Eddy-Current Effects In Magnetic Design (Part 1): The Skin Effect

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One of the more troublesome aspects of transductor (magnetic component) design is the winding design—the electrical aspect of transductor design. What complicates it the most are eddy-current effects: the skin and proximity effects. The goal of this six-part article series—a mini-course—is to explain and clarify eddy-current effects, provide some useful graphs and design equations, and explain how to use them in winding design for a more optimized transductor. This material is adapted from a book by the author, Power Magnetics Design Optimization, available at www.innovatia.com. Part 1 of this series explains the origin of the skin effect and analyzes some key expressions related to it that will aid designers in the sizing of wires for inductor and transformer designs.

Cause Of The Skin Effect

The skin effect is the crowding of a varying current to the periphery of a conductor. A varying current in a conductor such as a wire will induce into the wire a magnetic field that affects its own radial distribution of current and hence current density. As shown in Fig. 1, current flowing in a wire down the page will produce a B-field.

Fig. 1. Eddy currents are generated by a varying current, i, that produces a varying B-field into and out of page where the head of the vector (dot) is coming out of the page and the tail (x) is going into the page. B causes eddy currents, i_B, to flow in loops; current on the inside part of the loops oppose the original current, subtracting from it.

By Faraday’s (and Lenz’s) Law, the field flux, \( \phi \), of B will produce a voltage, \( v \).

\[
  v = -\frac{d\phi}{dt}
\]

The negative sign (according to the vector right-hand rule) indicates that this voltage will generate an eddy current that opposes the source current and subtracts from it.

As shown in Fig. 1, the currents, \( i_B \), from the induced-voltage loops of B oppose the source current, \( i \), at the center of the wire. Then \( i \) is affected by the total current, which can be decomposed conceptually into “filaments” of current distributed along the radial dimension \( r \), where \( r = 0 \) at the surface and \( r = r_c \) at the center; \( r_c \) is the conductor radius.
Outer filaments are not linked (as thoroughly) by the $B$-field generated by the inner filaments. Consequently, more opposing induced current flows at the center and subtracts from the center source current the most. As also shown, the outer currents aid the source current. The result is that most of the net source current flows near the surface (or “skin”) of the conductor; hence the name skin effect.

The radial current density exponentially decreases as the center is approached from the surface of the wire where the value of current density is highest. The current density, $j$, normalized by the static current, $I$, density, $J = I/A_c$, is

$$\frac{j(r)}{J} = e^{-\frac{r}{\delta}}$$

where $\delta$ is the skin depth, a characteristic length (length constant) and $A_c$ is wire conductive area. The skin depth varies inversely with the frequency of the current. From the Helmholtz diffusion equation of fields theory, skin depth is

$$\delta = \frac{1}{\sqrt{\mu \cdot \sigma \cdot \pi \cdot f}} = \sqrt{\frac{\rho}{\mu \cdot \pi \cdot f}}$$

where $\mu$ is the permeability of the conductor material, $\rho$ is its electrical resistivity (conductivity, $\sigma = 1/\rho$), and $f$ is frequency in Hz. The total amount of current is equal to that of a static current flowing in a ring (annulus or shell - the “skin”) of thickness $\delta$ between $r = 0$ (at the conductor surface) and $r = \delta$. The distribution and the equal current area are shown in Fig. 2, for which $\delta = (0.2) \cdot r_c$.

![Fig. 2. Current density $j(r)/J$ as a function of the normalized radius $r/r_c$. The area under the exponential curve is the current at a given skin depth (in this case, $\delta = (0.2) \cdot r_c$). It has the same area as the shaded area representing the static current flowing uniformly at the circumference of the wire to a depth of $\delta$.](image)

The skin depth at 80°C for copper (Cu) and aluminum (Al) simplify to the following working formulas when the materials’ parameters are substituted.

$$\delta_{Cu} = \sqrt{\frac{2.131 \mu \Omega \cdot \text{cm}}{1.257 \mu \text{H/m} \cdot \pi \cdot f}} = \frac{73.46 \text{ mm}}{\sqrt{f / \text{Hz}}} \approx \frac{73.5 \text{ mm}}{\sqrt{f / \text{Hz}}}$$

$$\delta_{Al} = \sqrt{\frac{3.487 \mu \Omega \cdot \text{cm}}{1.257 \mu \text{H/m} \cdot \pi \cdot f}} = \frac{93.97 \text{ mm}}{\sqrt{f / \text{Hz}}} \approx \frac{94 \text{ mm}}{\sqrt{f / \text{Hz}}}$$

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At the same frequency, Al skin depth is greater than Cu though its static resistivity is higher. Consequently, more of the Al cross-section is used than for Cu so that for a certain range of frequencies and with sufficient layers, Al wire can have lower winding resistance than Cu wire for the same cross-sectional area and frequency.

