

## **Determining Design Power Over An Input Voltage Range (Part 2): Inductor Design Power**

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In part 1, the maximum power handled by the inductor of a PWM-switch converter,  $\bar{P}_{L_{\max}}$ , was defined in relation to input power  $\bar{P}_g$  for the three PWM-switch configurations (see the reference).  $\bar{P}_{L_{\max}}$  tells us the maximum amount of power the inductor will carry over the  $V_g$  range, but this is not the power rating we can use to optimally design the inductor for size. To that end, we need a new parameter, which I have dubbed *design power*.

Why? Because the inductor has a fixed structure that must accommodate the ranges of both the voltage and the current it sees. The maximum values in each of these ranges,  $V_{g_{\max}}$  and  $\bar{i}_{g_{\max}}$ , must be considered. While  $V_{g_{\max}}$  and  $\bar{i}_{g_{\max}}$  never occur together, the inductor must be designed for both. This suggests for design a maximum input power of

$$\bar{P}_{g_{\max}} = V_{g_{\max}} \cdot \bar{i}_{g_{\max}}$$

where  $V_{g_{\max}}$  is the maximum input voltage and  $\bar{i}_{g_{\max}}$  is the maximum average input current (both at the input to the switch).

However, this  $\bar{P}_{g_{\max}}$  is a worst-case (maximum) value for calculating the *inductor design power*,  $\bar{P}_{L_d}$ . The power-rating question needs further investigation because  $\bar{P}_{g_{\max}}$  is not necessarily the optimal value of *inductor design power*. As we'll see, that will depend on the PWM-switch configuration.

### **Design Power Vs. Input Power**

We can relate  $\bar{P}_{g_{\max}}$  to average inductor input power as follows. At minimum  $V_g$ ,

$$\bar{P}_g = V_{g_{\min}} \cdot \bar{i}_{g_{\max}} = (V_{g_{\max}} / r) \cdot \bar{i}_{g_{\max}} = \bar{P}_{g_{\max}} / r$$

Or equivalently, at the maximum- $V_g$  operating-point,

$$\bar{P}_g = V_{g_{\max}} \cdot \bar{i}_{g_{\min}} = V_{g_{\max}} \cdot (\bar{i}_{g_{\max}} / r) = \bar{P}_{g_{\max}} / r$$

At both extremes,

$$\bar{P}_{g_{\max}} = r \cdot \bar{P}_g = \left( \frac{V_{g_{\max}}}{V_{g_{\min}}} \right) \cdot \bar{P}_g$$

The corresponding maximum on-time power amplitude is

$$\begin{aligned} P_{g_{\max}} &= V_{g_{\max}} \cdot I_{g_{\max}} = (r \cdot V_{g_{\min}}) \cdot I_{g_{\max}} = V_{g_{\max}} \cdot (r \cdot I_{g_{\min}}) \\ &= r \cdot P_g = \left( \frac{V_{g_{\max}}}{V_{g_{\min}}} \right) \cdot P_g \end{aligned}$$

These power quantities are graphed in Fig. 1. Each pair of power curves on the same graph differ by a factor of  $r$ .

