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PFC (Part 2): How Current Harmonics Cause Distortion And The Role of the Delta-Wye Transformer

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The first part of this article[1] reviewed the evolution of power factor correction (PFC) requirements and discussed the IEC 61000-3-2 PFC standard. Here in part 2, the relationship between line frequency harmonics and distortion is analyzed, and we explain how the delta-wye transformer corrects this distortion on the power grid.

Wattage Equation For Distortion

We am starting with wattage or the use of electrical energy to perform work. This wattage equation is shown in equation 1. It consist of a direct current component, V_{DC} times I_{DC} plus the second term which is a Fourier series of ac terms multiplied by the phase angel, Θ, between the voltage and current of that harmonic. These equations were published in IEEE conferences [2,3].

$$Watts = V_{DC} I_{DC} + \sum_{k=1}^{\infty} V_k I_k \cos\theta_k \qquad (1)$$

as *k* goes from 1 to ∞, and Θ_k is the phase angle between the voltage and current of the particular harmonic being evaluated. In most cases only the odd harmonics are seen. All of the even harmonics are 0 (zero). Equation 2 may be an easily understood form of equation 1.

$$Watts = V_1 I_1 \cos\theta_1 + V_3 I_3 \cos\theta_3 + V_5 I_5 \cos\theta_5 + V_7 I_7 \cos\theta_7 ..... \qquad (2)$$

All values in the above equations are in rms values. This is true for all equations in this article.

The ac power is a sine wave voltage for the purpose of the IEC 61000-3-2; there are no dc components of voltage or current. There are only odd components present. The 2005 IEC 61000-3-2 version in Annex A[2], section A.2 Supply source specifies the following in section c):

The harmonic rations of the test voltage (*U*) shall not exceed the following values with the EUT connected as in normal operation:

0.9% for the harmonic of order 3;

0.4% for the harmonic of order 5;

0.3% for the harmonic of order 7;

0.2% for the harmonic of order 9;

0.2% for the even harmonic of order from 2 to 10;

0.1% for harmonics of the order from 11 to 40.

The ac voltage has only a fundamental: V_1, and all the other harmonic voltages have zero (0) amplitude. This causes equation 2 to become equation 3:

$$Watts = V_1 I_1 \cos\theta_1 \qquad (3)$$

The current is not a sine wave but a distorted wave. Many of the current electronic wattmeters measure the harmonic current in terms of total harmonic distortion (THD). The THD will be developed with some algebra shown below.

$$I_{RMS} = \sqrt{I_1^2 + I_3^2 + I_5^2 + I_7^2 + I_9^2 + \cdots I_\infty^2} \qquad (4)$$

$$I_{RMS}^2 = I_1^2 + I_3^2 + I_5^2 + I_7^2 + I_9^2 .... \qquad (5)$$

$I_{RMS}^2 - I_1^2 = I_3^2 + I_5^2 + I_7^2 + I_9^2 ....$ This defines the nonfundamental current squared. (6)

$\sqrt{I_{RMS}^2 - I_1^2} = \sqrt{I_3^2 + I_5^2 + I_7^2 + I_9^2 ....}$ This defines the nonfundamental current . (7)

In equations 4, 5, 6 and 7 all values of current are rms values.

THD is the ratio of the nonharmonic current to the fundamental current as shown in equation 7.

$\text{Current } THD = \dfrac{Current - not - fundamental}{I_{1-Fundamental}}$ Stated in math terms below in equation 8.

$$THD = \frac{\sqrt{I_3^2 + I_5^2 + I_7^2 + I_9^2 ....}}{I_1} = \sqrt{\frac{I_{RMS}^2 - I_1^2}{I_1^2}} \qquad (8)$$

$$THD = \sqrt{\frac{I_{RMS}^2 - I_1^2}{I_1^2}} \rightarrow THD^2 = \frac{I_{RMS}^2 - I_1^2}{I_1^2} \rightarrow THD^2 = \frac{I_{RMS}^2}{I_1^2} - 1 \qquad (9)$$

$$THD^2 + 1 = \frac{I_{RMS}^2}{I_1^2} \rightarrow \sqrt{THD^2 + 1} = \frac{I_{RMS}}{I_1} \qquad (10)$$

$$\left(\frac{I_1}{I_{RMS}}\right) = \sqrt{\frac{1}{THD^2 + 1}} \qquad (11)$$

Equation 8 is the current that is not the fundamental divided by the fundamental current. This becomes important in the development of the THD equation for current. Multiply equation 3 by a new unity factor as shown in equation 9 and there is no change to the values.

