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Understanding Supercapacitor Discharge Into A Constant Power Load

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Supercapacitors are gaining popularity as backup devices in diverse power supplies. Many of these supplies are intended for delivering constant power to the load for some period of time i.e. microprocessors should be supplied for some time during the device turn-off, same as memory ICs should be reliably powered before the system de-energizes. In order to design the supercapacitor-based backup module correctly, engineers should clearly understand that when activated, the backup supercapacitors will discharge to the constant power load in a way that's absolutely different from the discharge into a regular resistor. Moreover, in this case the discharge will occur at a rate that is much faster than that into a regular resistor.

A constant-power load creates a specific load to a supercapacitor that can be modeled by a negative resistance: to keep the power steady this circuit should consume more current while the input voltage drops. Therefore the voltage across the supercapacitor at discharge drops more rapidly than in the case of a regular resistor load. This is a crucial effect, which makes a designer use a bigger supercapacitor than when discharging into a resistor. Any regular capacitor works the same way but only a supercapacitor stores enough energy to run a load of tens of watts for a few seconds.

The points stated above have not been clearly explained in the publications available online. Some of them[1,2] mostly pay attention to the *charging* of supercapacitors, while others, like reference [3], provide simulation results, but omit voltage and current graphs supporting relevant formulas that clearly describe the unusual process of a supercapacitor discharging into a constant power load. This article is intended to fill the void in the existing online literature by providing a thorough description of what happens when a supercapacitor supplies a constant-power load.

Defining Key Equations For Capacitor Discharge

To understand how a supercapacitor behaves under a constant power load, let's begin by considering a general supercapacitor backup circuit where the capacitor sees a constant load. Fig. 1 achieves this by employing a boost converter that maintains constant voltage to a resistive load, which in turn forces it to maintain a constant power input and, therefore a constant power load for the supercap.

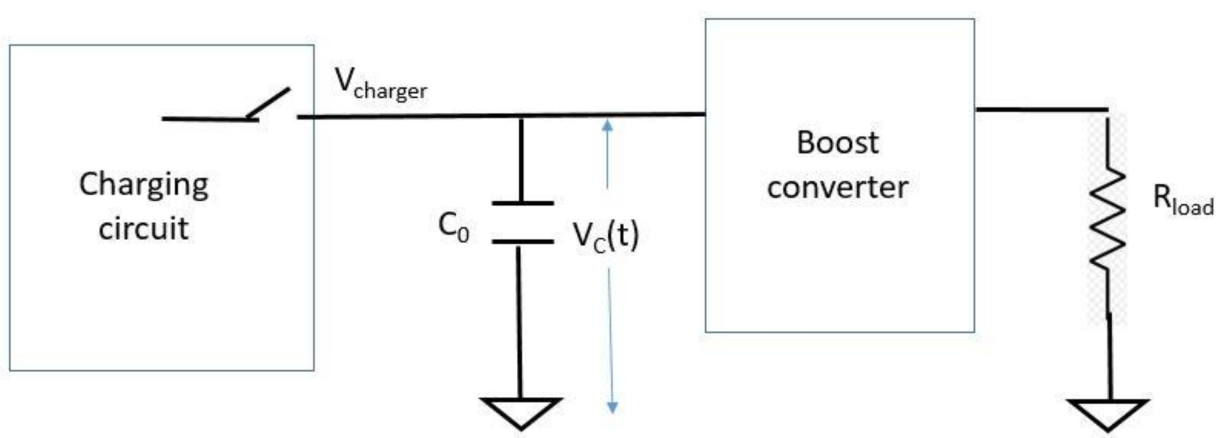

Fig. 1. A general supercapacitor-based backup circuit with constant-power load.

From this circuit we can designate the following terms:

P_{load} = constant power load associated with R_{load}

C_0 = supcapacitor capacitance value

$V_C(t)$ or V_C = instantaneous voltage across the supercapacitor

V_{C0} = Initial voltage across the supercapacitor

V_{Cmin} = minimum supercapacitor voltage that maintains operation of the step-up converter

η = step-up converter efficiency.