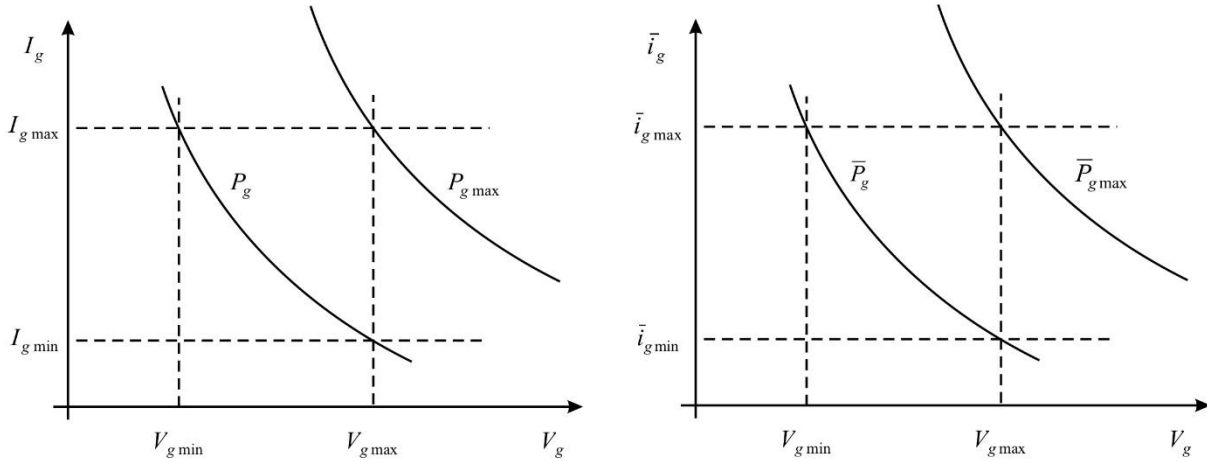


Fig. 1. Input-power plots at rated power,  $P_g$  and the greater design power,  $P_{gmax}$  (left graph). Similar plots for average  $P_g$  and average design  $P_g$  are shown on the right. The plot pairs are separated by the  $V_g$  (and  $I_g$ ) range of  $r$ .

The power curves within the operating region have maximum voltages,  $V_{gmax}$  that limit the inductor core flux, and also maximum current,  $I_{gmax}$  or  $\bar{i}_{gmax}$  that limits the winding current. Inductor wire size must be large enough for  $\tilde{i}_{Lmax} \approx \bar{i}_{Lmax}$  for a near-constant current waveform. ( $\bar{i}_{Lmax}$  varies parametrically in “design space” with  $V_g$  but not in time as a waveform.)  $D$  controls  $\bar{v}_L$  (and hence flux) over the  $V_g$  range. Because of  $D$ ,  $\bar{v}_L$  can be varied and inductor design power can be less than  $\bar{P}_{gmax}$  as

$$\bar{P}_{Ld} = \bar{i}_{Lmax} \cdot \bar{v}_{Lmax}$$

For the three PWM-switch configurations, inductor design power is defined as

$$\bar{P}_{Ld} = \bar{i}_{Lmax} \cdot \bar{v}_{Lmax} = \bar{i}_{gmax} \cdot (D \cdot V_g)_{max}, CA$$

$$\bar{P}_{Ld} = \bar{i}_{Lmax} \cdot \bar{v}_{Lmax} = (\bar{i}_{gmax} / D_{max}) \cdot (D' \cdot D \cdot V_g)_{max}, CP$$

$$\bar{P}_{Ld} = \bar{i}_{Lmax} \cdot \bar{v}_{Lmax} = (\bar{i}_{gmax} / D_{max}) \cdot (D \cdot V_g)_{max}, CL$$

Then the *design-power ratio*,

$$\frac{\bar{P}_{Ld}}{\bar{P}_g} = \frac{\bar{i}_{Lmax} \cdot \bar{v}_{Lmax}}{\bar{i}_{gmax} \cdot V_{gmin}} = \frac{\bar{i}_{Lmax}}{\bar{i}_{gmax}} \cdot \frac{\bar{v}_{Lmax}}{V_{gmin}}$$

The CA and CL  $\bar{v}_{Lmax}(V_g)$  is found from,

$$\frac{d\bar{v}_L}{dV_g} = \frac{d}{dV_g}(D \cdot V_g) = D + V_g \cdot \frac{dD}{dV_g} = 0$$

from which the values of  $V_g$  and  $D$  at which  $\bar{v}_L$  is maximum are used to find  $(D \cdot V_g)_{max}$ .

For the CA,

$$\frac{d}{dV_g}(D \cdot V_g) = D + V_g \cdot \frac{dD}{dV_g} = \left(1 - \frac{V_g}{V_o}\right) + V_g \cdot \left(-\frac{1}{V_o}\right) = 1 - 2 \cdot \frac{V_g}{V_o} = 0 \Rightarrow \frac{V_g}{V_o} = \frac{1}{2} \Rightarrow D(\bar{v}_{L\max}) = 1 - 1/2 = 1/2$$

Then the CA

$$\bar{v}_{L\max} = (D \cdot V_g)_{\max} = D(\bar{v}_{L\max}) \cdot V_g(D(\bar{v}_{L\max})) = \frac{1}{2} \cdot (V_o / 2) = V_o / 4$$

and  $\bar{i}_{L\max} = \bar{i}_{g\max}$ . Thus

$$\frac{\bar{P}_{Ld}}{\bar{P}_g} = \frac{1}{4} \cdot \frac{V_o}{V_{g\min}}, \text{ CA}$$