Please note: there is voltage THD that is the same concept as the current. The voltage THD was first used in audio amplifiers to describe how well an audio amplifier reproduced sound driving a speaker.

$$Watts = V_1 I_1 \cos\theta_1 \left(\frac{I_{RMS}}{I_{RMS}}\right) \rightarrow Watts = V_1 I_{RMS} \cos\theta_1 \left(\frac{I_1}{I_{RMS}}\right) \rightarrow Watts = V_{RMS} I_{RMS} \cos\theta_1 \left(\frac{I_1}{I_{RMS}}\right) \qquad (12)$$

Note V_1 and V_{RMS} are the same value since there are no voltage harmonics when making the measurement in the IEC 61000-3-2 specification. In equation 4, I_{RMS} is full of harmonics and can't be simplified.

Substituting equation 11 into equation 12 gives an equation that includes harmonics.

$Watts = V_{RMS} I_{RMS} \cos\theta_1 \sqrt{\dfrac{1}{THD^2 + 1}}$, the total power equation with distortion and displacement. (13)

The displacement factor is $\cos\theta$

The distortion factor is $\sqrt{\frac{1}{THD^2+1}}$

The system power factor is $\cos\theta_1\sqrt{\frac{1}{THD^2+1}}$ the product of the displacement factor and distortion factor.

The input EMI film capacitors will provide a small leading displacement power factor of 10^0, which results in a 0.984 power factor. A THD of 34% will result in a distortion factor of 0.946. The total power factor is 0.931. The IEC 61000-3-2 has four classes A, B, C and D. Only Class C lighting provides a limit as a percent of the fundamental. Classes A, B and D of the standard provided current values.

The electronic wattmeters from Voltech (now Tektronix), Newton's 4th, Yokogawa, etc. provide a single button to determine the distortion for either voltage or current. This feature allowed design engineers to determine what effect any power factor correction component had on their product.

Removing Harmonics From Power Distribution On the Grid

In this part of the article I will discuss how the delta-wye transformer can cancel third-order harmonics. Instead of mathematics, graphical analysis will be used. A typical electrical distribution system could look like that shown in Fig. 1. A similar figure was given in the part 1 of this article, but here Δ and Y symbols are added to indicate the configurations of the transformer windings.

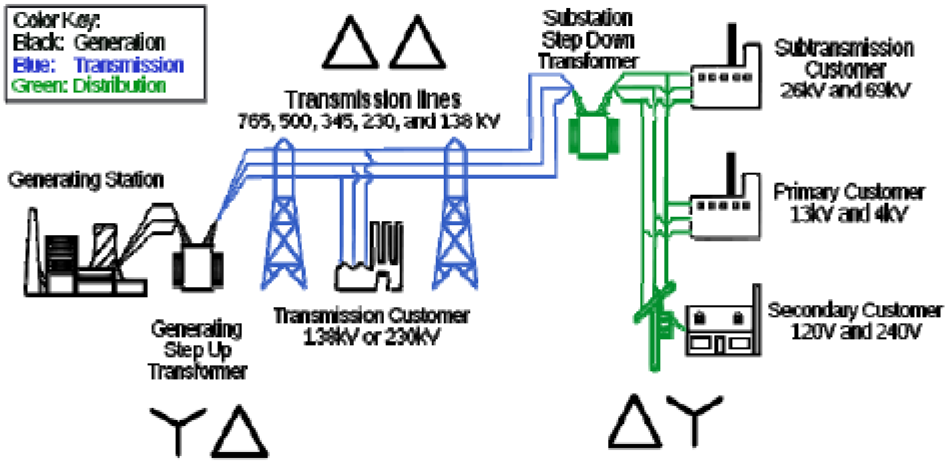


Fig. 1. Block diagram of a utility transmission system. Courtesy of Cirrus Logic's Power Meter presentation given by Alan (Ya Long) Zha in 2010. The Δ and Y transformer symbols were added here by Jim Spangler.

A graph of a three-phase voltage system with both the delta and wye voltages can be seen below in Fig 2. The graph of the various voltages are easy to generate using an Excel spreadsheet. The way to start is with the information below. Each is a column in the spreadsheet. The first column is degrees which starts at 0 (zero) and is incremented by 5 degrees for two complete cycles or 720 degrees. It is important to remember that Excel operates in radians and not degrees. A conversion must be made between degrees and radians.

