ISSUE: August 2016

A Practical Primer On Motor Drives (Part 7): Power Calculations In Three-Phase Systems

by Ken Johnson, Teledyne LeCroy, Chestnut Ridge, N.Y.

The previous installment in this series explained the various power calculations associated with single-phase ac systems and discussed voltage and current measurement requirements to enable accurate calculation of power-related parameters. In this part, the discussion moves on to power calculations in three-phase systems as encountered in motor drive applications.

After reviewing the basics of calculating power values in systems with resistive and non-resistive loads, techniques are explained for measuring power given various three-phase system configurations including four-wire and three-wire wye-connected systems, as well as three-wire delta-connected systems. Both line-to-neutral and line-to-line measurements techniques are discussed as they apply to these configurations as is the conversion of line-to-line voltage measurements to their line-to-neutral equivalents for the purpose of calculating power per phase.

Both the two-wattmeter and three-wattmeter methods of measuring power are explained. The former demand measurement of three voltages and three currents, while the latter require that only two voltages and two currents be measured. As in previous parts, the Motor Drive Analyzer is used to demonstrate the various measurement techniques on an example motor drive and how the MDA's built-in functions automate the power calculations described.

Power Calculations In Three-Phase Systems

Extending the power calculations from a single-phase system to a three-phase system is straightforward. In the simplest case for a three-phase ac system, the neutral is present (represented as N in Fig.1), voltages are sensed line-neutral, and the line currents are measured. If the load is purely resistive, the three-phase line current and voltage vector magnitudes would each be instantaneously multiplied and summed to get the total three-phase power, as shown in the figure.

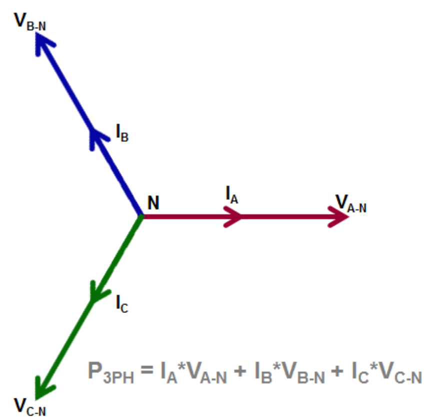

Fig. 1. Vector representation of voltage and current in a three-phase system with a purely resistive load. Magntitudes of currents and voltages can be multipled and summed to calculate three-phase power.

However, in the case of power consumed by and supplied to non-resistive loads, the phase angle complicates matters. Fig. 2 shows an inductive load with current lagging voltage:

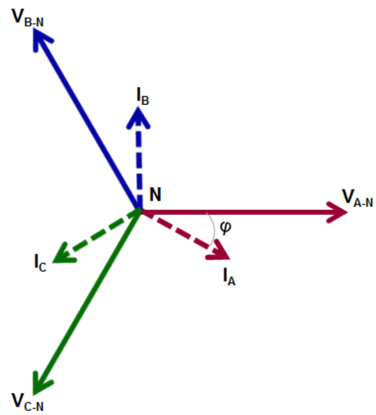

Fig. 2. Vector representation of voltage and current in a three-phase system with a non-resistive (inductive) load. In this case, calculation of three-phase power becomes more complex.

In this case, real (P), reactive (Q), and apparent (S) power must be computed on each phase and summed to get the total three-phase power, as shown in Fig. 3.

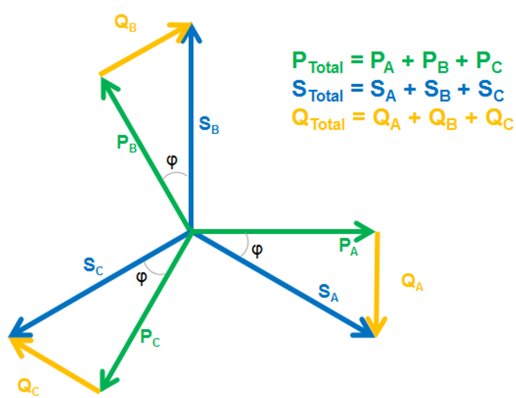

Fig. 3. In a three-phase system with a non-resistive load, the three forms of power (real, reactive and apparent) must be calculated for each phase and then summed.

The above cases show various power calculations based on an accessible neutral for voltage reference. However, the neutral may not be accessible, requiring line-to-line voltage sensing, or line-to-line voltage sensing may be preferred. Line-to-neutral voltage sensing is not possible on a delta winding because there is no neutral accessible.

Additionally, there are ways to perform total three-phase power calculations by using only two of the three line-to-line voltages and two of the three line currents (four signals total). Therefore, fewer signals are required, we conserve valuable inputs on measurement instruments, and other measurements may become possible (such as efficiencies or cross-correlation to other system signals.)

Thus, there are several different techniques for measuring power in a three-phase system, and it pays to understand them and the results they provide. However, no technique is inherently better or worse or more accurate than others are (with some minor exceptions).

Three-Phase, Four-Wire Wye-Connected Systems: Line-To-Neutral Voltage Sensing

For a three-phase system in which the neutral is present and voltages can be sensed line-to-neutral, the sum of the single-phase calculations for the three-phase system provides the total three-phase power (real, reactive, and apparent.)

Fig. 4 shows this case with a three-phase Wye-connected coil with an accessible neutral. This is referred to as a three-phase, four-wire system. Note that we are no longer representing current and voltage vectors in the figure, but rather showing a simplified schematic with probe sensing locations and/or direction of travel of the signal.

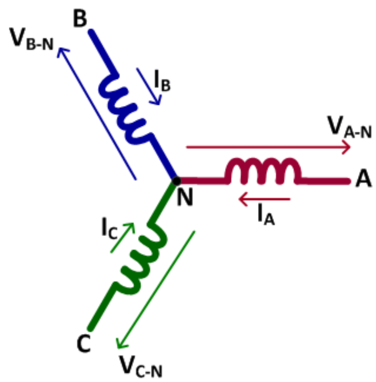

Fig. 4. Three-phase Wye-connected coil with an accessible neutral.