For etched-circuit-boards (ECBs), a 1 oz/ft² (35 μm thick) board trace has δ equal to the plating thickness (δ = h) at 60°C at a frequency of f = 4.3 MHz. The surface resistivity, ρ·(h·w)/l, is 530 μΩ/Ω where l is a square (l/w = 1) of the conductive surface.

Penetration Ratio, \( \xi_r \)

In the Fig. 2 graph, the area in the hatched rectangle of width \( \delta/r_c \) is the same as that under the exponential curve.

\[
\int_{0}^{\infty} e^{-i\delta} \cdot dr = -\delta \cdot e^{-i\delta}\bigg|_{0}^{\infty} = \delta
\]

The skin effect causes the wire-equivalent static cross-sectional area to be that of a ring at the surface of \( \delta \) thickness. Thus the skin-effect resistance, \( R_{w\delta} \), of the varying (ac) current varies inversely with \( \delta(f) \) and \( r_c \) whereas the static (dc) resistance, \( R_{w0} \), varies inversely with \( r_c^2 \). As wire circumference increases with \( r_c \), \( R_{w\delta} \) decreases.

Whereas \( R_{w0} \) applies to the whole conductor cross-sectional area, \( \pi \cdot r_c^2 \), \( R_{w\delta} \) effectively applies to the ring of area \( [2\cdot\pi\cdot(r - \delta/2)]\cdot\delta \). \( R_{w\delta} \), is not a reactance but is nevertheless a dynamic (time-dependent) resistance. It does not affect the phase of the source current but depends on its frequency.

The skin effect in a round wire of radius \( r_c \) is expressed as the round-wire penetration ratio, \( \xi_r \), which is the conductor radius normalized to skin depth.

\[
\xi_r = \frac{r_c}{\delta}
\]

Just like \( \delta \), \( \xi_r \) is a characteristic length or “length constant”. It is to length what time is to a circuit time constant and is the constant in the exponential function.

Because of the skin effect, wire size for switching converters cannot simply be chosen for ampacity: static current-carrying capacity. Instead, multiple strands of smaller wire with the combined ampacity of one larger wire each have a smaller \( \xi_r \) and conductive utilization of their area is effectively increased.

If the equivalent of the total current flows at a depth of \( \delta \), then if the wire radius is chosen to be \( r_c = \delta \), or \( \xi_r = 1 \), its conductive area is ~ 63% at the given frequency of current. Because current distribution is exponential with a tail going to infinity and not a finite \( r_c \), the utilization is not exactly one, though \( \xi_r = 1 \) is an approximate optimization criterion for isolated (single) wires.

The skin-depth frequency, \( f_\delta \), at which \( \delta = r_c \) is found by solving the skin-depth equation for \( f \).

\[
f_\delta = \left(\frac{73.5 \text{mm}}{r_c}\right)^2 \cdot \text{Hz}, \text{ Cu, 80°C}
\]

\[
f_\delta = \left(\frac{94 \text{mm}}{r_c}\right)^2 \cdot \text{Hz}, \text{ Al, 80°C}
\]

A #28 AWG, Cu wire has \( r_c = 0.16 \text{ mm} \) and \( f_\delta = 211 \text{ kHz} \) at 80°C.

The \( f_\delta \) values are useful additions to a wire table. Given \( f_\delta \) for each wire size and knowing the fundamental frequency \( f \) of the current, then \( \delta \) is proportional to \( 1/\sqrt{f} \) and in general, the penetration ratio can be expressed as a function of \( f_\delta \).
\[ \xi_r = \frac{f}{f_\delta} \]

(In a wire table, \( f_\delta \) is based on \( r_c \) for round wire; hence the subscript \( r \) of \( \xi_r \).) Current in a #24 AWG Cu wire at \( f = 150 \text{ kHz} \) has \( f_\delta = 81.15 \text{ kHz} \) and

\[ \xi_r = \frac{150 \text{ kHz}}{81.15 \text{ kHz}} = 1.86 \]

**Resistance Factor, \( F_R \)**

The static-resistance multiplier that accounts for the increased resistance from eddy-current effects is \( F_R \). An expression for \( F_R \) can be derived for an isolated (single) round wire as the following ratio.

\[ F_{rw} = \frac{R_w(f)}{R_{wo}} = \frac{\rho \cdot l / A_r(f)}{\rho \cdot l / A_0} = \frac{A_0}{A_r(f)} \]

At 0 Hz, current is uniformly distributed in the wire having an area expressed in terms of \( r_c \).

\[ A_0 = \pi \cdot r_c^2 \]

At frequency \( f \), the conductor is reduced effectively to that of a ring of thickness \( \delta \) with the following cross-sectional area.

\[ A_r(f) = \pi \cdot r_c^2 - \pi \cdot (r_c - \delta)^2 = 2 \cdot \pi \left( r_c - \frac{\delta}{2} \right) \cdot \delta = \pi \cdot \left( 2 \cdot r_c \cdot \delta - \delta^2 \right) \]