Phase A graph SIN(A4*PI()/180)

Phase B graph SIN((A4+120)*PI()/180)

Phase C graph SIN((A4-120)*PI()/180)

Phase A-Phase B Graphic = SIN(A4*PI()/180)- SIN((A4+120)*PI()/180)

Phase B-Phase C Graphic = SIN((A4+120)*PI()/180)- SIN((A4-120)*PI()/180)

Phase C-Phase A Graphic = SIN((A4-120)*PI()/180)- SIN((A4-120)*PI()/180)

A4 is in degrees from 0 to 720 in 5-degree steps.

PI() is the excel method to describe pi (π) 3.14259. . .

The Excel method to change degrees to radian is (degrees (A4) * PI()/180).

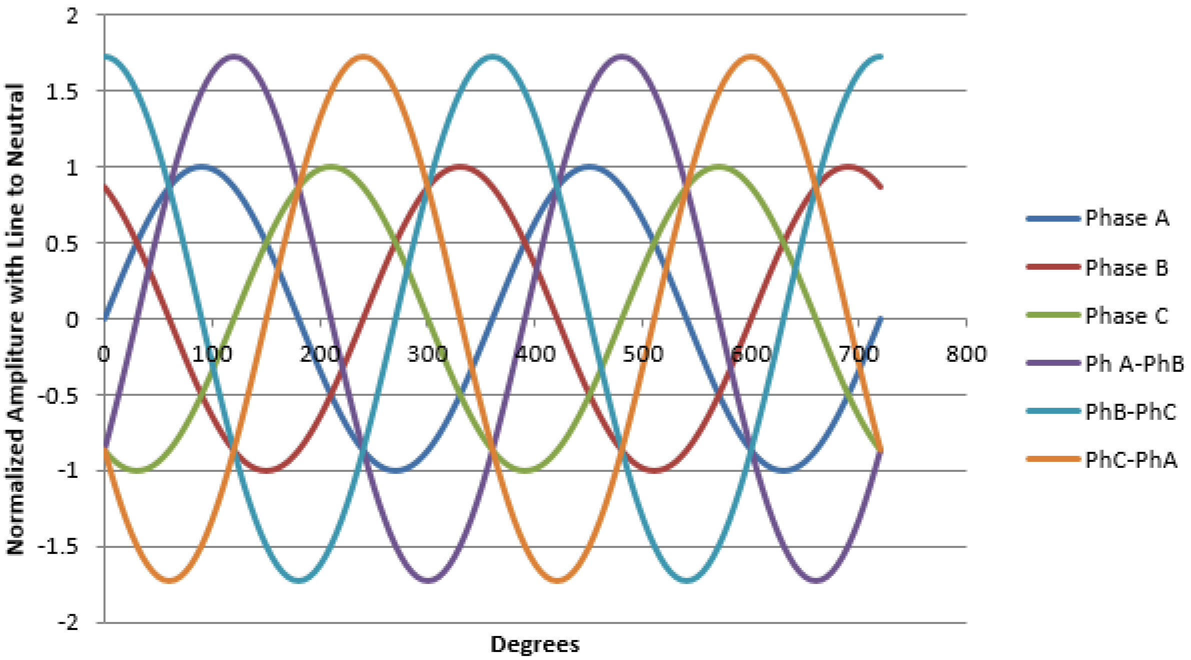


Fig 2. Three-phase voltage waveforms in wye and delta. Note that the delta is 1.73 times the wye line-neutral voltage and there is a 30 degree shift between the two.

At universities where electrical machines are taught there is often a machine lab. In this course both induction motors and generators are taught. In the lab, one interesting point is that the voltages generated by the ac generators are not perfect sine waves. The voltages could have third- and other-order harmonics. The third-order harmonic, which can be seen, could be additive or subtractive. If the generator has an additive third-order harmonic you would see a flat top.

An important note: The graph waveforms shown here were generated using an Excel spreadsheet. A copy of this spreadsheet is available online.[4]

The graph in Fig. 3, which was generated using the Excel spreadsheet with a 50% third-order harmonic added, shows an exaggerated additive effect with double humps seen as the red trace. If the generator produces a subtractive third order, you would see a peak like the green trace in Fig. 3. Again, this exaggerated effects is caused by a 50% third-order harmonic.

These negative third-order harmonics can be eliminated in the distribution wye-delta transformer. This occurs at the power generating station where the generator output terminals are connected to a wye primary transformer as in Fig. 1 which shows the locations of the various transformers.