Note that the figure above shows the convention of line currents traveling into the coil towards the neutral. This would be typical of a coil within a machine (e.g., a motor) that consumes power. However, at any given time, some of these currents are flowing out of the neutral (in the opposite direction shown), because the currents at neutral must sum to zero. Additionally, even though it would appear from the directions shown that power is negative (because the voltage polarity is opposite that of the current polarity), the power measurement system typically calculates positive power.

In this case, we measure the voltages across the same coil through which the currents are traveling, identically to the vector-based power calculation examples provided above. This is known as a three-wattmeter, four-wire method for power calculations using three voltage and three current signals, and is an ideal method to use if there is suspicion that the load is unbalanced because per-phase values for power (real, apparent, and reactive) can be calculated from direct voltage and current measurements in each winding.

The example in Fig. 5 from Teledyne LeCroy's Motor Drive Analyzer shows the connection schematic diagram indicating the electric utility three-phase ac supply input to a motor drive, represented schematically as a rectifier circuit. The three currents (I_A, I_B , and I_C) are represented as flowing into the load, and the three voltages are probed line-to-neutral (V_A, V_B, and V_C). Note the "+" designation on the V_A, V_B, and V_C elements indicating the polarity of the voltage measurements made at these points (both line-to-neutral).

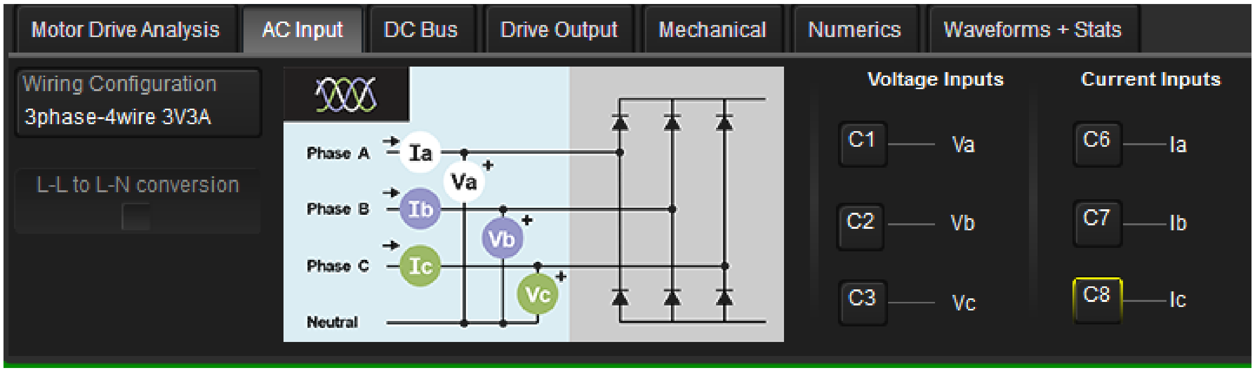

Fig. 5 Three-phase, four-wire ac input to a motor drive represented as a rectifier circuit in Teledyne LeCroy's MDA.

The eight input channels of the Motor Drive Analyzer facilitate capture of the three voltage and three current signals. Channels 1 (yellow), 2 (magenta), and 3 (cyan, or light blue) on the left side display the V_{A-N}, V_{B-N}, and V_{C-N} line-to-neutral voltages respectively, and Channels 5 (light green), 6 (purple) and 7 (red) display the I_A, I_B, and I_C line currents, respectively, as shown in Fig. 6.

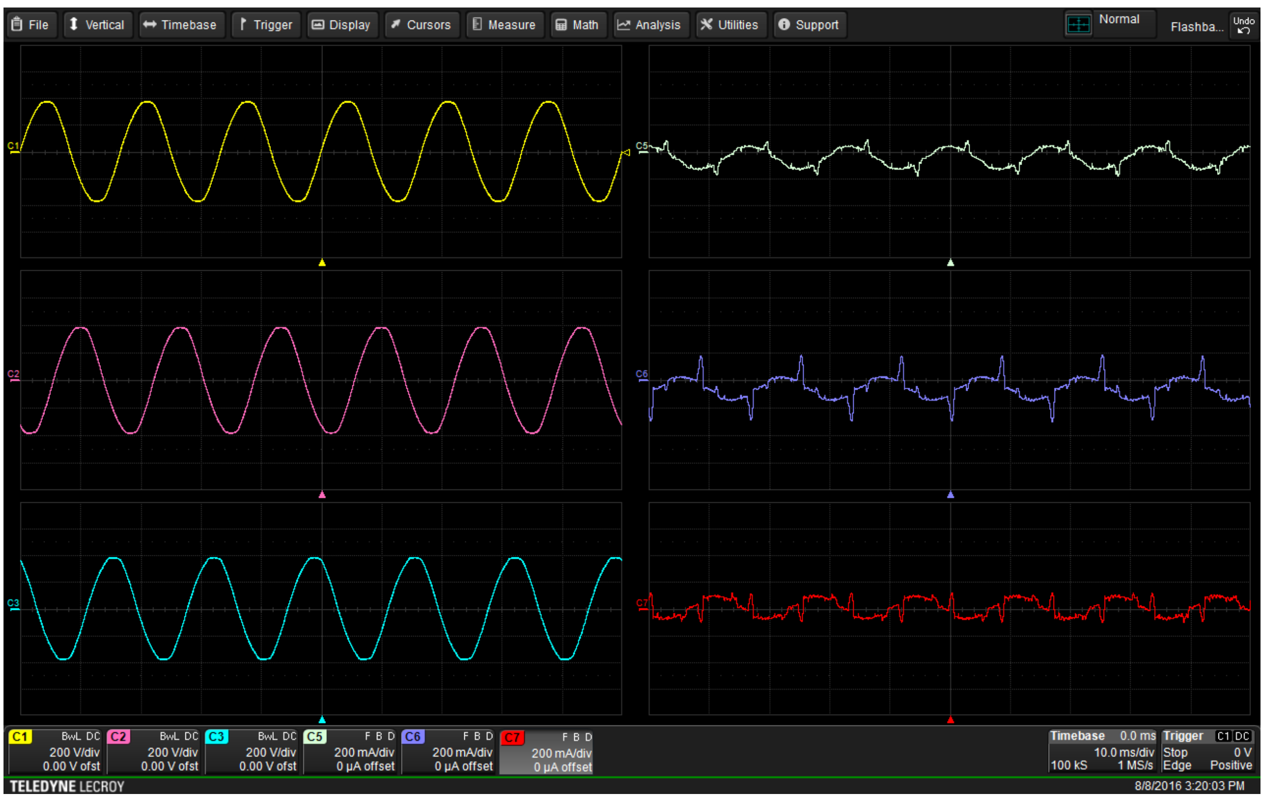

Fig. 6. Measurement of voltage and current signals shown in Fig. 5 for a three-phase, four-wire ac input to a motor drive.

