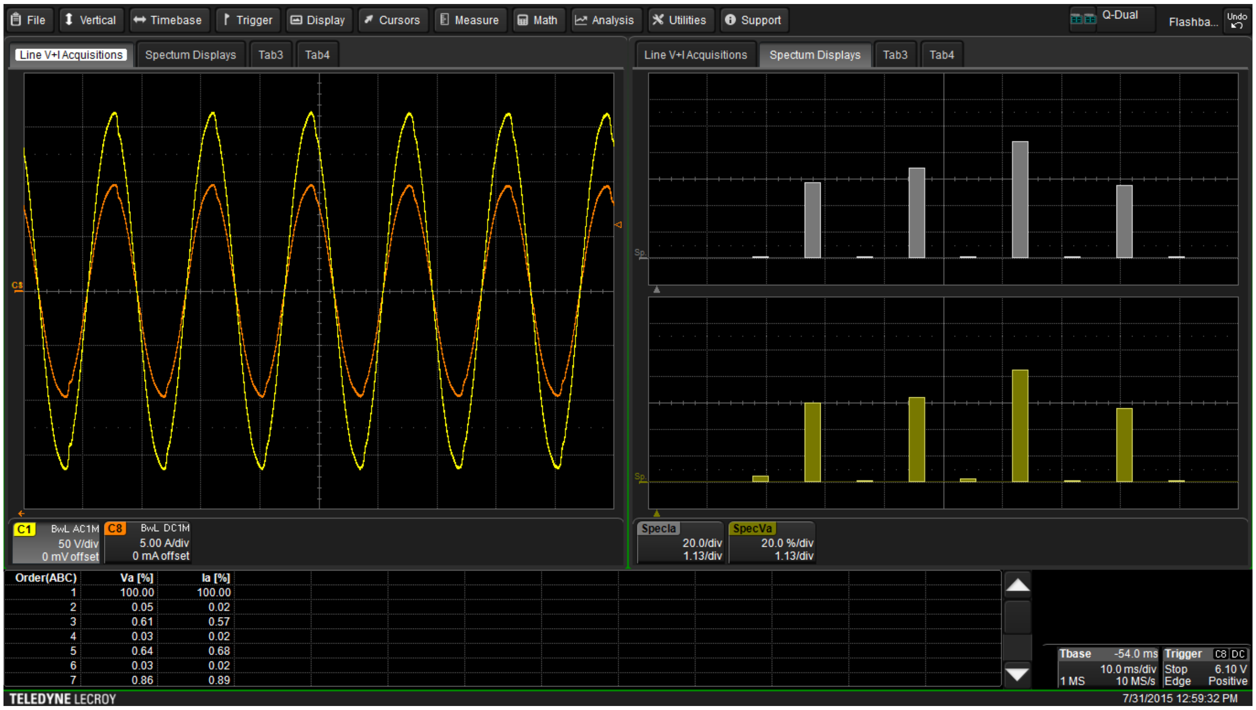

Fig 16. Harmonic order calculations (as percentages of the fundamental) and spectral displays for the voltage and current waveforms.

By changing the scaling in the Harmonic Order table from percentages to volts and amps (Fig. 17), we learn that the value of the fundamental matches the calculated value from Figs. 14 and 15.

Order(ABC)	Va [V]	Ia [A]
1	113.60	6.74
2	0.06	0.00
3	0.69	0.04
4	0.03	0.00
5	0.73	0.05
6	0.03	0.00
7	0.97	0.06

Fig 17. The Motor Drive Analyzer can also display the values of the fundamental and the harmonics in volts and amps.

Single-Phase Wiring Configurations

As described previously, a single-phase system could contain either two wires (a single current-carrying conductor and a neutral return path, with the voltage referenced to neutral) or three wires (two current-carrying conductors with two voltages, each referenced to the neutral return path). In both cases, voltage is sensed line-to-neutral.

One-Phase, Two-Wire Systems (One Voltage, One Current)

The example below from Teledyne LeCroy's Motor Drive (Power) Analyzer (MDA) in Fig. 18 shows the connection schematic diagram indicating the electric utility single-phase ac supply input to a motor drive, represented schematically as a rectifier circuit. The current (Ia) is represented as flowing into the load, and the voltage is probed line-to-neutral (Va).