From the Energy Conservation law we can write:

$$C_0 \cdot \frac{\left(V_{C0}^2 - v_C^2\right)}{2} = \frac{P_{load}}{\eta} \cdot t \quad (1)$$

From (1) we can derive an expression for the instantaneous voltage across the supercapacitor:

$$v_C(t) = \frac{\sqrt{C_0 \cdot \left(C_0 \cdot V_{C0}^2 - 2 \cdot \frac{P_{load}}{\eta} \cdot t\right)}}{C_0} \quad (2)$$

Equation 2 allows us to determine the time necessary for the supercapacitor to discharge down to V_{Cmin}:

$$t_0 = -\frac{C_0 \cdot \left(V_{Cmin}^2 - V_{C0}^2\right)}{2 \cdot \frac{P_{load}}{\eta}} \quad (3)$$

We can also define the supercapacitor discharge current vs. time as

$$i_{load}(t) = \frac{P_{load}}{\eta \cdot v_C(t)} \quad (4)$$

When it is necessary to define a supercapacitor value for driving a specific load, one can use the following formula, which stems from (1):

$$C_0 = \frac{2 \cdot P_{load} \cdot t}{V_{C0}^2 \cdot \eta - \eta \cdot v_C^2} \quad (5)$$

A Design Example

If we assign component values and some operational values to the supercapacitor circuit described above, we can plot voltage and current waveforms for the supercapacitor. We begin by designating the following values:

C_0 = 0.5 F

V_{C0} = 16.8 V

P_{load} = 34 W

V_{Cmin} = 5 V

η = 0.9

With these values, we can plot equation 2 for $V_C(t)$, the instantaneous voltage across the supercapacitor, which is repeated here:

$$v_C(t) := \frac{\sqrt{C_0 \cdot \left(C_0 \cdot V_{C0}^{\ 2} - 2 \cdot \frac{P_{load}}{\eta} \cdot t\right)}}{C_0}$$

As a reference, we can plot a graph of supercapacitor current for the case where this supercap discharges through a resistor R_{ref}. This resistor value will be chosen to represent the load at the time the supercapacitor starts to discharge:

$$R_{ref} := \frac{V_{C0}^{\ 2} \cdot \eta}{P_{load}} = 7.471\,\Omega \tag{6}$$

With a resistive load, the voltage across a supercapacitor decreases according to the following equation:

$$v_{C\_ref}(t) := V_{C0} \cdot e^{\frac{-t}{R_{ref} \cdot C_0}} \tag{7}$$

Now, returning to the case of the constant load, which in this case is 34 W, we can apply equation 3 to determine the time t_0 required to discharge to the minimum operating voltage of the boost converter, V_{Cmin} = 5 V:

$$t_0 := -\frac{C_0 \cdot V_{Cmin}^{\ 2} - C_0 \cdot V_{C0}^{\ 2}}{2 \cdot \frac{P_{load}}{\eta}} = 1.702\,s$$

This term t_0 represents the time that the supercap can supply the specified load.

Finally, we can plot capacitor current during discharge per equation 4:

$$i_{load}(t) = \frac{P_{load}}{\eta \cdot v_C(t)}$$

The plots of $V_C(t)$, $V_{C\_ref}(t)$ and $i_{load}(t)$ are shown in Fig. 2. From these plots, we see that a supercapacitor discharging into a constant-power-consuming load discharges much faster than when it discharges into a resistor.