Note that this is a 480-V, three-phase input to a 1990s vintage motor drive in the off state. We observe some distortion in the line-current input signals due to the nature of the load, but the sinusoidal nature of these signals is evident.

If the line-to-neutral voltage signals are placed on the same display grid as the corresponding line-current signals, it will be easy to visually observe the phase relationship. At the same time, we may use the built-in three-phase power analysis capabilities in the Motor Drive Analyzer to calculate voltage and current; real, apparent, and reactive power; and power factor and phase angle, as shown in Fig. 7.

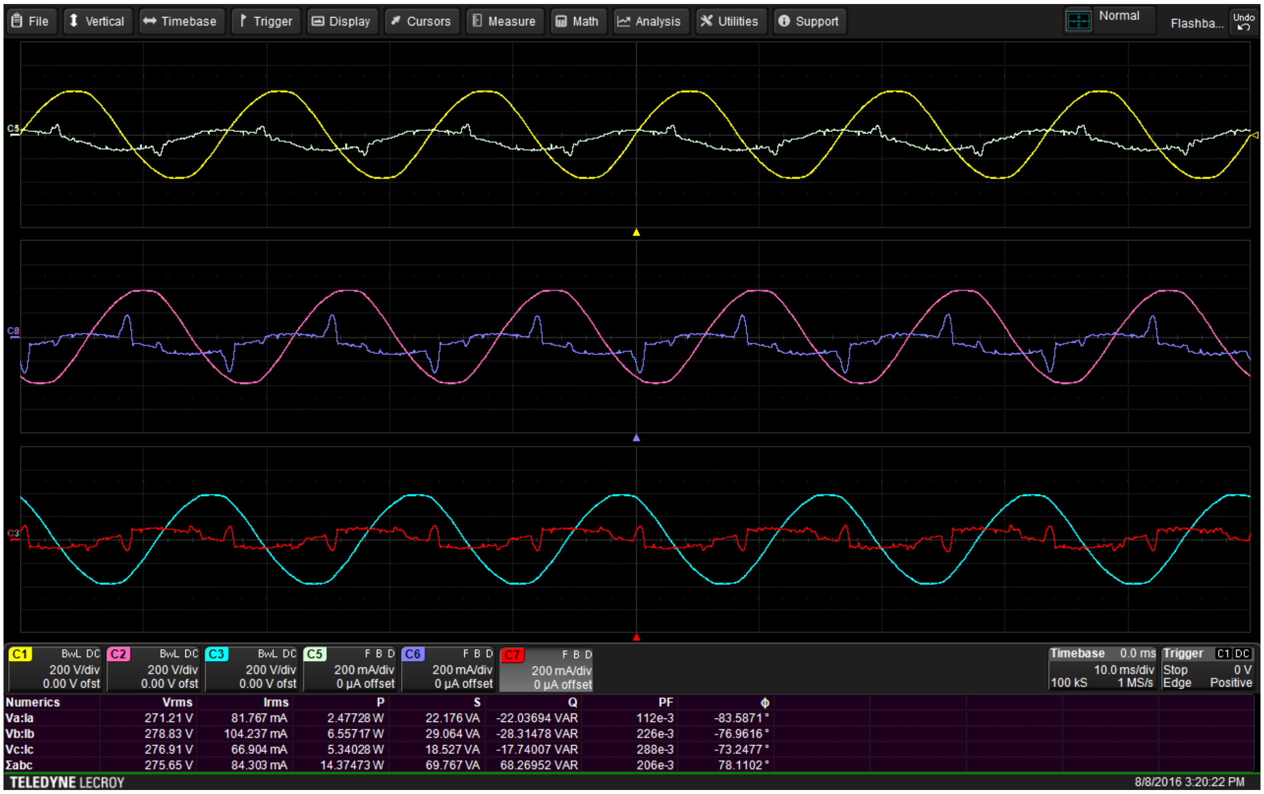

Fig. 7. Displaying voltage and current measurements (from Figs. 5 and 6) for each of the three phases concurrently on the same time scale reveals the phase relationships between these signals. These measurements are performed under a no-load condition (the motor drive is off.) The associated power parameters calculated by the MDA are shown in the Numerics table in the lower left of the display.

The Numerics table appears below in Fig 8. (same as in the image above but larger).

Numerics	Vrms	Irms	P	S	Q	PF	ϕ
Va:Ia	271.21 V	81.767 mA	2.47728 W	22.176 VA	-22.03694 VAR	112e-3	-83.5871 °
Vb:Ib	278.83 V	104.237 mA	6.55717 W	29.064 VA	-28.31478 VAR	226e-3	-76.9616 °
Vc:Ic	276.91 V	66.904 mA	5.34028 W	18.527 VA	-17.74007 VAR	288e-3	-73.2477 °
Σabc	275.65 V	84.303 mA	14.37473 W	69.767 VA	68.26952 VAR	206e-3	78.1102 °

Fig. 8. A closeup of the Numerics table from Fig. 7.

Note that the three line-to-neutral voltages average out to 275.35 V (close to the expected 277 V), and the line-to-neutral voltages are balanced within a couple of percent. The three-phase power values are not balanced in all three phases because this is a no-load situation with very low levels of current. So, any small imbalance in the load will be more apparent under a no-load condition.

