Waveform-Based Design: Key Parameters And Figures Of Merit For Power Circuit And Magnetics Optimization

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Magnetics and power-converter design are related by common design variables that are electrical functions of time, or *waveforms*.[1] This article offers a brief tutorial on waveforms commonly encountered in both magnetics and circuit design, their performance parameters, and why they are important in design optimization.

The waveform characteristics that concern us here are parameters such as duty-ratio (duty-cycle) and duty-ratio complement for switched (discontinuous) waveforms and terms such as peak ($\hat{x}$), average ($\bar{x}$), rms ($\tilde{x}$) and variation or ripple ($x_{\sim} = \Delta x$) of continuous waveforms ($x(t)$). These parameters, in turn, are used to define figures of merit such as utilization (U) as well as figures of demerit such as form factor (κ) and crest factor (χ), which can be used to optimize waveforms in both power circuits and magnetic components.

Ripple factors (γ) represent another set of figures of merit or demerit (or simply, *performance parameters*) which are given special attention here and these can be related back to the other performance parameters. All of these various terms permit design optimization without regard to scale (i.e. signal amplitude).

While I have used these waveform parameters and figures of merit throughout my writings on power circuits and magnetics, in this article, I take a step back and define these terms more methodically, while also noting where these terms come into use. For those interested in reading more about how these terms can be applied in power circuit and magnetics design, the list of references [2-13] brings you back to my previous works.

Waveform Parameters

A *waveform* is an electrical function, usually of voltage, current, or power, expressed as a function $x(t)$, of time, t. Circuit behavior is described primarily by waveforms and much of design optimization involves waveforms, making them central to power-electronics design.

In switched power electronics, switching causes discontinuous waveforms in both x and t. Such waveforms are *digital*. Waveforms with two values in their range are *square-waves*. The values are labeled *high* or *on*, and *low* or *off*. Additionally, square-waves change value at discrete times.

In switched power circuits, square-waves or other *switched waveforms* that are periodic have two states in their cycle and are often characterized by the parameter D, the *duty-ratio*, defined as the fraction of the *switching period*, T_s, that the waveform level is in the high or *on* state;

$$D = \frac{t_{on}}{T_s}$$

Its complement is

$$D' = 1 - D = \frac{t_{off}}{T_s}$$

For all waveforms, the *waveshape* is the waveform without regard to scaling. The waveshape of a periodic $x(t)$ can be expressed as invariant in x and t by normalizing both;

$$\text{Waveshape of } x(t) = \frac{x(t/T)}{\hat{x}}$$

where T is the waveform period and $\hat{x}$ is the peak or maximum value of $x(t)$.

Waveforms are often too complicated in themselves to use for analysis or design. Instead, parameters are extracted from them that simplify and focus attention on some aspect of circuit design or behavior. One common parameter is the *average*, defined for periodic waveform $x(t)$ as

$$\bar{x} = \frac{1}{T} \cdot \int_0^T x(t) \cdot dt$$

over the time interval from zero to *T*. The average value is often used to characterize power-circuit performance because it corresponds to the desired goal for many converter input and output waveforms: constant voltage or current values. No actual waveform is truly constant but the average of a near-constant waveform can be regarded as though it is its value as an ideal constant waveform.

In the linearization of circuit models, waveforms are decomposed into a constant or *static* operating-point value, X, and a *varying* incremental quantity, $X_{\sim}(t)$. As shown for the waveform in Fig. 1, $X = I_g$ is the average value of the total waveform, $x(t) = i_g(t)$.

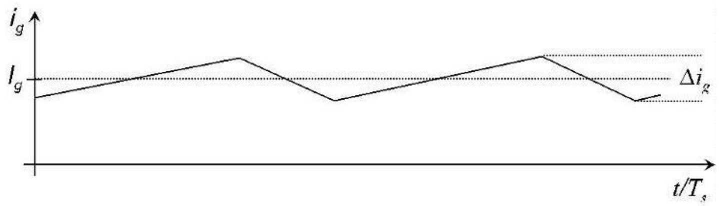

Fig. 1. Waveform $i_g(t)$ is the total variable composed of a constant average, I_g added to a varying component with a range of variance of Δi_g. The average approximates the ideal of a constant value.