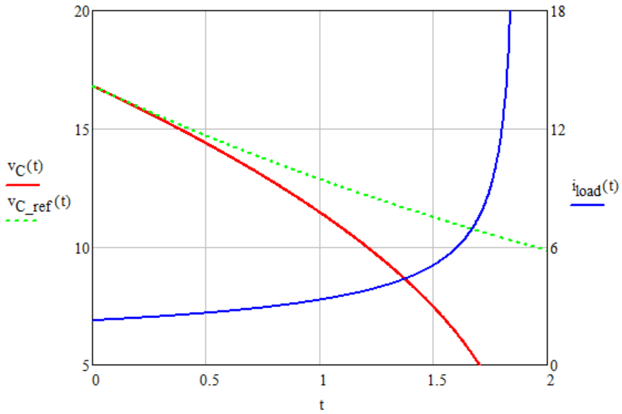

Fig. 2. The voltage across the 0.5-F supercapacitor $V_C(t)$ (red curve) and the current $i_{load}(t)$ consumed by the constant 34-W load (as seen at the input to the boost converter) (blue curve) are plotted along with the voltage across the supercapacitor with a resistive load $V_{C\_ref}(t)$ (green curve). Comparing the two voltage plots illustrates how a constant-power load drains the supercapacitor energy faster than a resistor.

If we had to determine what supercapacitor value C_{0X} would supply the load for a pre-determined time t_X, we could use formula 5. For the conditions given in our example, let's assume a value of 6 sec for t_X, which leads to the following value for

$$C_{0x} := \frac{2 \cdot P_{load} \cdot t_x}{V_{C0}^{\,2} \cdot \eta - \eta \cdot V_{Cmin}^{\,2}} = 1.762\,\mathrm{F}$$

Those who want to make sure all the above is correct, can simulate the schematic for the supercapacitor-based backup circuit shown in Fig. 3—presuming the IC model is correct and the IC internal limits (overcurrent protection, duty-cycle threshold etc.) are not violated. An example of a simulation result for this circuit is shown in Fig. 4. Due to graphical considerations, the circuit conditions simulated are not the same as those plotted in Fig. 2. However, the general shape of the simulated capacitor voltage curve is similar to that plotted above.

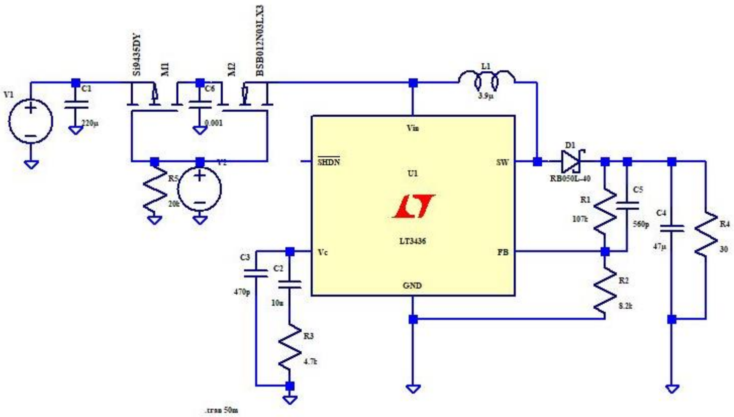

Fig. 3. Simulation schematic. Supercapacitor C6 discharges energy into the constant-power boost converter that keeps power dissipation by the load R4 steady.

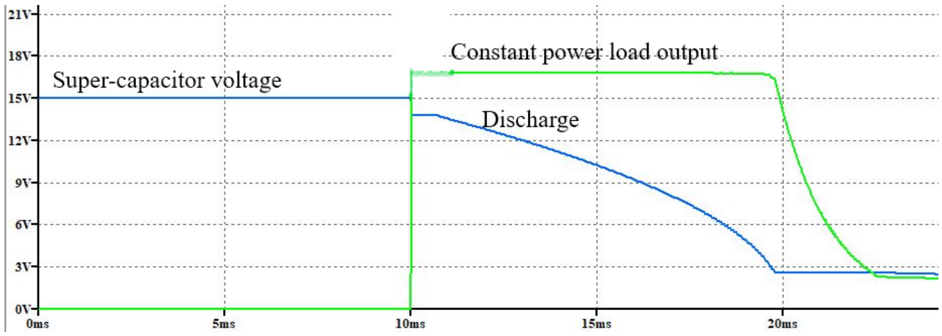

Fig. 4. Simulation of the schematic in Fig. 3. Voltage across C6 drops fast while the supercapacitor is discharging, keeping load power steady.

References

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3. "Calculation of Discharge Time," Elna capacitors document, published by Mouser.

About The Author

Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.