The total three-phase power values are the sum of the per-phase power values. The load is capacitive, as can be seen in the phase relationships between the line-to-neutral voltage and line current waveforms (the current is leading the voltage). This is expected, because the input supplies the large dc filter and energy-storage capacitor in this drive, even when the drive is off. Therefore, we show the per-phase reactive power (Q) values as negative, as is the phase angle (φ). The power factor is also fairly low. Again, this is expected because power factors for equipment like this usually increase with load.

If the drive is energized and we apply a load to the connected motor (in this case, an ~2 N·m load from the dynamometer), the current consumption on all three phases increases, as does the distortion of the input current waveform (the drive load is very non-linear) as seen in Fig. 9. In addition, the power factor improves and the phase angle decreases.

Note that there is still significant imbalance of current and power across all three phases, even with a load. Also, note that the applied load is only about 10% of the rated load. If the motor was loaded to 100%, three-phase balance and power factor would likely improve further.

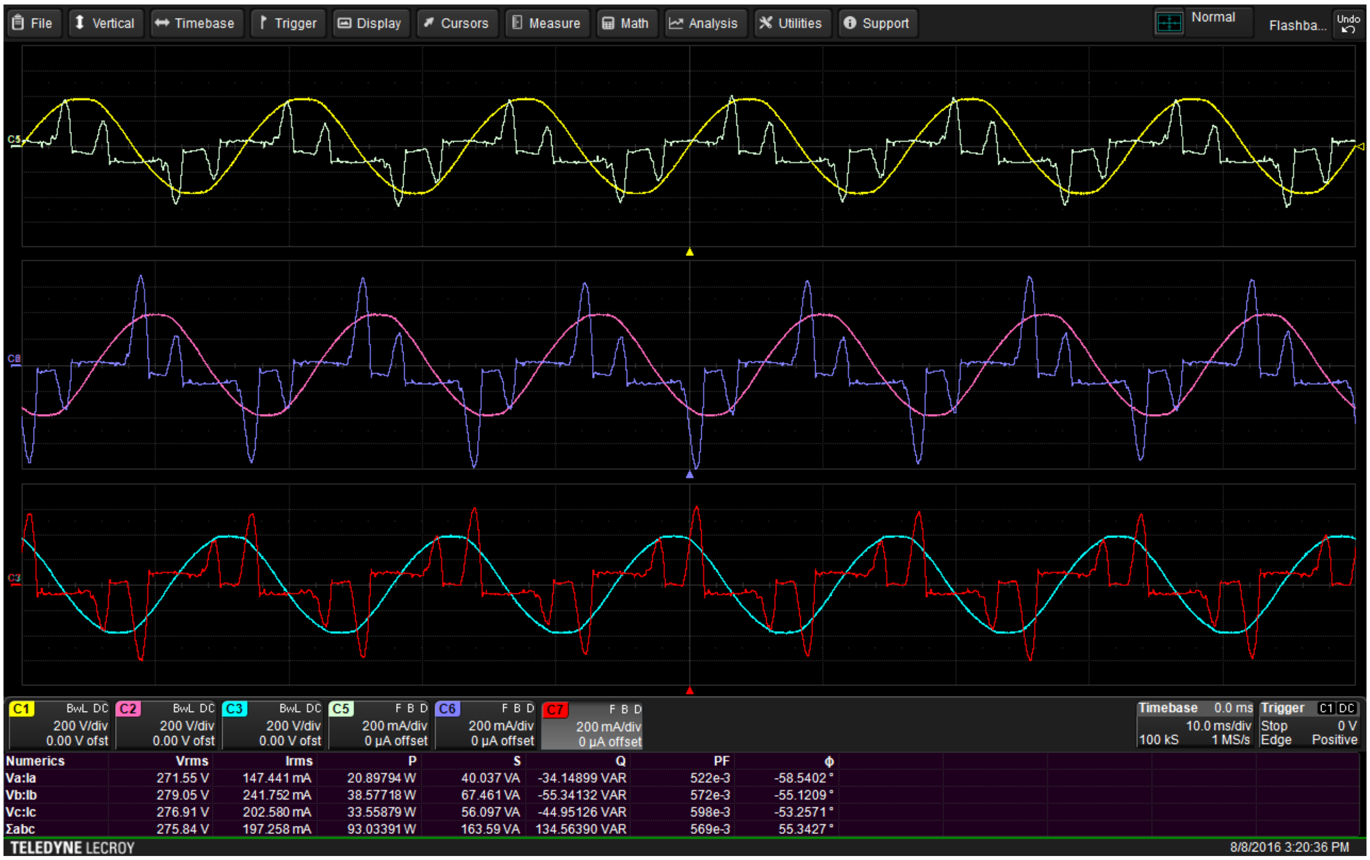

Fig. 9. Voltage and current measurements for the motor drive in Fig. 5 with the drive energized and the motor loaded at 10%. Compared with the motor drive off measurements, there is greater distortion of input current waveforms, while power factor is higher and phase angle between voltage and current is lower.

The Numerics table appears below in Fig. 10 (same as in the image above but larger).

Numerics	Vrms	Irms	P	S	Q	PF	φ
Va:Ia	271.55 V	147.441 mA	20.89794 W	40.037 VA	-34.14899 VAR	522e-3	-58.5402 °
Vb:Ib	279.05 V	241.752 mA	38.57718 W	67.461 VA	-55.34132 VAR	572e-3	-55.1209 °
Vc:Ic	276.91 V	202.580 mA	33.55879 W	56.097 VA	-44.95126 VAR	598e-3	-53.2571 °
Σabc	275.84 V	197.258 mA	93.03391 W	163.59 VA	134.56390 VAR	569e-3	55.3427 °

Fig. 10. A closeup of the Numerics table from Fig. 9.

Three-Phase, Three-Wire Wye-Connected Systems: Line-To-Line Voltage Sensing

If the neutral is not accessible for probing, or if the user prefers to measure line-to-line and not line-to-neutral voltages, then the measurement becomes more complex. In this case, the voltages are sensed line-to-line and are not referenced across the same points of the winding, as are the line currents (Fig. 11.)

