Exclusive Technology Feature

ISSUE: May 2018

Analyzing The Effect Of Voltage Drops On The DC Transfer Function Of The Buck Converter

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Switching converters combine passive elements such as resistors, inductors, capacitors but also active devices like power switches. When you study a power converter most of these components are considered ideal: when switches close they do not drop voltage across their terminals, inductors do not feature ohmic losses and so on. In reality, all these elements, either passive or active, are far from being perfect. In this article, we will study their effects on the dc transfer function of a buck converter. We will also apply this analysis to a forward converter, which is a buck-derived topology.

Ohmic Losses

A closed switch exhibits a certain resistance (the $r_{DS(on)}$ for a MOSFET) and drops voltage across its terminals when current flows. As the switch toggles from one state to another, it transitions through a linear mode during which it also dissipates power that affects efficiency (switching losses).

When conducting, a diode can be modeled by a voltage source V_{T0} in series with a dynamic resistance r_d. When current flows in this network (the diode is turned on), you also observe a voltage drop across its terminals, the forward drop noted V_f and equal to $V_{T0} + i_d(t) r_d$. A diode does not instantaneously block either: depending on the technology, the device conducts current in the reverse direction before it starts recovering its blocking state. This is true for silicon PN junctions and efficiency suffers in continuous conduction mode (CCM): power is dissipated when the diode and the switch are conducting together for a brief instant and create a fleeting short circuit across V_{in} in a buck converter.

Schottky diodes do not exhibit recovery losses and conduction losses are significantly lower than their silicon counter parts. However, their parasitic capacitance can contribute to efficiency reduction in high-frequency applications. These phenomena will not be included here.

Regarding passive elements, rms currents flowing in inductors and capacitors generate heat when crossing equivalent series resistances (ESRs) respectively noted as r_L and r_C for these components. Other phenomena such as magnetic losses or off-state leaks also degrade efficiency but will not be considered here. Fig. 1 shows a simplified representation of these parasitic elements.

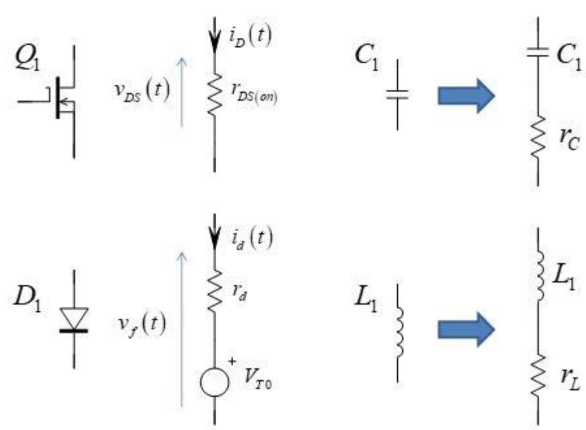

Fig. 1. Components we use in power conversion are not perfect and host parasitic terms.

The Perfect Case

These various drops affect the dc and ac transfer functions of a converter. Dc because the presence of ohmic paths create various drops that must be compensated at some point (the loop will take care of these) but also ac as (1) resistive drops create dividers affecting the gain and (2) energy dissipation means damping so it is likely that sharp resonant peaks are affected by the presence of these parasitic elements. While their impact is less important in high-voltage applications, for instance a 1-V V_f in a 24-V application, you cannot neglect them in low-voltage circuits such as those encountered in portable battery-powered applications.

Calculating the output voltage of a buck converter with or without consideration of these parasitic terms can be performed in different ways. The simplest method is to calculate the average voltage across the inductor using the so-called volt-second balance law. It says that, at steady-state (meaning the converter has reached its output target and is stabilized), the average voltage across the inductor is 0 V. Mathematically, you write it this way:

$$\left\langle v_L(t)\right\rangle_{T_{sw}} = \frac{1}{T_{sw}}\int_0^{T_{sw}} v_L(t)\,dt = 0 \qquad (1)$$

Graphically, you represent the inductor voltage during the on-state (while the series switch is turned on) and during the off-state (when the diode freewheels). Then, as shown in Fig. 2, you compute the area under the on- or off-state line by multiplying the rectangle height by its base length. Calculating an area is actually the same as integrating the variable—here $v_L(t)$—during the on- or off-times.