X is sometimes referred to as the *quiescent* ($x_{\sim} = 0$) value of the *total* variable, x, where x is the sum of the static and incremental quantities;

$$x = X + x_{\sim}$$

$x_{\sim}$ represents a small variation of $x(t)$ around the constant X value. Thus $x_{\sim}$ is a change in x, or Δx. If the change occurs equally on both sides of X, then $x_{\sim}$ is *bipolar* (positive and negative in range). As a dependent variable, for small Δx, it is approximately the differential of x, or

$$x_{\sim} = \Delta x \approx dx$$

The quantity $x_{\sim}$ is the *ripple* of x. Whenever $x_{\sim}$ can be considered small relative to X, the waveform satisfies the *small-ripple approximation*:

$$\text{Small-ripple approximation: } x_{\sim} << X$$

Then the waveform average is approximately the static value, as shown for the waveform in Fig. 2.

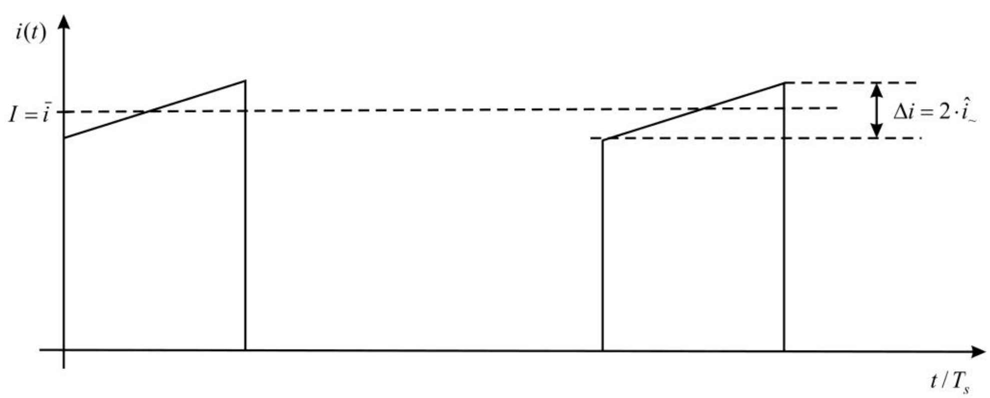

Fig. 2. Current waveform that is discontinuous in mode (DCM) because it is zero for a finite interval in the cycle.

Power waveforms often have a zero value for part of a switched cycle. The *amplitude* is the waveform maximum value while non-zero or *on*, and can be approximated as the average on-time value of the waveform if the on-time ripple is small, as illustrated in Fig. 2. I is the average value of $i(t)$ whenever $i \neq 0$ A, or is the average amplitude of $i(t)$. I is thus the average of the on-time $i(t)$.

A compact waveform parameter notation that is used here is given in the following table.

Table. Waveform parameter notation.

$\overline{x}$	average x
$\hat{x}$	peak (maximum) x
$\tilde{x}$	rms value of x

The peak value of $x(t)$ is its maximum. The *valley* value (not denoted in the table) is its minimum. The value of x representing voltage or current that dissipates the same average power in a resistance as that of a constant voltage, V, or current, I, is the *rms* value.

Waveform Performance Parameters

Waveform parameters are more meaningful when related to the functions to be achieved by power circuits. Often the desired goals for power circuits are similar for a wide class of different circuit functions. For instance, it is almost always desirable to maximize circuit efficiency, for this makes circuit behavior closer to that of the desired or *ideal* function. In engineering, *figures of merit* or *demerit* are *performance parameters*, quantities that make such ideals more explicit. They guide design as parameters to be maximized or minimized in the search through conflicting criteria for an optimal solution.