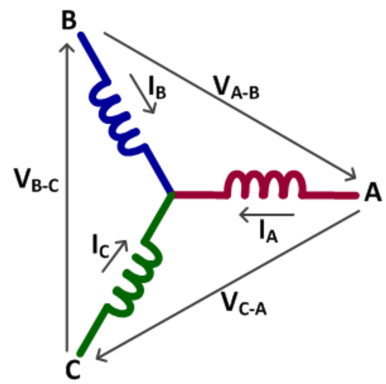

Fig. 11. Three-phase Wye-connected coil where the neutral is not accessible.

The example in Fig. 12 from Teledyne LeCroy's Motor Drive Analyzer shows the connection schematic diagram indicating the electric utility three-phase ac supply input to a motor drive, represented schematically as a rectifier circuit. The three currents (I_A, I_B, and I_C) are represented as flowing into the load, and the three voltages are probed line-to-line (V_{A-B}, V_{B-C}, and V_{C-A}).

Note the "+" designation on the V_{A-B}, V_{B-C}, and V_{C-A} elements indicating the polarity of the voltage measurements made at these points, and note specifically in Fig. 11 above and the Fig. 12 schematic below that the third line-to-line voltage is referenced from C to A phase.

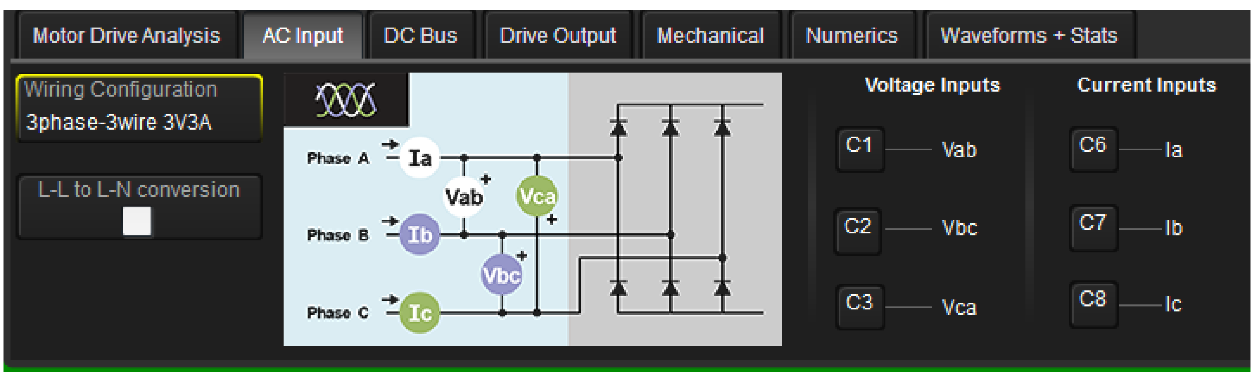

Fig. 12. Three-phase, three-wire ac input to a motor drive represented as a rectifier circuit in Teledyne LeCroy's MDA. In this case voltages are probed line-to-line.

The eight input channels of the Motor Drive Analyzer facilitate capture of the three voltage and three current signals. Channels 1 (yellow), 2 (magenta), and 3 (cyan, or light blue) on the left side display the V_{A-B}, V_{B-C}, and V_{C-A} line-to-neutral voltages, respectively; and channels 5 (light green), 6 (purple), and 7 (red) display the I_A, I_B, and I_C line currents, respectively.

Again, this is a 480-V, three-phase input to a 1990s vintage motor drive with a load applied to the connected motor (in this case, an ~2 N·m load from the dynamometer). The measured line currents essentially match those measured in the previous case, but the voltage signals are of higher amplitude and are phase-shifted by 30°. At the same time, we can use the built-in three-phase power analysis capabilities in the Motor Drive Analyzer to calculate voltage and current; real, apparent, and reactive power; and power factor and phase angle for the three-phase system, as shown in Fig. 13.