Inductor voltages integrated over time (volt-seconds, V-s) describe the inductor magnetic core flux activity during the on- and off-times. At equilibrium, as the net volt-seconds value over a switching cycle must be zero (the flux excursion during the on-time must return to its starting point during the off-time, otherwise saturation may occur), both areas must be equal.

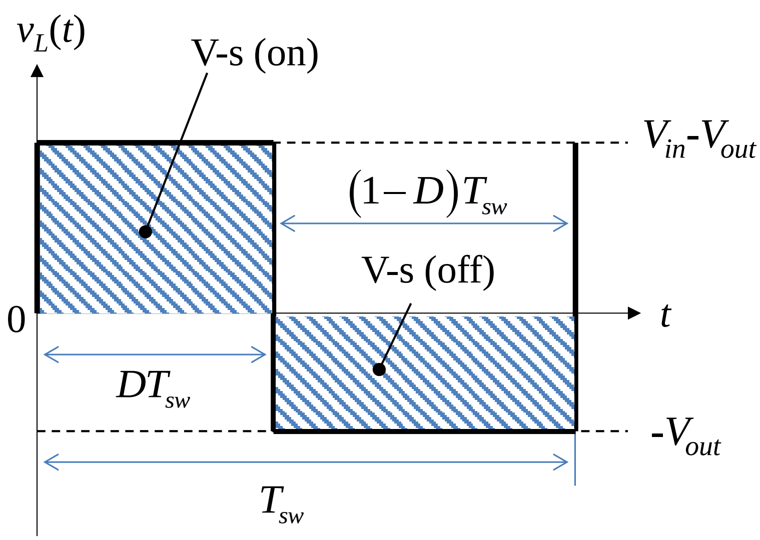

Fig. 2. Flux balance in the inductor implies that areas above and below 0 are equal. In this example, the converter operates in continuous conduction mode (CCM).

Let's now run the exercise while considering perfect elements, no ohmic losses and drops. In a buck converter, when the switch closes during t_{on}, at steady-state, one inductor terminal receives V_{in} while the second is at V_{out}. The V-s are computed as:

$$\left\langle v_L(t)\right\rangle_{t_{on}} = t_{on}\left(V_{in}-V_{out}\right) = DT_{sw}\left(V_{in}-V_{out}\right) \qquad (2)$$

In this expression, D is the duty ratio and T_{sw} the switching period. During the off-time, the inductor current circulates in the same direction as during t_{on} but finds a path through the diode now conducting. The inductor terminal previously biased to V_{in} dips to 0 V as the diode is considered perfect. The instantaneous inductor voltage reverses and we can write the following area expression:

$$\langle v_L(t)\rangle_{t_{off}} = t_{off}V_{out} = (1-D)T_{sw}V_{out} \tag{3}$$

At equilibrium, the subtraction of (3) from (2) must return 0:

$$D(V_{in}-V_{out})-(1-D)V_{out}=0 \tag{4}$$

Solving for D in the above equation returns the classical dc transfer value noted M for a perfect buck converter:

$$M=\frac{V_{out}}{V_{in}}=D \tag{5}$$

This is "un cas parfait" (pardon my French) where no parasitics are considered.

Adding Resistive Paths

Let's now complicate the circuitry by adding the $r_{DS(on)}$, the inductor ohmic loss r_L and the diode forward drop V_f. During the on-state, we have the circuit in Fig. 3 in which R represents the load:

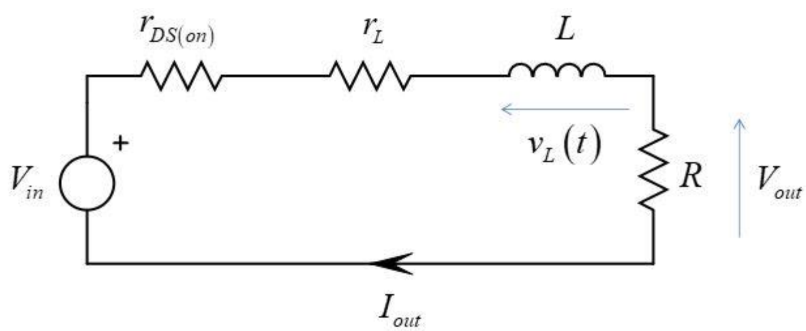

Fig. 3. During the on-time, current circulates through the MOSFET and other ohmic paths.