In power-circuit design, quantities to be minimized, or *figures of demerit*, are applied to the central behavioral element: waveforms. Two of these performance-related, optimizing quantities are used frequently. The first is the *form factor*, κ, defined as

$$\kappa = \frac{\tilde{x}}{\overline{x}}$$

As the ratio of waveform rms to average, it is the fraction of the quantity related to circuit power losses (the rms) to that which is desired (the average). The rms value is related to circuit loss from Watt's Law;

$$\overline{P} = \tilde{i}^2 \cdot R = \tilde{v}^2 / R$$

The rms values of i or v waveforms are related by their square to dissipative losses in circuit resistance. To normalize the extent of the losses, they are scaled by the average, which is the value of the waveform that expresses circuit scale—the "sizing" of the circuit. This scaling makes κ a general figure of demerit in that it is invariant to circuit scaling and applies as much to a 1-W as to a 1-kW converter. It is an inefficiency measure of voltage or current waveforms and is generally to be minimized in design wherever the average is desired, as is common for converters.

The second figure of demerit is the *crest factor*,

$$\chi = \frac{\hat{x}}{\tilde{x}}$$

This waveform parameter is mainly useful for optimizing inverter design, where the output is a bipolar waveform, $x(t)$. In this case, the desired and specified (or given) quantity is the rms value of the output waveform. It is desired that the peak value of x be minimized relative to the rms value because this minimizes the ratings (or electrical "sizing") required of power components, and hence their cost and volume (or mechanical size).

Components are rated by their maximum (peak) or rms current and voltage values. The ratings must in either case exceed, by some reliability margin, the actual limiting values that occur in the circuits. To deliver a given rms quantity with minimal sizing of parts, χ is minimized in design. Sine waves have $\chi^2 = 2$. Square-waves with $D > ½$ have $\chi^2 < 2$. Thus, inverters can be designed more cheaply if they output square waves instead of sine waves. For constant x, both χ and κ equal one.

The third performance factor is a *figure of merit* and is useful in assessing the extent to which a component is being used. The *utilization* is defined as

$$U = \frac{\bar{x}}{\hat{x}}$$

The average of voltage or current associated with a component relates to the benefit from using it or to its functional sizing. The peak relates to the rating of the part and its required sizing. The desired value of U is maximum whenever maximum capability can be achieved by the part. $U = 1$ when $\bar{x} = \hat{x}$ and this implies a constant waveform. A constant waveshape is optimal for all three performance figures though it conflicts with the intended behavior of transformers and coupled inductors (or *transductors*) which require varying waveforms and results in one of the fundamental tradeoffs in power-circuit design.

These performance-optimizing quantities guide engineering search and decision-making but are usually not determinative in circuit design because multiple optimizations must be satisfied with a global solution that might not optimize any one of them. Yet they offer the engineer a simplifying insight into optimization activity and are useful in guiding it. They are covered more thoroughly in reference [1].

Ripple Characterization

The centrality of waveforms in both power circuit and magnetics design leads to a further development of how to characterize them, namely the ripple component.

Useful quantities for characterizing the amount of ripple of waveforms are the *ripple factors*, denoted generally by γ. The *peak ripple factor* is defined as

$$\hat{\gamma} = \frac{\Delta x}{\hat{x}}$$

The peak-to-peak ripple, Δx, is scaled by the peak x and expresses the fraction of the total waveform value that is ripple.

A generally more useful quantity is the *average ripple factor*,

$$\bar{\gamma} = \frac{\Delta x / 2}{\bar{x}} = \frac{\hat{x}_{\sim}}{\bar{x}}$$

where $\Delta x/2 = \hat{x}_{\sim}$ is the amplitude or peak (^) of the ripple (~) as a component waveform, scaled by the average x.