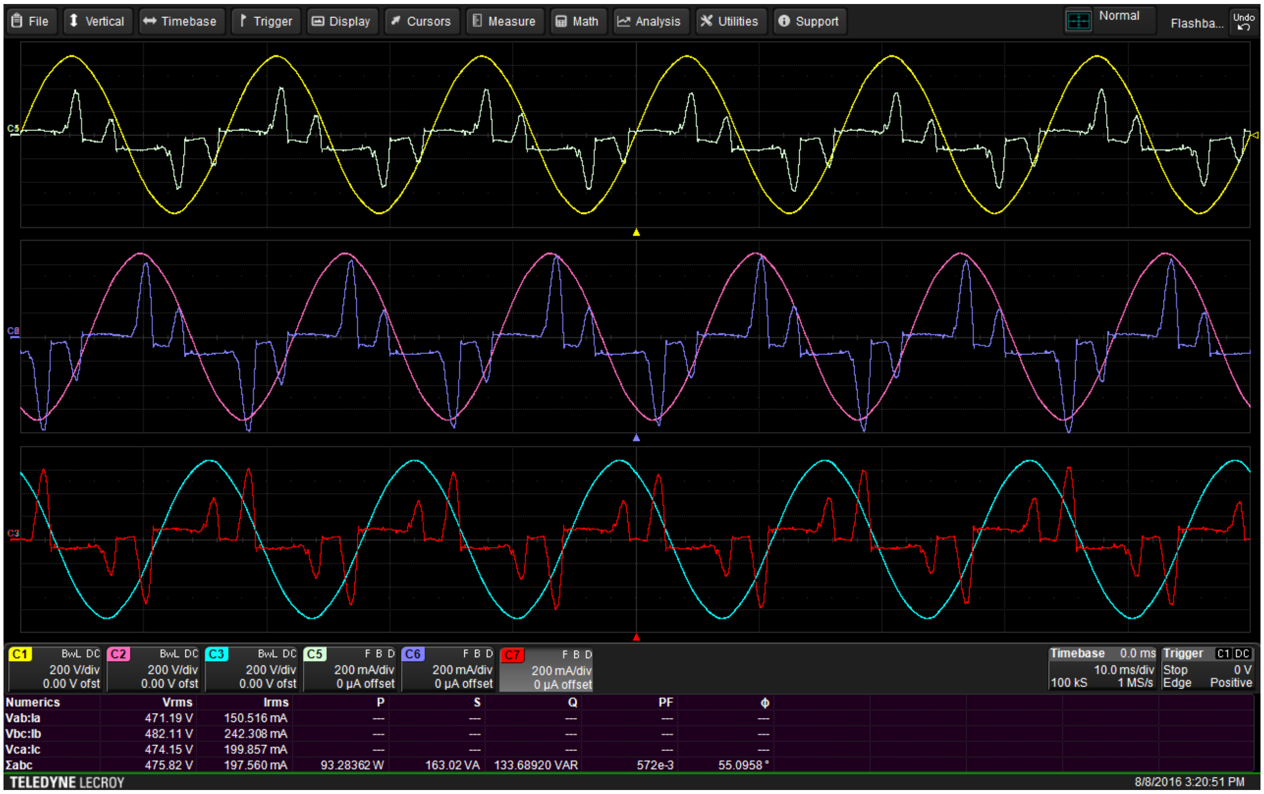

Fig. 13. Voltage and current measurements for the motor drive in Fig. 12 with the drive energized and the motor loaded at 10%. The measured line currents are close to those measured in the four-wire case (see Figs. 5 and 9), but the voltage signals are larger and are phase-shifted by 30°.

The Numerics table appears in Fig 14 (same as in the image above but larger).

Numerics	Vrms	Irms	P	S	Q	PF	Φ
Vab:Ia	471.19 V	150.516 mA	---	---	---	---	---
Vbc:Ib	482.11 V	242.308 mA	---	---	---	---	---
Vca:Ic	474.15 V	199.857 mA	---	---	---	---	---
Σabc	475.82 V	197.560 mA	93.28362 W	163.02 VA	133.68920 VAR	572e-3	55.0958 °

Fig. 14. A closeup of the Numerics table from Fig. 13.

Note that while voltage and current RMS values can be calculated on all three measurement phases, we display only total three-phase power because the voltage and current vectors are out of phase. Thus, calculation of individual phase power values is impossible. However, given that the three voltages, whether probed line-to-line or line-to-neutral, must always vector sum to zero, one may transform both the magnitude and phase to convert the line-to-line voltages to a line-to-neutral voltage at a virtual neutral point, as shown in Fig. 15.

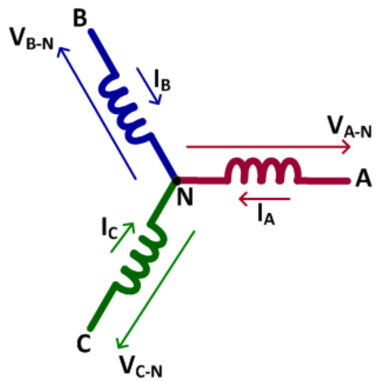

Fig. 15. Three-phase Wye-connected coil. Given that the three voltages must always vector sum to zero, line-to-line voltages may be transformed to line-to-neutral voltages at a "virtual" neutral point, which ultimately makes it possible to calculate phase power values.

These transformed line-to-neutral voltages are then used to calculate power as in the three-phase, four-wire Wye-connected line-to-neutral voltage sensed case above. This is known as "line-line to line-neutral conversion." Performed by the Motor Drive Analyzer, this converts both the voltage vector magnitude and phase (as shown below with the "checkbox" selection for L-L to L-N conversion):

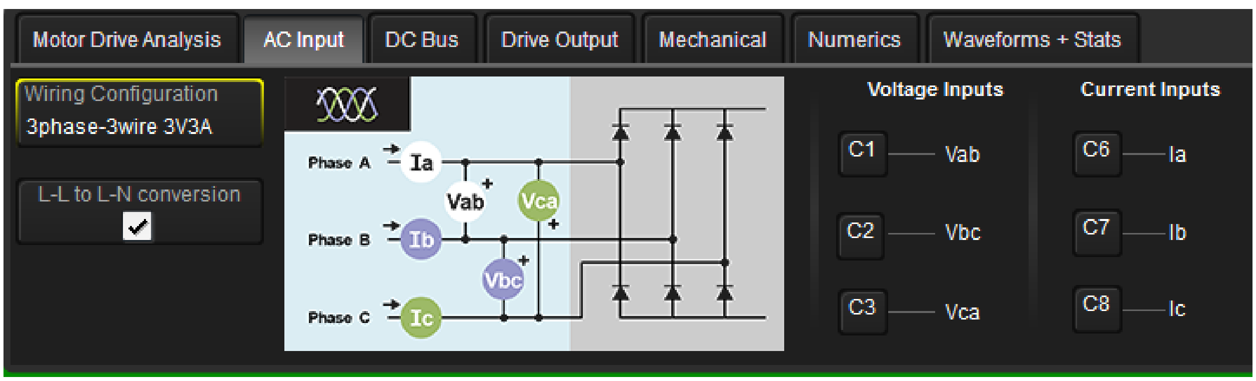

Fig. 16. The MDA is configured to convert the sensed line-to-line voltages to their line-to-neutral equivalents.

Then power calculations are made on each individual phase, as in the three-phase, four-wire configuration described earlier with results given in the Numerics table in Fig. 17.

Numerics		Vrms	Irms	P	S	Q	PF	Φ
Va:Ia	LL to LN	270.28 V	150.516 mA	21.08219 W	40.681 VA	-34.79045 VAR	518e-3	-58.7911 °
Vb:Ib	LL to LN	276.08 V	242.308 mA	39.16566 W	66.897 VA	-54.23177 VAR	585e-3	-54.1637 °
Vc:Ic	LL to LN	276.62 V	235.773 mA	32.54526 W	65.220 VA	-56.51842 VAR	499e-3	-60.0640 °
Σabc	LL to LN	274.33 V	209.532 mA	92.79311 W	172.80 VA	145.76518 VAR	537e-3	57.5207 °

Fig. 17. Closeup of Numerics table showing phase power calculations after the MDA's line-line to line-neutral conversion of the voltages sensed in Fig 13.