The inductor volt-seconds during the on-time are no longer described by (2) and need an update. The circulating current during the on-time is I_{out} equal to V_{out}/R. Therefore

$$\langle v_L(t)\rangle_{t_{on}} = DT_{sw}\left[V_{in}-\frac{V_{out}}{R}\left(r_{DS(on)}+r_L\right)-V_{out}\right] \tag{6}$$

During the off-time, the inductor current keeps circulating in the same direction through the diode that is now freewheeling. The inductor voltage reverses and Fig. 4 shows the updated current path while the power MOSFET is turned off.

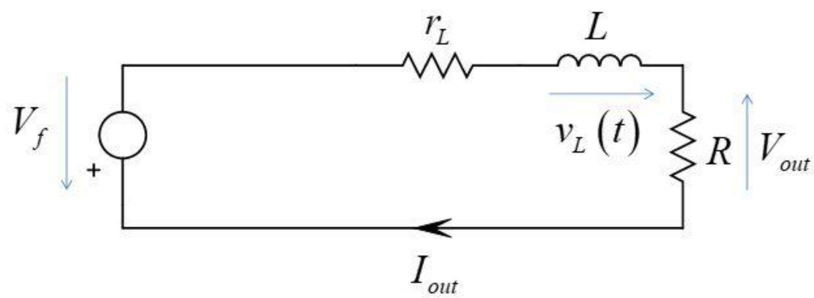

Fig. 4. During the off-time, the diode conducts and pulls the inductor left terminal to –V_f.

We can calculate the inductor volt-seconds during the off-time by considering the inductor right terminal biased at V_{out} while its left terminal is biased to $-\left(V_f + r_L I_{out}\right)$. Therefore, we have:

$$\left\langle v_L(t)\right\rangle_{t_{off}} = (1-D)T_{sw}\left[V_{out} + V_f + \frac{V_{out}}{R} r_L\right] \quad (7)$$

If we subtract (7) from (6) then solve for M to obtain 0, we have:

$$\frac{V_{out}}{V_{in}} = D\left[\frac{1}{1+\frac{r_{DS(on)}}{R}D+\frac{r_L}{R}+\frac{V_f}{V_{out}}D'}\right] \quad (8)$$

In this expression we can see that the $r_{DS(on)}$ average contribution is weighted by the duty ratio D while the diode forward drop V_f depends on $D' = 1-D$. In CCM-operated converters featuring low duty ratios (12 V to 1.2 V conversion for instance), it is therefore preferable to concentrate the effort on the diode characteristics (because D' is large) and minimize its effects perhaps by selecting a low-V_f Schottky or implementing synchronous rectification. As D is small, the $r_{DS(on)}$ contribution will be less important.

On the other hand, for larger duty ratios, the $r_{DS(on)}$ will have a greater impact on efficiency. But regardless of the duty ratio, the inductor ohmic loss r_L is present during the on- and off-times and must be kept at its lowest value.

From (8), we can extract the duty ratio value that will be adjusted by the control loop to keep V_{out} on target:

$$D = \frac{V_f + V_{out}\left(1+\frac{r_L}{R}\right)}{V_f + V_{in} - V_{out}\frac{r_{DS(on)}}{R}} \quad (9)$$

Assume a buck converter supplied from a 12-V source that must deliver 5 V precisely at a 5-A output current (R = 1 Ω). The MOSFET $r_{DS(on)}$ is 56 mΩ, the diode forward drop at this current is 787 mV and the inductor ESR is 70 mΩ. What is the duty ratio value to exactly deliver 5 V? Applying equation (9) we have

$$D = \frac{0.787 + 5\times\left(1+\frac{0.07}{1}\right)}{0.787+12-5\times\frac{0.056}{1}} = 0.491 \quad (10)$$

In this example, equation (5) would return 0.417, a lower value. We can test (10) by using a lossy averaged model such as the one described in reference [1]. The schematic for this model appears in Fig. 5. The operating bias points are displayed in the schematic (1 V = 100%) and confirm the result delivered by (10).