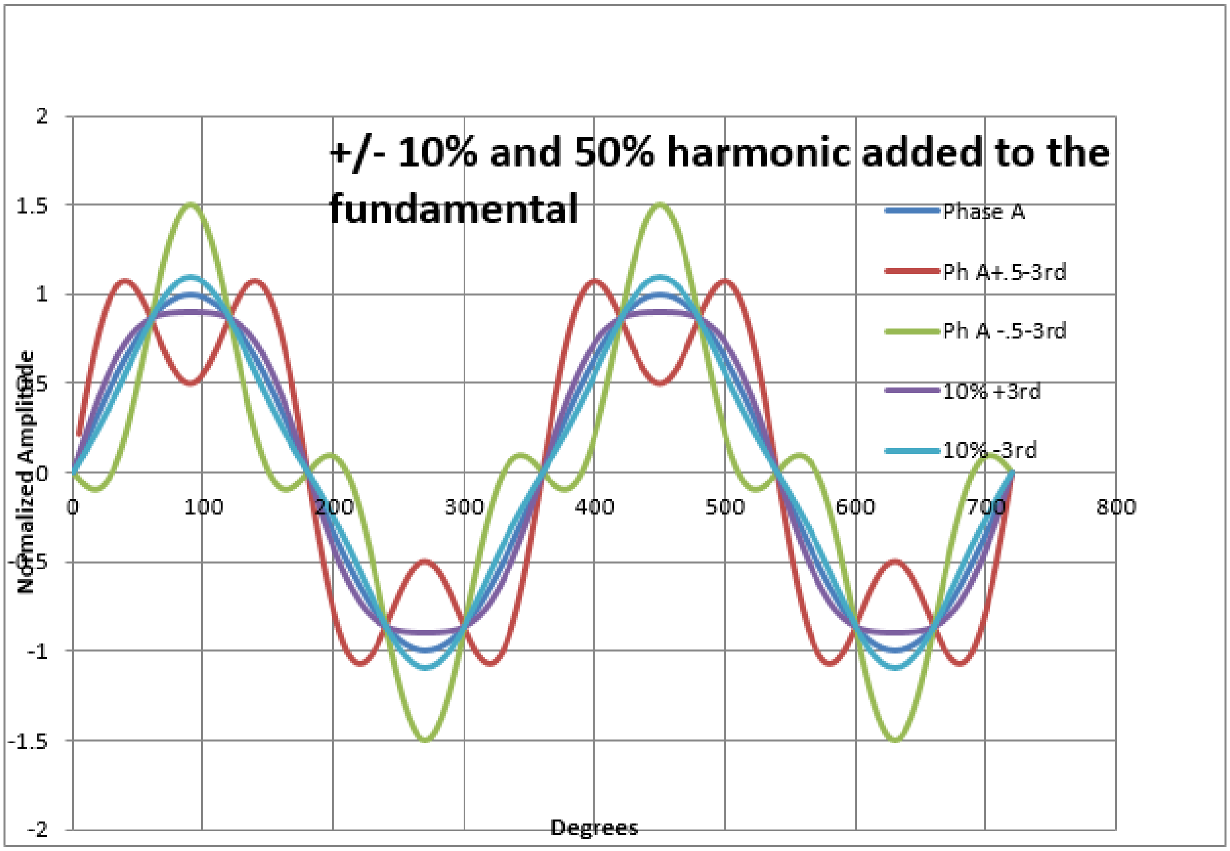


Fig 3. Harmonics created in a generator. Note that both additive and subtractive third order can be present.

SIN(A4*PI()/180)	Fundamental
J4-0.5*SIN(3*I4*PI()/180)	Fundamental minus 0.5 of the third harmonic
J4+0.5*SIN(3*PI()*I4/180)	Fundamental plus 0.5 of the third harmonic
J4+0.1*SIN(3*PI()*I4/180)	Fundamental plus 0.1 of the third harmonic
J4-0.1*SIN(3*PI()*I4/180)	Fundamental minus 0.1 of the third harmonic

The secondary of the transformer is connected to the transmission lines. The high-voltage transmission lines do not need a neutral line. This saves power lines as shown in Fig. 1. The simple diagram for the transformer is shown in Fig 4, while a corresponding winding diagram is shown in Fig. 5.

The beauty of this arrangement is seen by looking at Fig. 6, which has five traces. This is to show that if there are harmonics in the wye-connected A and B phase wye, the third-order harmonic will cancel out.