Note that to enforce the vector sum to zero, the phase C current values are adjusted—the larger the imbalance (which is considerable in this case), the larger the adjustment of the C phase's current value. While the real power (P) calculation after L-L to L-N conversion for the total three-phase system remains essentially the same, the apparent power (S), reactive power (Q), power factor (PF), and phase angle (φ) calculations for the three-phase system differ, primarily due to the system imbalance.

Three-Phase, Three-Wire Delta-Connected Systems

In a delta winding, the neutral is not accessible for probing, so voltage must be measured line-to-line. Again, the three voltages must vector sum to zero, so we may transform the line-to-line voltages to line-to-neutral voltages.

However, the line currents into the winding are terminal currents, not coil currents—the terminal currents could flow into either coil adjacent to the terminal connection (Fig. 18.) While this poses no problem for calculating total three-phase power, we do not directly measure the power in each individual coil because there is no direct measurement of the current flowing through the coil.

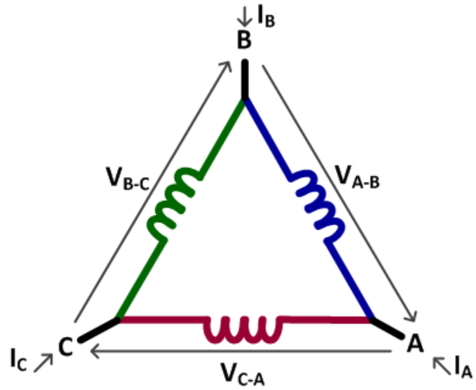

Fig. 18. Three-phase, three-wire delta-connected system. The sensed currents are terminal currents rather than coil currents, so power in individual coils may not be measured directly.

Three-Phase, Three-Wire Wye- or Delta-Connected Systems: Two-Wattmeter Method

In cases where a neutral point is not available (e.g., a delta winding, or an inaccessible neutral in a wye winding), or one wants to conserve input channels for other measurements, the best alternative is the two-wattmeter method. The two-wattmeter method uses two line-to-line voltage measurements to a common line reference (e.g., line C) with two line-current measurements (lines A and B) that do not include the current flowing into the line voltage reference (e.g., phase C).

It can be mathematically proven that this method returns the same calculated total three-phase power (real, apparent, and reactive) value(s) as the three-wattmeter method. However, because there is disassociation of the normal phase relationships between the voltage and current signals, the calculated per-phase power values for the two measured phases will not represent the coil powers, though they will correctly sum to the total three-phase power. The measured signals are as shown in Fig. 19.

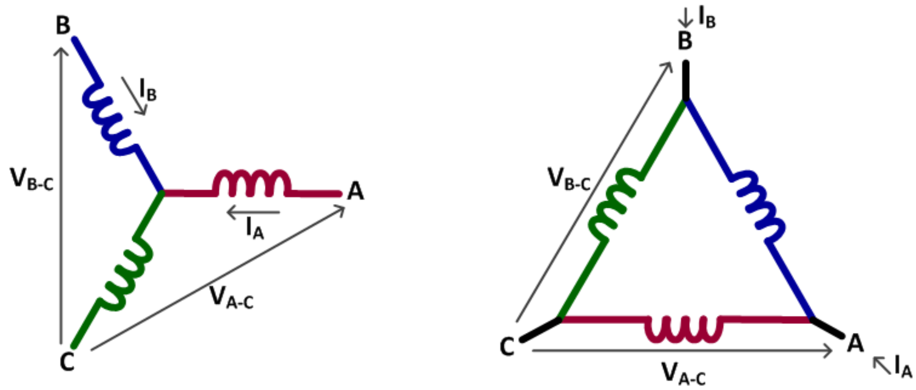

Fig. 19. Three-phase, three-wire wye-connected system (left) and delta connected system (right) showing voltages and currents sensed using the two-wattmeter method.

The example below from Teledyne LeCroy's Motor Drive Analyzer shows the connection schematic diagram indicating the electric utility three-phase ac supply input to a motor drive, represented schematically as a rectifier circuit. The two currents (I_A and I_B) are represented as flowing into the load, and the two voltages are probed line-to-line (V_{A-C} and V_{B-C}).

Note the "+" designation on the V_{A-C} and V_{B-C} elements, which indicate the polarity of the voltage measurements made at these points. Also, note specifically in Fig. 19 above and the Fig. 20 schematic below that the first line-to-line voltage is referenced from line A to line C (which is different from the earlier line-to-line voltage sensing case in which all three voltages and all three currents were measured).

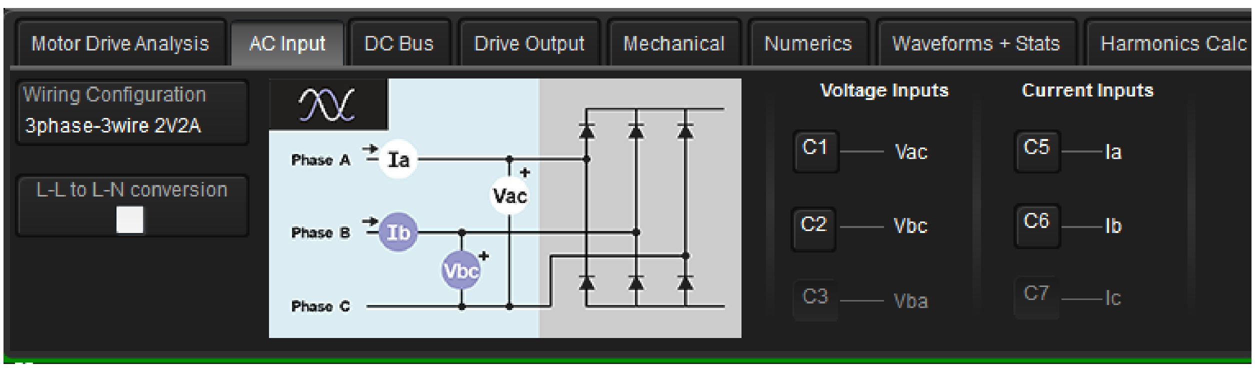

Fig. 20. Three-phase, three-wire ac input to a motor drive represented as a rectifier circuit. In this case two voltages are probed line-to-line and only two currents are measured in keeping with the two-wattmeter method.