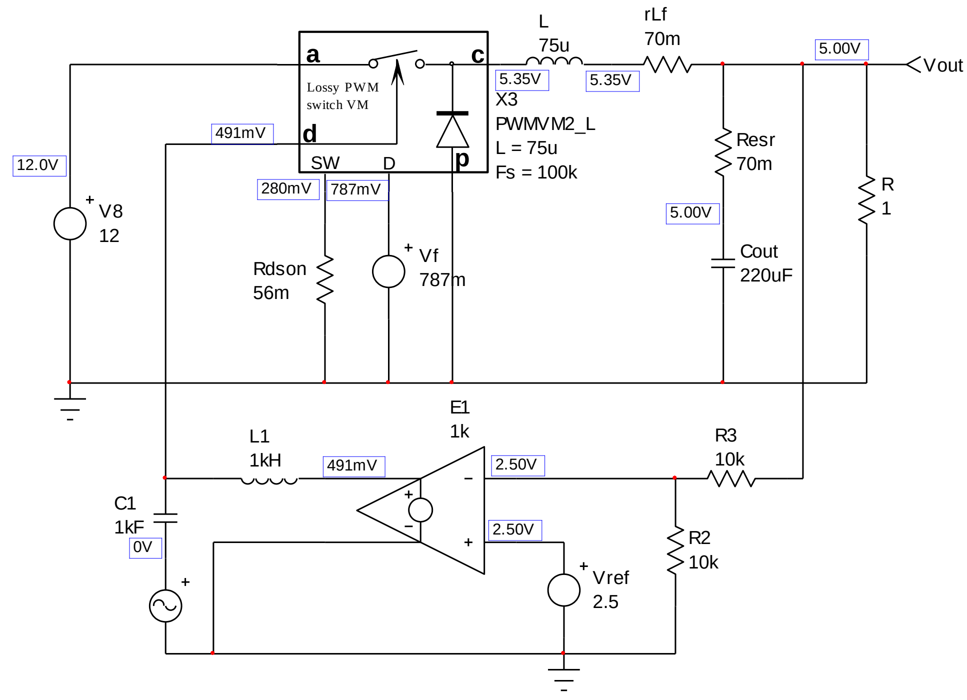

Fig. 5. A lossy average model accounts for effects brought by the various ohmic paths.

The Forward Converter

The forward converter is a buck-derived structure: a buck converter to which an isolation transformer has been added. Ensuring cycle-by-cycle core demagnetization is necessary for the forward converter and many variations exist to implement this mechanism.

Fig. 6 illustrates the simplest way associating a third transformer winding with diode D_3. Assuming a 1:1 turns ratio with the primary side, this extra winding imposes a demagnetizing slope on the magnetizing inductance L_{mag} identical to that when Q_1 conducts. The maximum duty ratio must thus be less than 50% to make sure core reset is ensured in worst-case situations. More elaborate structures such as the active clamp forward lifts this duty cycle limit to 60% or 65% but it won't be studied here. The classical dc transfer function formula for the ideal forward converter is defined as

$$V_{out} = NDV_{in} \tag{11}$$

This expression resembles that of a buck converter but with V_{in} replaced by NV_{in}, which reflects the transformer scaling action of the forward converter.

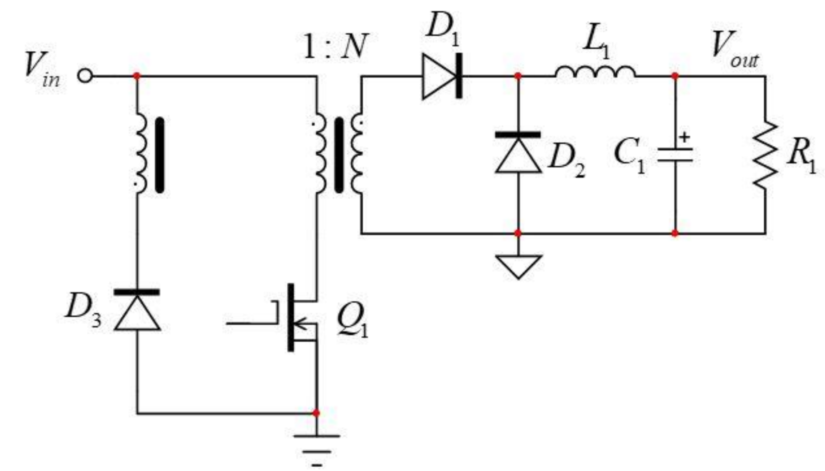

Fig. 6. The forward converter requires an auxiliary winding for the core demagnetization.

Without getting into details of the transformer operation, we can explore the turn-on and turn-off phases of this switching converter. When the controller instructs the power switch to conduct, the voltage applied across the transformer primary is V_{in} minus Q_1's voltage drop.