The delta voltage will have a $\sqrt{3}$ larger magnitude than the line-neutral voltage plus a phase shifted by 30^{o} like that shown in Fig.2. These harmonics are contained in the generation station and will not be transmitted down the delta transmission lines. It is important that the third-order harmonic produce circulating currents in the transformer wye which has the neutral wire that can cause the transformer to heat up.

The easiest way to show the three phases of a delta and a wye is in an Excel spread sheet graph.

When the transformer is connected to the generator, their third-order harmonics cancel. The delta is normally on the transmission side of the transformer. The wye (Y) is normally on the load or generator side of the transformer. The transmission side is normally three wires while the load and generator side is four wires and the neutral can carry a large amount of current. The neutral wire is not shown in Figs. 4 and Fig. 5 nor is the resistance of the windings.

The line-to-line voltage is the delta, while the line-to-neutral voltage is the wye (Y). A delta-wye transformer is seen in Fig 5.

Note V_{LN} is V_{AN}, V_{BN}, and V_{CN} as shown in Fig. 2; while the phase-to-phase voltage shown in Fig. 2 and Fig. 4, are described by equations 14 and 15 with a -30^{O} shift.

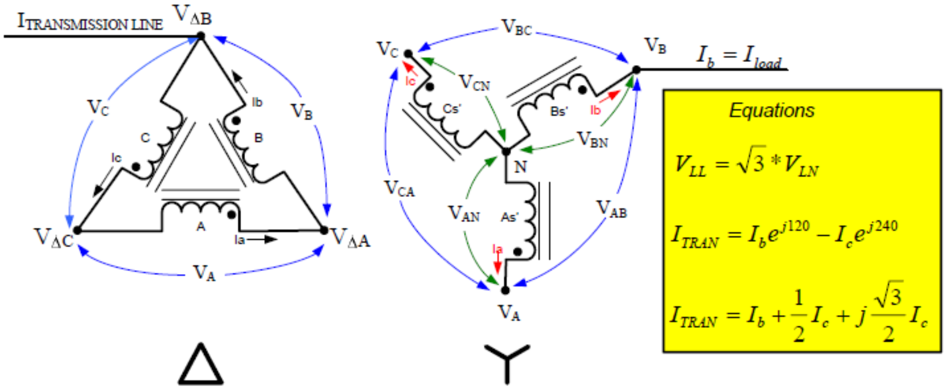


Fig. 4. The delta winding is normally on the transmission side of the transformer. The wye (Y) is normally on the load or generator side of the transformer. The transmission side is normally three-wire while the load and generator side is four wires and the neutral can carry a large amount of current.

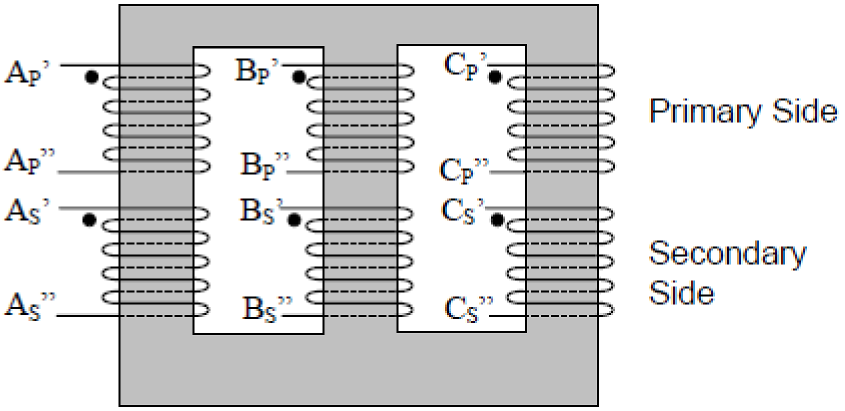


Fig 5. Three-phase transformer with both primary and secondary windings shown. The dots are shown because of phasing. The left side is associated with phase A, while the right side is associated with phase C. The dot notation is shown in Fig. 4 for connection to obtain the correct phasing.