The signal capture, motor load, and display of Numerics calculation is similar to the previous cases, and appears in Fig. 21 with the Numerics table enlarged in Fig. 22.

Fig. 21. Voltage and current measurements for the motor drive in Fig. 20, illustrating the two-wattmeter method.

The Numerics table appears below (same as in the image above but larger):

Numerics	Vrms	Irms	P	S	Q	PF	ϕ
Vac:Ia	468.84 V	147.482 mA	15.98435 W	69.146 VA	-67.26990 VAR	231e-3	-76.6393 °
Vbc:Ib	482.18 V	245.540 mA	77.48008 W	118.40 VA	89.52156 VAR	654e-3	49.1250 °
Σabc	475.51 V	196.511 mA	93.46443 W	162.42 VA	132.82412 VAR	575e-3	54.8694 °

Fig. 22. Closeup of the Numerics table showing voltage and current measurements and the associated power calculations.

Note that these voltage and current pairs do not represent a phase or coil measurement of power because the voltage and current vectors are not correctly associated. Thus, it is impossible to draw any conclusions from a visual inspection of the voltage and current waveform pairs. However, the total three-phase power does sum correctly from these two values.

If the gulf between the two power values spawns confusion, consider the vector diagrams of the voltage and current associations used in the two-wattmeter method (Fig. 23 on the left) and the three-wattmeter method using line-to-neutral voltage references (Fig. 23 on the right).

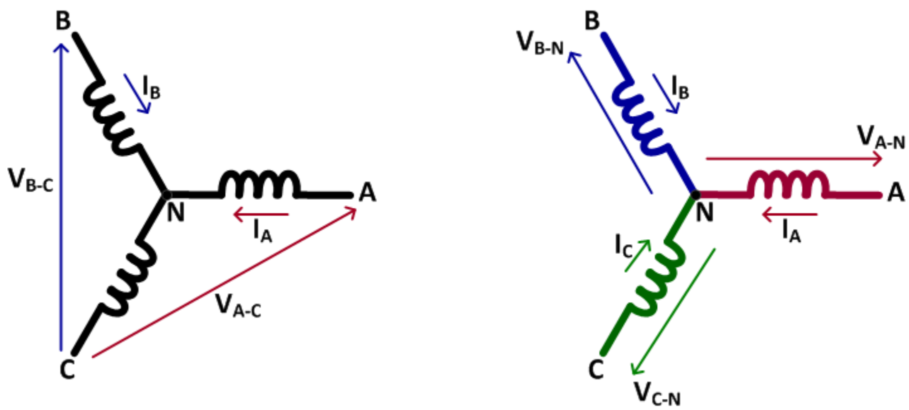

Fig. 23. Vector diagrams for three-phase, four-wire wye-connected systems compare the voltage and current measurements performed using the two-wattmeter method (left) versus the three-wattmeter method (right).

In the Fig. 23 diagram on the left, a line-to-line voltage is associated with a line current in each of the two wattmeters ("blue" and "red" voltage and current pairs.) In the diagram on the right, a line-to-neutral voltage is associated with a line-to-neutral current in each of the three wattmeters. If the system is balanced, then the total three-phase wattmeter calculations will be the same in each case.

We may also perform a line-line to line-neutral conversion on the data acquired for the two-wattmeter method. If we do so, we can then calculate a per-phase value for each of the three phases, again assuming a balanced three-phase system. This Numerics table with the resulting calculations appears below in Fig. 24.

Numerics		Vrms	Irms	P	S	Q	PF	ϕ
Va:Ia	LL to LN	270.22 V	147.482 mA	20.85228 W	39.853 VA	-33.96128 VAR	523e-3	-58.4539 °
Vb:Ib	LL to LN	277.94 V	245.538 mA	39.86112 W	68.245 VA	-55.39257 VAR	584e-3	-54.2614 °
Vc:Ic	LL to LN	274.79 V	236.366 mA	32.75037 W	64.951 VA	-56.08737 VAR	504e-3	-59.7185 °
Σabc	LL to LN	274.32 V	209.796 mA	93.46377 W	173.05 VA	145.63413 VAR	540e-3	57.3096 °

Fig. 24. Closeup of Numerics table listing calculated values of current and power produced by the two-wattmeter method and line-line to line-neutral conversion on the data.

Conclusion

This section reviewed the measurement techniques available for making power calculations for the various popular configurations encountered in three-phase ac systems associated with motor drives. The use of the Motor Drive Analyzer to perform the necessary measurements and calculations was also demonstrated. In the next installment in this series, part 8, the topic shifts to power semiconductor device operation, which is necessary background for understanding motor drive circuit operation. For a full list of topics that will be addressed in this series, see part 1.

About The Author

Kenneth Johnson is a director of marketing and product architect at Teledyne LeCroy. He began his career in the field of high voltage test and measurement at Hipotronics, with a focus on <69-kV electrical apparatus ac, dc and impulse testing with a particular focus on testing of transformers, induction motors and generators. In 2000, Ken joined Teledyne LeCroy as a product manager and has managed a wide range of oscilloscope, serial data protocol and probe products. He has three patents in the area of simultaneous physical layer and protocol analysis. His current focus is in the fields of power electronics and motor drive *test solutions, and works primarily in a technical marketing role as a product architect for new solution sets in this area. Ken holds a B.S.E.E. from Rensselaer Polytechnic Institute.*

For further reading on motor drives, see the How2Power [Design Guide](), locate the "Power Supply Function" category, and click on the "Motor drives" link.