The drop occurs because current flows in the resistive path offered by the switch when conducting. This current is made of two components: the magnetizing current and the reflected output current imposed by the transformer turns ratio N. At the junction of D_1 and D_2 cathodes, the primary voltage appears reduced by D_1's forward drop. Finally, the output current I_{out} induces a voltage drop across r_L as shown in Fig. 7 in which we have neglected the magnetizing current contribution.

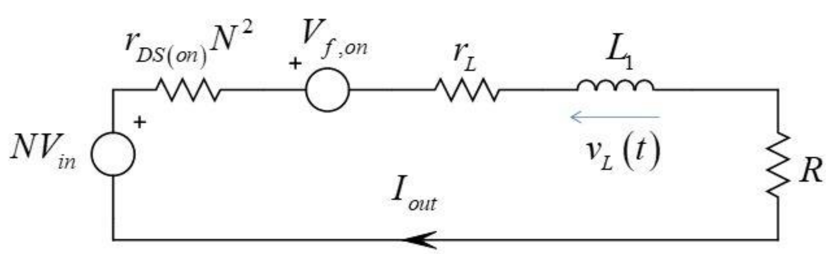

Fig. 7. The input voltage scaled by the transformer turns ratio is further reduced by the various drops. This representation is valid during the on-time without the magnetizing current contribution.

The inductor volt-seconds during the on-time are thus expressed as

$$\langle v_L(t)\rangle_{t_{on}} = DT_{sw}\left[\left(V_{in} - \frac{V_{out}}{R} r_{DS(on)}N\right)N - V_{f,on} - r_L\frac{V_{out}}{R} - V_{out}\right] \qquad (12)$$

During the off-time, freewheel diode D_2 conducts and the voltage across inductor L_1 reverses. The situation is similar to that of the buck converter described in Fig. 4 and the inductor volts-second expression is

$$\langle v_L(t)\rangle_{t_{off}} = (1-D)T_{sw}\left[V_{out} + V_{f,off} + \frac{V_{out}}{R} r_L\right] \qquad (13)$$

If we subtract equation (13) from (12) and solve for M to obtain 0, we have:

$$D = \frac{V_{out} + V_{f,off} + \frac{V_{out}}{R} r_L}{V_{f,off} - V_{f,on} + N\left(V_{in} - \frac{V_{out}}{R} N \cdot r_{DS(on)}\right)} \qquad (14)$$

The diode forward drops can be extracted from the data sheet knowing their nominal average operating current. It is DI_{out} for D_1 while it is $(1-D)I_{out}$ for D_2. These drops can also be replaced by the voltage drops across synchronous switches if any.

We can still use the lossy model in the forward converter. However, during the on-time, we now have the drop incurred by the MOSFET located on the primary side combined with the effect of D_1 from the secondary side. This requires the addition of a simple in-line expression in the form of source B_1 in Fig. 8.

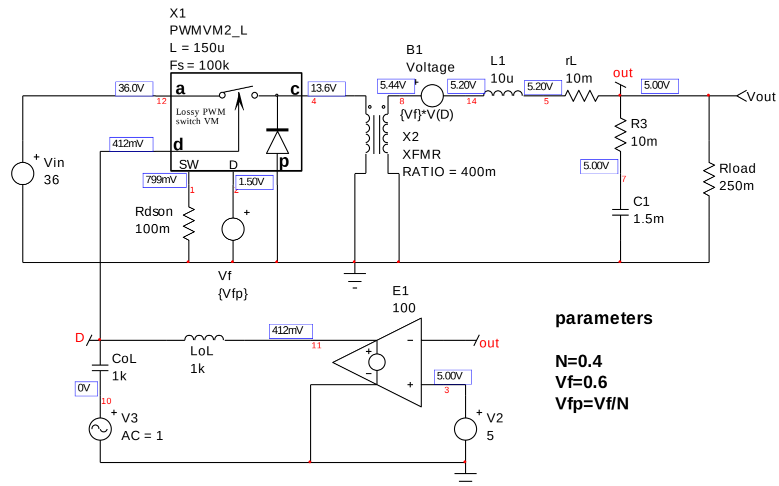

Fig. 8. The lossy model lends itself well to simulating a forward converter affected by ohmic losses.