Both ripple factors are defined so that the boundary between a DCM and CCM waveform is at a value of $\gamma = 1$. Then these regions can be defined by the value of γ:

$\gamma \leq 1 \Rightarrow$ CCM

$\gamma = 1 \Rightarrow$ CCM at the DCM/CCM boundary (BCM)

$\gamma > 1 \Rightarrow$ DCM

Other definitions of ripple factor have been given elsewhere in the literature. The above definitions are preferred because of their relationship to DCM and CCM, placing $\gamma = 1$ at the boundary.

Waveform parameters can be expressed in γ as follows.

For $\hat{\gamma}$,

$$\hat{x} = \Delta x / \hat{\gamma} \ ; \ \Delta x = \hat{\gamma} \cdot \hat{x} \ ; \ \bar{x} = \Delta x \cdot \left(\hat{\gamma} - \tfrac{1}{2}\right) = \hat{x} \cdot (1 - \hat{\gamma}/2)$$

For $\bar{\gamma}$,

$$\hat{x} = \bar{x} \cdot (1 + \bar{\gamma}) \ ; \ \Delta x = 2 \cdot \bar{\gamma} \cdot \bar{x} \ ; \ \bar{x} = \Delta x / (2 \cdot \bar{\gamma}) = \hat{x} / (1 + \bar{\gamma})$$

The defined ripple factors are convertible to each other;

$$\hat{\gamma} = \frac{\Delta x}{\bar{x} + \Delta x / 2} = \frac{2 \cdot \bar{\gamma}}{1 + \bar{\gamma}} \ ; \ \bar{\gamma} = \frac{\hat{\gamma}/2}{1 - \hat{\gamma}/2}$$

The expressions for $\bar{\gamma}$ are somewhat simpler than for $\hat{\gamma}$ and are one reason to prefer $\bar{\gamma}$ as a default ripple factor, hereafter denoted more simply by γ:

$$\text{default } \gamma \equiv \bar{\gamma}$$

Another reason is that waveforms related to specifications are expressed commonly in $\bar{x}$, and these quantities appear in the design equations involving waveform ripple.

Form factor can be expressed in ripple factor as

$$\kappa = \frac{\tilde{x}}{\bar{x}} = \frac{\sqrt{\bar{x}^2 + \tilde{x}_\sim^2}}{\bar{x}} = \sqrt{1 + \left(\frac{\tilde{x}_\sim}{\bar{x}}\right)^2}$$

Substitute the rms expression for CCM triangle-waves;

$$\tilde{x}_\sim = \frac{\Delta x / 2}{\sqrt{3}} = \frac{\hat{x}_\sim}{\sqrt{3}}$$

Then for triangle-waves,

$$\kappa = \sqrt{1 + \left(\frac{\hat{x}_\sim / \sqrt{3}}{\bar{x}}\right)^2} = \sqrt{1 + \frac{\gamma^2}{3}}$$

Utilization can also be expressed in γ as

$$U = \frac{\bar{x}}{\hat{x}} = \frac{1}{1 + \gamma}$$

For a waveform, $x(t)$, for which $\bar{x} = \hat{x}_{\sim} = \Delta x/2$, then γ = 1 and U = $\frac{1}{2}$. Waveforms for which the small-ripple approximation is valid have $\gamma \approx 0$ and $U \approx 1$. It would seem therefore that minimization of ripple is desirable, that γ is a figure of demerit, and that $\gamma = 0$ is a design goal. In magnetics design, we will find that this is not always true and depends on which design criteria are to be optimized.

Waveforms are central to the description of circuit behavior, and their characterization establishes foundational concepts that are used throughout power electronics.

References

All the works cited in this section are by the author, Dennis Feucht.

1. *Power Magnetics Design Optimization* (*PMDO*), see chapter 1, "Waveforms". This work is available in laminated-paper book form at www.innovatia.com or contact the author for a free PDF copy at www.innovatia.com/Inquiry.htm.

The following references provide example uses of the various figures of merit and demerit defined in this article.

Form factor (κ), ripple factors (γ), crest factor (χ) and utilization U:

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About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

For more on magnetics design, see these How2Power Design Guide search results.