For physical significance, \( \delta \leq r_c \) and \( \xi_r \geq 1 \). The exact \( F_{rw}(\xi_r) \) is derived from fields theory. Based on a skin-depth ring of uniform current over its width, we can derive the following approximation for \( F_{rw} \).

\[ F_{rw} \approx \frac{A_0}{A_r(f)} = \frac{r_c^2}{2 \cdot r_c \cdot \delta - \delta^2} = \frac{\left( \frac{r_c}{\delta} \right)^2}{2 \cdot \left( \frac{r_c}{\delta} \right)} - 1 = \frac{\xi_r^2}{2 \cdot \xi_r - 1} \]

The change of \( F_{rw} \) with a change in \( \xi_r \) is found by taking the derivative of \( F_{rw} \).

\[ \frac{dF_{rw}}{d\xi_r} \approx \frac{2 \cdot \left( 1 - \frac{1}{\xi_r} \right)}{\left( 2 - \frac{1}{\xi_r} \right)^2} \Rightarrow \frac{dF_{rw}}{d\xi_r}(\xi_r \to \infty) = \frac{1}{2} \]

As \( \xi_r \) becomes large, the plot of \( F_{rw} \) approaches an asymptotic line with a slope of \( \gamma_2 \); \( F_{rw}(\xi_r \to \infty) = \frac{\xi_r}{2} \).

The asymptote, \( F_{rw}(\infty) \), is plotted as a dotted line against \( \xi_r \) on a log-log plot in Fig. 3 with some tabulated values. The approximated \( F_{rw} \) is minimum for \( \xi_r = 1 \); \( F_{rw}(1) \approx 1 \).
The skin-effect $F_{RW}$ of an isolated single wire can approximated as follows.

$$F_{RW}(\xi_r) \approx \frac{\xi_r^2}{2 \cdot \xi_r - 1} \approx \frac{r_c}{2 \cdot \delta}$$

Thus the skin effect increases as $\xi_r$ increases. As frequency decreases, $\delta$ increases monotonically until the resistance is the static resistance at 0 Hz. For $\xi_r < 1$, $\delta$ exceeds the radius of the wire.

The increase in $F_{RW}$ for $\xi_r < 1$ expands the shell of current beyond the center of the core itself in the approximation, though the exact $F_{RW}$ continues to reduce. The $F_{RW}$ equation was derived on the assumption of uniform current in the shell but it actually decreases exponentially toward the center. Because it is not uniform in density as was assumed, the exact $F_{RW}$ for decreasing $\xi_r$ asymptotically decreases toward one and is the solution of a modified Bessel's equation. The current density at the center of a round conductor spans its range from all to none over the range of about $1 \leq \xi_r \leq 10$ from this approximation.

### Strands Reduce Skin Effect

Wires bundled together as a single, collective conductor are called *strands*. To demonstrate that strands of reduced wire size can increase conduction and decrease $F_{RW}$, consider the simple case of comparing a wire of conductive radius $r_c$ and area $A_{c1}$ with two wires, each of area $A_{c2} = A_{c1}/2$ and radius $r_c / \sqrt{2}$. The two smaller wire strands have the same total static conductive area as the larger wire.

$$\pi \cdot r_c^2 = 2 \left( \pi \cdot \left( \frac{r_c}{\sqrt{2}} \right)^2 \right)$$
The effective areas with skin depth $\delta$ are calculated for $A_{c1}$ and $A_{c2}$.

$$A_{c1} = \pi \cdot r_c^2 - \pi \cdot (r_c - \delta)^2 = \pi \cdot (2 \cdot r_c - \delta) \cdot \delta$$

$$A_{c2} = 2 \cdot \pi \cdot \left[ \left( \frac{r_c}{\sqrt{2}} \right)^2 - \left( \frac{r_c - \delta}{\sqrt{2}} \right)^2 \right] = 2 \cdot \pi \cdot \left( \sqrt{2} \cdot r_c - \delta \right) \cdot \delta$$

A larger cross-sectional area has lower resistance. To compare, divide out $\delta^2$ from each, and the area ratio reduces as follows.

$$\frac{A_{c2}}{A_{c1}} = \frac{2 \cdot (\sqrt{2} \cdot \xi_r - 1)}{2 \cdot \xi_r - 1}, \quad \xi_r = \frac{r_c}{\delta}$$

A ratio of greater than one for $r_c/\delta$ favors the smaller wires. At $f_0 (r_c)$, $\xi_r = 1$, the ratio is about 0.828, and the larger wire has lower resistance. Crossover is at $\xi_r = 1/2 \cdot (\sqrt{2} - 1) \approx 1/0.828 \approx 1.207$ where $F_{RW} \approx 1.030$. For increasing $f_0$, $\delta$ decreases, $\xi_r$ increases, and the smaller wires have lower resistance. A lower limit on resistance for reduced wire size is reached because wire packing factor, $k_p =$ conductive area/winding area, decreases sublinearly with $r_c$ from an increase in number of strands.

In Part 2, we consider the proximity effect and how it differs from the skin effect.

**About The Author**

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