In this simulation circuit, the component values correspond to a 100-kHz forward converter supplied from a 36-V to 72-V telecom network delivering 5 V with a 20-A nominal current. The diodes' total drop is 0.6 V on average and is equal for both components. The transformer turns ratio is 1:0.4 and the power switch $r_{DS(on)}$ is 100 mΩ. With a 10-mΩ value for r_L, equation (14) gives a duty ratio of 41.2% while (11) would lead to a duty ratio D of 34.7%. As indicated by the bias points reflected on the schematic, SPICE also determines a duty ratio of 41.2%, confirming the formula we have derived.

To refine our simulation, we have captured the same circuit using SIMPLIS Elements, the demonstration version from SIMetrix Technologies.[2] This schematic appears in Fig. 9 and the simulation is completed in a few seconds. The operating waveforms are given in Fig. 10.

For a 5-V output, the on-time is measured to be 4.115 µs, which over a 10-µs switching period corresponds to a 41.15% duty ratio, very close to what we have calculated. In reality, magnetic losses but also input line drops (through a filter for instance) will also distort the calculation and it is very likely that the final duty ratio will be slightly above this calculated value. However, you won't see a difference as large as what (11) would give you.

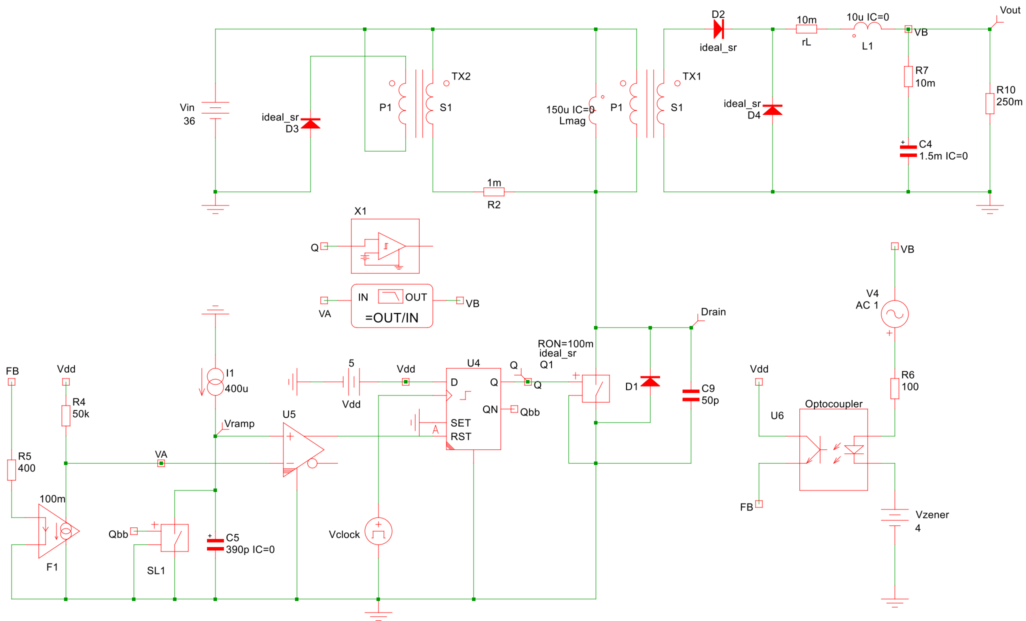

Fig. 9. The demo version of SIMPLIS Elements lets you simulate the example forward converter from Fig. 8 using the circuit shown here. The results of this simulation (shown in Fig. 10) confirm our calculations.