For the CL,

$$\frac{d}{dV_g}(D \cdot V_g) = D + V_g \cdot \frac{dD}{dV_g} = \left(\frac{V_o}{V_o + V_g}\right) + V_g \cdot \left(-\frac{V_o}{(V_o + V_g)^2}\right) = \left(\frac{V_o}{V_o + V_g}\right)^2 = 0$$

$$\Rightarrow [D(\bar{v}_{L\max})]^2 = 0 \Rightarrow D(\bar{v}_{L\max}) = 0 \Rightarrow V_g \rightarrow \infty$$

That is, the function continues to increase with increasing  $V_g$ . It is maximum within the range at  $V_{g\max}$  and  $D_{\min}$ ;

$$\bar{v}_{L\max} = (D \cdot V_g)_{\max} = D_{\min} \cdot V_{g\max} = \frac{V_o}{V_o + V_{g\max}} \cdot V_{g\max} \Rightarrow$$

$$\frac{\bar{P}_{Ld}}{\bar{P}_g} = \frac{D_{\min}}{D_{\max}} \cdot \left(\frac{V_{g\max}}{V_{g\min}}\right) \Rightarrow \frac{\bar{P}_{Ld}}{\bar{P}_g} = \frac{V_o + V_{g\min}}{V_o + V_{g\max}} \cdot \left(\frac{V_{g\max}}{V_{g\min}}\right), \text{ CL}$$

For the CP,

$$\frac{d\bar{v}_L}{dV_g} = \frac{d}{dV_g}(D' \cdot D \cdot V_g) = \frac{d}{dV_g} \left( \left(1 - \frac{V_o}{V_g}\right) \cdot V_o \right) = \left(\frac{V_o}{V_g}\right)^2 = 0 \Rightarrow$$

$$[D(\bar{v}_{L\max})]^2 = 0 \Rightarrow D(\bar{v}_{L\max}) = V_o/V_g = 0 \Rightarrow V_g \rightarrow \infty$$

As for the CL, the CP  $\bar{v}_L(V_g)$  monotonically increases over the  $V_g$  range to a maximum at  $V_{g\max}$  and  $D_{\min}$ . Then

$$\bar{v}_{L\max} = (1 - D_{\min}) \cdot D_{\min} \cdot V_{g\max}$$

$$\frac{\bar{P}_{Ld}}{\bar{P}_g} = \frac{\bar{i}_{L\max}}{\bar{i}_{g\max}} \cdot \frac{\bar{v}_{L\max}}{V_{g\min}} = (1 - D_{\min}) \cdot \frac{D_{\min}}{D_{\max}} \cdot \frac{V_{g\max}}{V_{g\min}} \Rightarrow$$

$$\frac{\bar{P}_{Ld}}{\bar{P}_g} = 1 - \frac{V_o}{V_{g \max}}, \text{ CP}$$

The design formulas are summarized in the table.

Table. PWM-switch inductor design-power ratios.

Config	$D$	$D(\bar{V}_{L \max})$	$\bar{V}_{L \max}$	$\frac{\bar{P}_{Ld}}{\bar{P}_g}$	$\frac{\bar{P}_{Ld}}{\bar{P}_{L \max}}$
CA (boost)	$1 - \frac{V_g}{V_o}$	$1/2$	$\frac{V_{g \max}}{2}$	$\frac{1}{4} \cdot \left( \frac{V_o}{V_{g \min}} \right)$	$\frac{(V_o/2)^2}{V_{g \min} \cdot (V_o - V_{g \min})}$
CP (buck)	$\frac{V_o}{V_g}$	$D_{\min}$	$V_{g \max}$	$1 - \frac{V_o}{V_{g \max}}$	1
CL (boost-buck)	$\frac{V_o}{V_o + V_g}$	$D_{\min}$	$V_{g \max}$	$\frac{V_o + V_{g \min}}{V_o + V_{g \max}} \cdot \left( \frac{V_{g \max}}{V_{g \min}} \right)$	$\frac{V_o + V_{g \min}}{V_o + V_{g \max}} \cdot \left( \frac{V_{g \max}}{V_{g \min}} \right)$

Plots of the design-power ratios as a function of  $V_g$  and  $V_o$  for the three configurations are shown in Fig. 2, with  $V_{g \min} = 10 \text{ V}$ ,  $V_{g \max} = 30 \text{ V}$ , and for the same midrange ( $V_g = 20 \text{ V}$ ) values of  $D = 0.333$  as in the graph of  $D(V_g)$  in Fig. 1. The  $\bar{P}_{Ld} / \bar{P}_g$  are 0.75 at  $V_{g \max}/2$  (CA), 0.778 at  $V_{g \max}$  (CP), and 1.5 at  $V_{g \max}$  (CL). The CL configuration has the most power-intensive requirement for the inductor for which  $\bar{P}_{Ld} / \bar{P}_g$  is always greater than one.