$$V_{LL} = \sqrt{3} * V_{LN} \tag{14}$$

$$V_{LN} = \frac{1}{\sqrt{3}} * V_{LL} \tag{15}$$

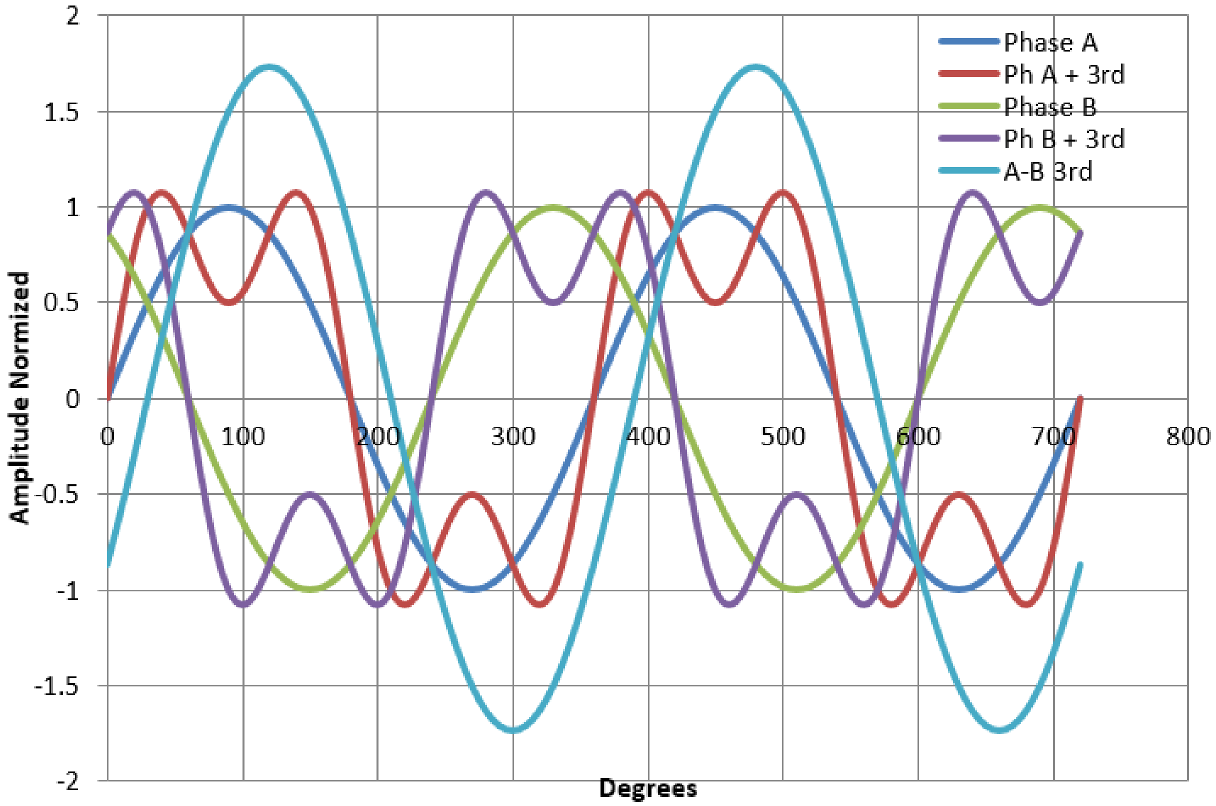


Fig 6. Primary and secondary voltages with the largest voltage A-B. Note the large blue trace is shifted 30^0 degrees from the A phase. The third-order harmonics have been cancelled. This is what occurs in all delta-wye transformers, there is a phase shift.

Motor Drives

A simple three-phase input to a motor drive is shown in Fig 7. The single-phase rectified input voltage and current are shown in Fig. 8, where current is shown as the red trace. When a power analyzer is used and the current is analyzed, the input current is seen to be composed of nearly equal amplitude of fundamental and third-order harmonic.

The normal voltage on the secondary can be a line-to-line voltage of 208 Vac to 480 Vac. The secondary transformer is a wye while the primary connected to the transmission lines is a delta. The input current is shown as the red trace while the input voltage is the blue trace.

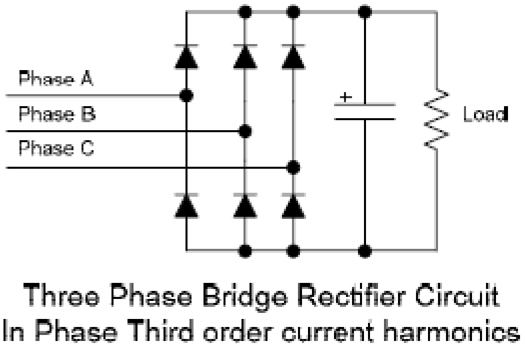


Fig 7. Motor control input rectification. The load could be any number and type of motors.

