Stored Energy In A Saturating Inductor Is Not Constant

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The behavior of magnetic components in saturation is a topic that is not always well understood. One misconception that I encountered recently was that the energy stored in a saturating inductor remains constant regardless of the magnetic core's status.

However, in a previous work, I derived formulas that allow us to quantify the dependencies of both inductance and stored energy on magnetizing current (see the reference). From those equations, we can demonstrate that the energy stored in the inductor does not remain constant in saturation as we'll show in this article.

This behavior is worth analyzing because inductors in various applications (such as solenoids and chokes) are frequently driven into saturation. The energy stored in the inductor is ultimately released and affects other components.

Inductance Versus Magnetizing Current

From the reference, we have equation (1) below, which expresses the dependence of inductance on magnetizing current.

$$L(I_m) = \frac{A_L \cdot N_t^2}{\cosh\left[\left(\frac{I_m \cdot N_t \cdot \mu_0 \cdot \mu_r}{B_{sat} \cdot l_{mag}}\right)^2\right]} \qquad (1)$$

where A_L is inductance factor, N_t is the inductor's number of turns, μ_0 is magnetic permeability of free space, μ_r is the relative magnetic permeability of the core material, B_{sat} is flux density and l_{mag} is the core's central magnetic line length.

To get a better sense of what this equation means, it helps to plot L vs I_m for an example inductor. Let's consider the following example characteristics:

$A_L = 1343 \times 10^{-9}$ H

N_t – 23

$\mu_r = 2500$

$B_{sat} = 0.4$ T

$L_{mag} = 268$ mm

If we plot equation (1) for this sample inductor, as shown in Fig. 1, we see that when a magnetic core saturates, it does so in an exponential way (if the core has a homogenous structure) and inductance decays exponentially.

("Homogenous structure" refers to a well-mixed material like ferrite whose properties are the same in all directions and whose behavior can be described by an exponential function that has an unlimited number of derivatives. A material that is not homogenous, has particle inclusions with different permeability, leading to a different type of core behavior.)

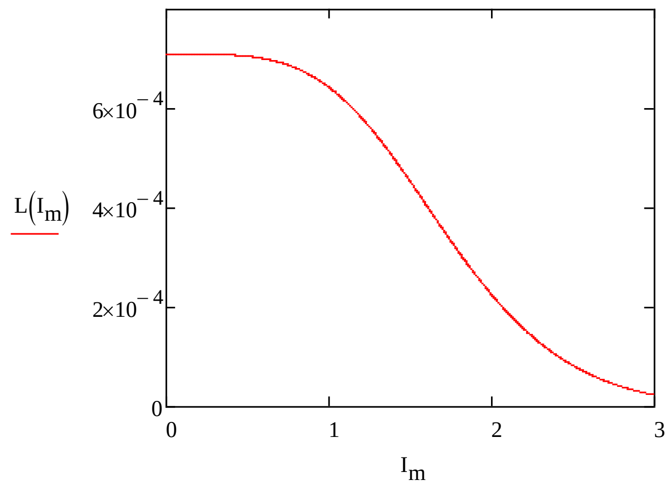

Fig. 1. Inductance of an inductor decays exponentially with magnetizing current rise. (Inductance is plotted here in henries and current is in amps.)

Just as a reminder, saturation occurs due to the loss of relative permeability (μ_r) in the magnetic core. We know that energy stored in the inductor is proportional to its inductance and the square of the current through the inductor. We also know that when the core saturates, inductance goes down, but current goes up. From Fig. 1, we see that the slope of the L vs. I_m curve in Fig. 1 (the second derivative of the function $L(I_m)$) changes its sign, somewhere in the 1.5-A to 2-A range.

Specifically, this second derivative of the inductance function is

$$F3(I_m) = \frac{d^2}{dI_m^{\,2}} L(I_m)$$

and if we plot this function using our example inductor values, we obtain the plot in Fig. 2, which shows the sign change noted above. This sign change implies an extreme value for stored energy. Further analysis will confirm this.

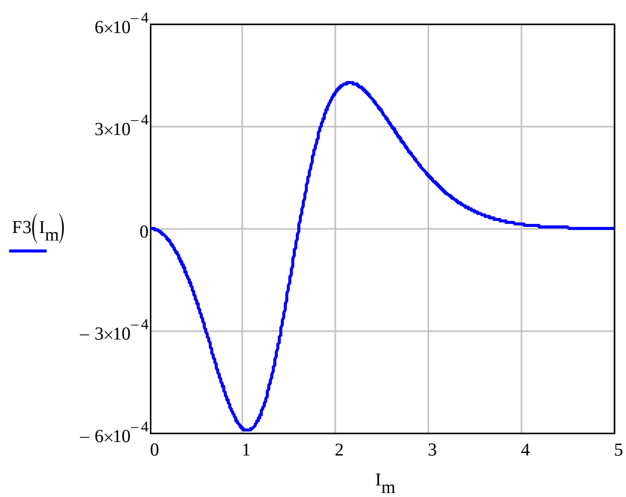

Fig. 2. A plot of the slope derivative of equation (1), inductance as a function of magnetizing current. The sign change seen here in the portion of the curve where the inductor is saturated, suggests a peak in the stored energy that occurs during saturation.

From the reference, we also have the following formula for energy stored in an inductor as a function of magnetizing current

$$E(I_m) = \frac{\dfrac{A_L \cdot N_t^2}{\cosh\left[\left(\dfrac{I_m \cdot N_t \cdot \mu_0 \cdot \mu_r}{B_{sat} \cdot l_{mag}}\right)^2\right]} \cdot I_m^2}{2} \qquad (2)$$

If we again consider the characteristics of the inductor example cited above, we can apply equation (2) to generate a plot illustrating the dependence of stored energy on magnetizing current, as shown in Fig. 3. Clearly, this plot shows a maximum for stored energy that occurs around 1.7 A. (If we now look back at the Fig. 2 plot, we see that 1.7 A is the point in that curve where the sign change occurred.)

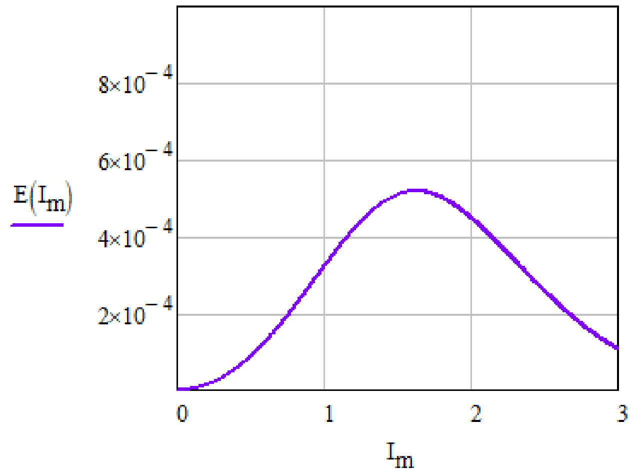

Fig. 3. Energy (in joules) of an inductor in saturation has a maximum.

The maximum on the energy characteristic should be taken into account by the designers of circuits employing saturable inductors like solenoids, ignition systems, relays, chokes, filters, etc. since the energy stored in the saturated core can damage sensitive components of the schematic or recuperating diodes while the device is turning off or during its regular operation.

Reference

"Magnetic-Core Modeling Offers Insight into Behavior, Operating Range, Saturation" by Gregory Mirsky, Electronic Design, September 9, 2015.

About The Author

Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

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