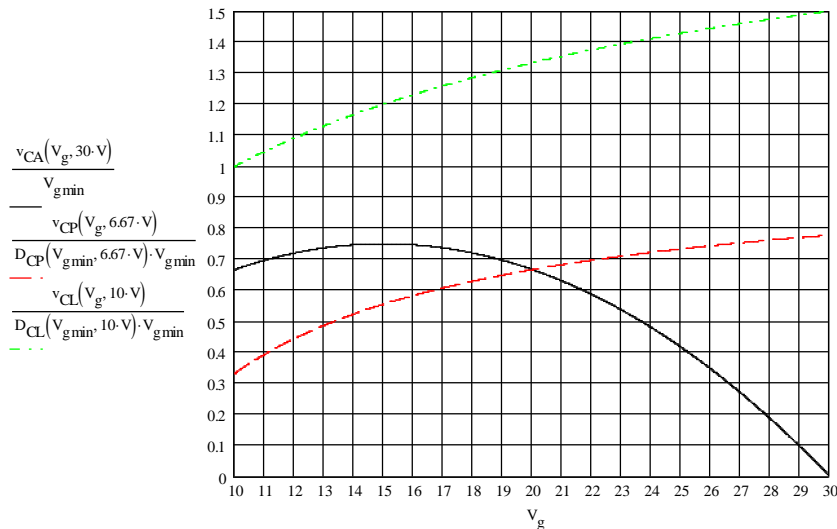


Fig. 2. Plots of the design-power ratios ( $\bar{P}_{Ld} / \bar{P}_g$ ) for the three PWM-switch configurations.

How does the design-power ratio,  $\bar{P}_{Ld} / \bar{P}_g$  affect inductor size? One indicator of size is the required fraction of the total window area,  $A_w$  of the winding area,  $A_{ww} = N \cdot A_{cwp}$ , where  $A_{cwp}$  is the insulated wire conductor area,  $A_{cw}$ , expanded by the packing factor of the wire for each of  $N$  turns. Then  $A_{ww} \leq A_w$  and for a winding of full window occupancy  $A_{ww} = A_w$ . For maximum core transfer power,  $N$  is

$$N_{opt} = \sqrt{N_\lambda \cdot N_i} = \sqrt{\frac{V_g \cdot t_{on}}{\Delta\phi} \cdot \frac{N \bar{i}_{sat}}{\bar{i}_g}} = \text{constant} \cdot \sqrt{\frac{V_g}{\bar{i}_g}}$$

Conductor area,  $A_c$  at constant current density is proportional to  $\tilde{i}_g = \kappa_g \cdot \bar{i}_g$ . The minimum required window area is

$$A_{ww} = N \cdot A_{cwp} \propto \sqrt{\frac{V_g}{\bar{i}_g}} \cdot \bar{i}_g = \sqrt{V_g \cdot \bar{i}_g} = \sqrt{\bar{P}_g}$$

At worst,  $\bar{P}_{Ld} \leq \bar{P}_{g \max} = r \cdot \bar{P}_g$  replaces  $\bar{P}_g$  for design, and  $A_{ww} \propto \sqrt{r} \cdot \sqrt{\bar{P}_g}$ . As the  $V_g$  range increases,  $A_{ww}$  increases by  $\sqrt{r}$ . This is the size and cost tradeoff for the benefit of a wide  $V_g$  range. It explains why converters with a fixed input voltage have a higher power density, and why the universal power-line voltage range has not become widely accepted for cost-driven offline converter designs.

In part 3, we take this same kind of formula derivation for design to transformers.

### Reference

"[Determining Design Power Over An Input Voltage Range \(Part 1\): Maximum Inductor Power](#)" by Dennis Feucht, How2Power Today, November 2020.

### About The Author



*Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.*

For more on magnetics design, see these How2Power Design Guide search [results](#).