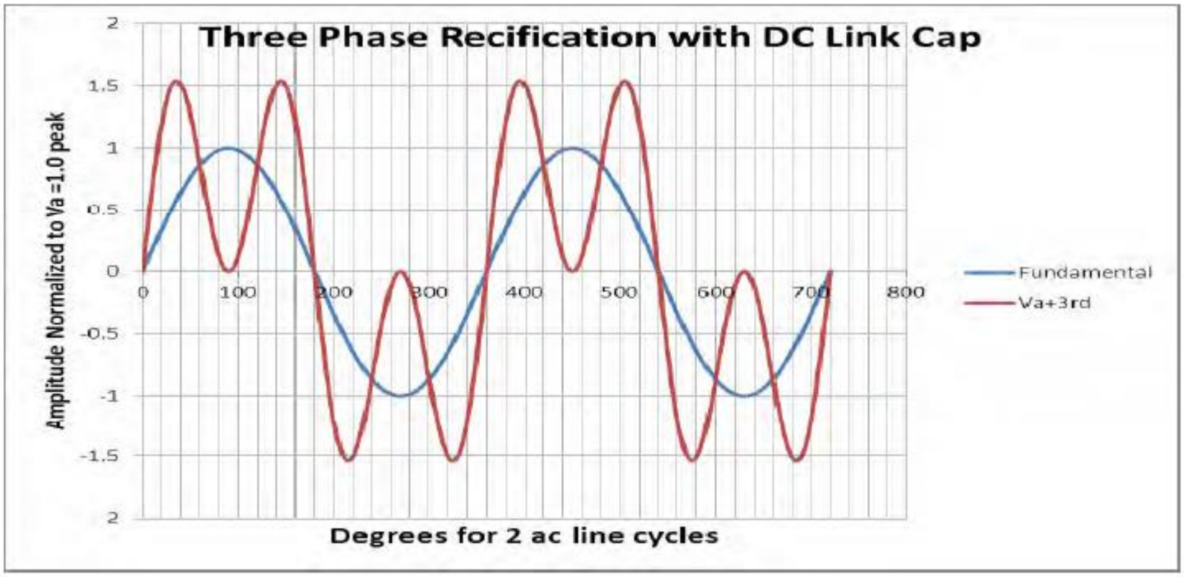
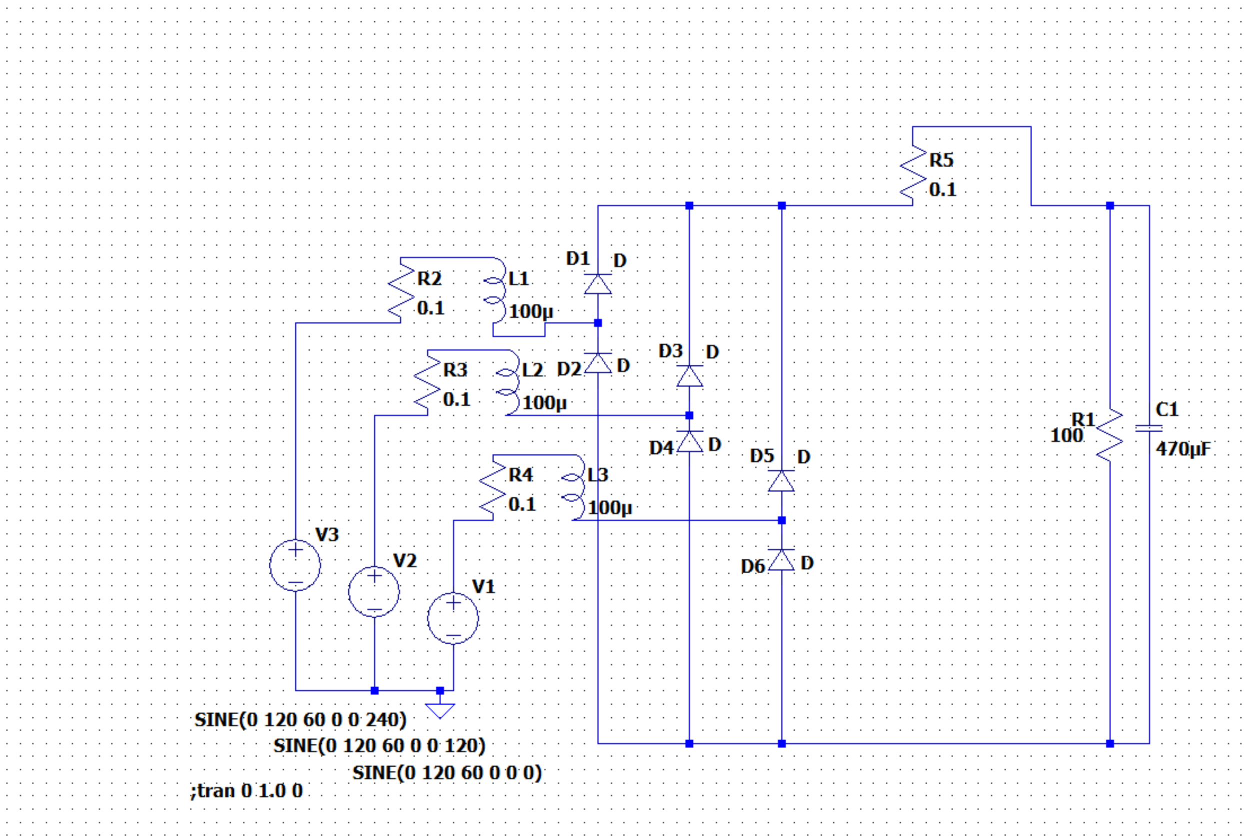