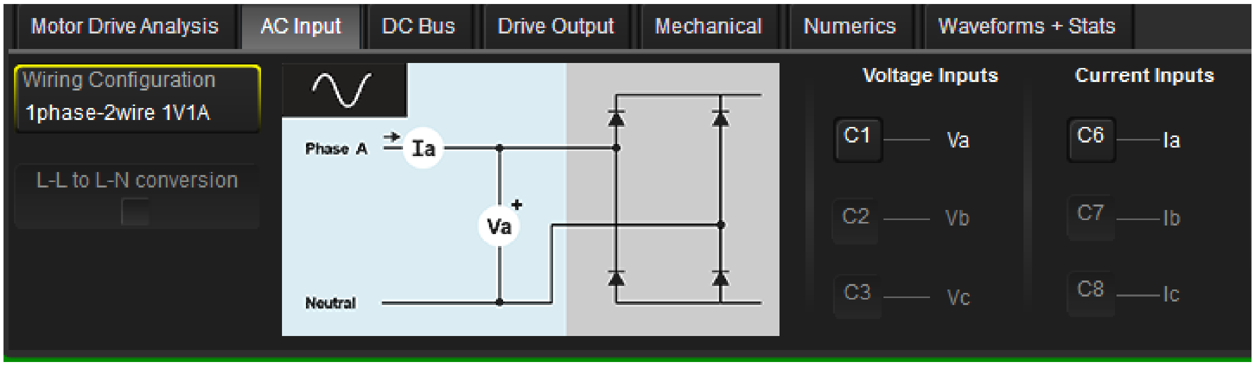

Fig 18. To aid motor drive analysis, users can designate the connection of the ac supply input to a motor drive, representing the motor drive as a rectifier circuit. In this case, a single-phase, two wire system feeds power to the motor drive.

One-Phase, Three-Wire Systems (Two Voltages, Two Currents)

The next example in Fig. 19 shows the connection schematic diagram indicating the electric utility single-phase ac supply input to a motor drive, represented schematically as a rectifier circuit. The two currents (Ia and Ib) are represented as flowing into the load, and the two voltages are probed line-to-neutral (Va and Vb).

Note the "+" designation on the Va and Vb elements indicating the polarity of the voltage measurements made at these points (both line-to-neutral). The total power measurements for the complete system are simply the sum of the separately measured A and B lines.

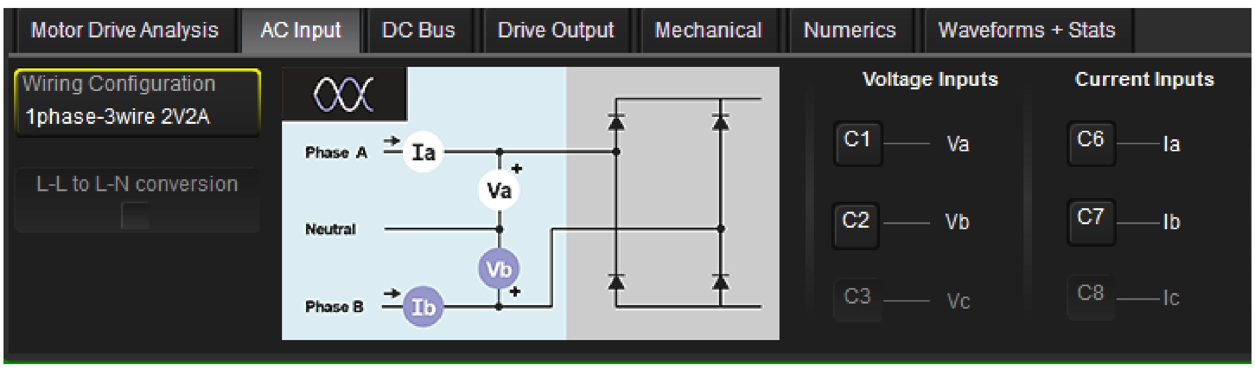

Fig 19. Here the Motor Drive Analzyer is configured to show the connection of the ac supply input to a motor drive, where a single-phase, three-wire system is used.

Conclusion

This section reviewed the basics of line power calculations in single-phase ac systems in applications with linear as well as nonlinear loads. The use of the Motor Drive Analyzer to perform these calculations was demonstrated. The upcoming part 7 extends this discussion to three-phase ac systems. For a full list of topics that will be addressed in this series, see part 1.

About The Author

Kenneth Johnson is a director of marketing and product architect at Teledyne LeCroy. He began his career in the field of high voltage test and measurement at Hipotronics, with a focus on <69-kV electrical apparatus ac, dc and impulse testing with a particular focus on testing of transformers, induction motors and generators. In 2000, Ken joined Teledyne LeCroy as a product manager and has managed a wide range of oscilloscope, serial data protocol and probe products. He has three patents in the area of simultaneous physical layer and protocol *analysis. His current focus is in the fields of power electronics and motor drive test solutions, and works primarily in a technical marketing role as a product architect for new solution sets in this area. Ken holds a B.S.E.E. from Rensselaer Polytechnic Institute.*

For further reading on motor drives, see the How2Power Design Guide, locate the "Power Supply Function" category, and click on the "Motor drives" link.