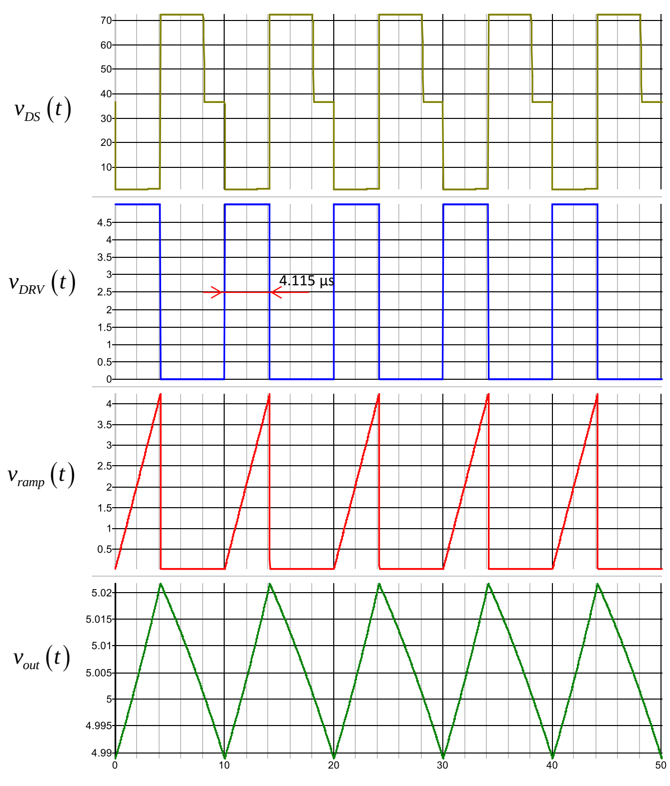

Fig. 10. Using Simplis Elements, operating waveforms for the example forward converter are generated in a few seconds and confirm the duty ratio calculated above using equation (14).

Finally, SIMPLIS can extract the small-signal response from the switching circuit owing to its piecewise-linear approach. The second-order response appears in Fig. 11. In contrast with an average model, with the piecewise linear model you can enrich the electrical circuit and see how extra losses such as diode t_{rr} or magnetic losses will affect the quality factor Q and other parameters.

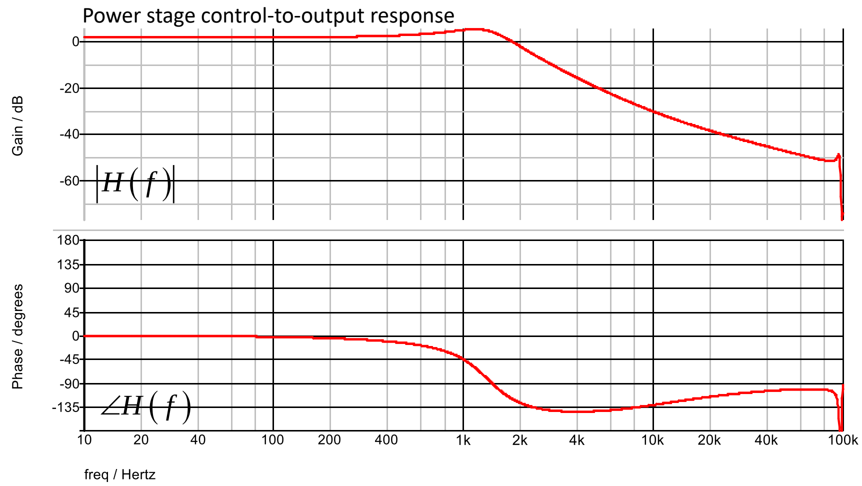

Fig. 11. SIMPLIS delivers the dynamic response of the example forward converter without the need to resort to an average model as with SPICE.

Conclusion

This short article shows how various drops can affect the dc transfer function of a buck converter operated in CCM. While the voltage drops can often be neglected for large input/output voltages, this is no longer the case when the input source is of low value or if the regulated output reaches a few volts. Considering these losses is important to compute the exact duty ratio, especially in those instances where it tunes a resonant network as in the active clamp forward case. An average model including conduction losses can nicely predict conduction loss effects on the operating point. SIMPLIS is also of great help especially if no average model is readily available for the converter you design.

References

1. "Switch-Mode Power Supplies: SPICE Simulations and Practical Designs" 2nd edition by Christophe Basso, McGraw-Hill, New-York 2014 , ISBN 978-0071823463
2. SIMPLIS Elements, demonstration version,

About The Author

Christophe Basso is a technical fellow at ON Semiconductor in Toulouse, France. He has originated numerous integrated circuits among which the NCP120X series has set new standards for low standby power converters. SPICE simulation is also one of his favorite subjects and he has authored two books on the subject. Christophe's latest work is "Linear Circuit Transfer Functions: An Introduction to Fast Analytical Techniques."

Christophe received a BSEE-equivalent from the Montpellier University, France and an MSEE from the Institut National Polytechnique de Toulouse, France. He holds 18 patents on power conversion and often publishes papers in conferences and trade magazines.

For further reading on power supply compensation, see the How2Power Design Guide, and do a keyword search on "compensation."