Fig 8. Three-phase motor drive input voltage (blue) and current (red).

An LT Spice circuit can be built using Fig. 7 as shown in Fig 9. Each of the phase currents can be measured. In addition, an analysis can be performed on the input phase currents, which includes a spectrum analysis to show harmonics. This method is used to obtain results without building circuits.



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SINE(0 120 60 0 0 240)
SINE(0 120 60 0 0 120)
SINE(0 120 60 0 0 0)
;tran 0 1.0 0
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Fig. 9. LT Spice of a three-phase-input bridge rectifier.

References

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2. "Electronic Fluorescent Ballast using Power Factor Correction Techniques for Load greater than 300 Watts" by James Spangler and Behera Hussain, IEEE APEC Conf. Proc., Mar 4-8, 2001, Anaheim, CA, pp.393-399.
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4. http://www.how2power.com/newsletters/2003/Sine-Wave-three-phase.xlsx.
5. IEC 61000-3-2. 2005 standard page 39 Annex A.

About The Authors

Kevin Parmenter is an IEEE Senior Member and has over 20 years of experience in the electronics and semiconductor industry. Kevin is currently director of Field Applications Engineering North America for Taiwan Semiconductor. Previously he was vice president of applications engineering in the U.S.A. for Excelsys, an Advanced Energy company; director of Advanced Technical Marketing for Digital Power Products at Exar; and led global product applications engineering and new product definition for Freescale Semiconductors AMPD - Analog, Mixed Signal and Power Division.

Prior to that, Kevin worked for Fairchild Semiconductor in the Americas as senior director of field applications engineering and held various technical and management positions with increasing responsibility at ON Semiconductor and in the Motorola Semiconductor Products Sector. Kevin also led an applications engineering team for the start-up Primarion.

Kevin serves on the board of directors of the PSMA (Power Sources Manufacturers Association) and was the general chair of APEC 2009 (the IEEE Applied Power Electronics Conference.) Kevin has also had design engineering experience in the medical electronics and military electronics fields. He holds a BSEE and BS in Business Administration, is a member of the IEEE, and holds an Amateur Extra class FCC license (call sign KG5Q) as well as an FCC Commercial Radiotelephone License.

Jim Spangler is a Life Member of the IEEE with over 40 years of electronics design experience and is president of Spangler Prototype Inc. (SPI). His power electronics engineering consulting firm's priority is helping companies to place products into production, assisting them to pass government regulations and agency standards such as UL, FCC, ANSI, IES, and the IEC.

For many years, he worked as a field applications engineer (FAE) for Motorola Semiconductor, On Semiconductor, Cirrus Logic, and Active Semiconductor, assisting customers in using semiconductors. He published numerous application notes and conference papers at a variety of conferences: APEC, ECCE, IAS, and PCIM. Topics included power factor correction, lighting, and automotive applications. As an FAE, he traveled internationally giving switch-mode power supply seminars in Australia, Hong Kong, Taiwan, Korea, Japan, Mexico, and Canada.

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Jim is a member of the IEEE: IAS, PELS, PES; the Illuminating Engineering Society (IES), and the Power Sources Manufacturers Association (PSMA) where he is co-chair of the Safety and Compliance Committee.

For further reading on power supply-related safety and compliance issues, see How2Power's special section on Power Supply Safety and Compliance.