A Simplified Winding Design Procedure For Transformers

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An alternative transformer winding design procedure to that given in references [1 - 5] is presented here that is somewhat simpler and more like procedures typically found in textbook literature. In many cases this procedure should result in a sufficiently optimized design. It is iterative, minimizes eddy-current effects, and provides turns limits from static conditions.

The rationale is to achieve a desired winding resistance R_w by using the eddy-current Dowell plots in reverse—by calculating the fixed-frequency resistance ratio F_r and selecting a plot minimum from the constant-layer plots. This results in a wire strand radius r_c at the plot number of layers M, and from M and core geometry finding the number of wire strands N_s and winding length l_w. Then comparing N_s to the limits of its acceptable range to prevent overheating N_{si} or overfilling the winding window with wire N_{sw}, and keeping N_s within these bounds, calculate a new F_r and iterate from the Dowell graph.

Unlike more comprehensive procedures, no distinction is made between strands in turns bundles and windings without bundled strands. The total number of strands, whether bundled or not, is calculated from Dowell plot variables, wire size, and layers without regard to bundling or its effects on eddy-current behavior.

Optimal Number Of Strands Under Static Condition

Winding design starts from the magnetic design determination of the optimal number of turns of the bundle $N_b = N_{opt}$ and core geometry. In general, such design decisions about core selection and optimal number of turns, $N_b = N_{opt}$, are made prior to winding design.

In this simplified winding design procedure, strands in bundles are counted as $N_s = N_b \cdot N_{sb}$ where N_{sb} is the number of strands in a turn bundle. Same-size strands, equally divided between primary and secondary (opposite-polarity) windings, achieve uniform power loss density in the total winding. The widest constraints on N_s are the required ampacity (lower limit) and the window area (upper limit):

$$1 \le \frac{\tilde{i}_p}{(\tilde{J}/\tilde{J}_0)\cdot \tilde{i}_{\max}(r_{cw}^2)} = N_{si} \le N_s \le N_{sw} = \frac{k_{ww}(\max)\cdot A_w}{N_b \cdot A_{cwp}(r_{cw}^2)}$$

where r_{cw} = insulated strand radius, $\tilde{i}_p$ = RMS primary winding current, $\tilde{i}_{\max}(r_{cw}^2)$ = strand ampacity, and $\tilde{J}/\tilde{J}_0$ = current density sizing factor, normalized to a reference value of $\tilde{J}_0$ of a core area-product of 1 cm^4; $k_{ww}(\max)$ is the maximum allotted fraction of winding area A_{ww} of the winding window area A_w, and $A_{cwp}(r_{cw}^2)$ is insulated strand area A_{cw} including bundle packing factor, or $A_{cwp} = A_{cw}/k_p$. Both of these limits are functions of wire area, vary by about $1/r_{cw}^2$, and decrease together in proportion to A_{cw}.

As wire size decreases, the range of allowable N_s increases. For each current polarity of winding, N_s is about half the total N_s because each winding has $A_{ww} \approx A_w/2$ and $k_{ww}(\max) \approx ½$ for each winding polarity. Then each polarity of winding is allotted a winding area of $A_{ww} \approx A_w/2$.

Table 1. Size adjustment factors for ampacity is given below, with $\tilde{J}_0 = 4.5$ A/mm², Cu.

$A \cdot A_w$ /cm⁴	core size	J/(4.5 A/mm²)		$A \cdot A_w$ /cm⁴	core size	J/(4.5 A/mm²)
0.01		1.778		0.608	T90	0.940
0.1		1.334		1.084	T106	0.990
1		1		2.15	T130	0.909
10		0.750		1.85	T131	0.926
100		0.562		3.22	T150	0.864
1000		0.422		8.62	T184	0.764
0.00091	T20	2.401		30.6	T250	0.652

0.0051	T26	1.937		63.7	T300D	0.595
0.0147	T37	1.711		0.229	E100	1.202
0.0263	T44	1.576		0.759	E137	1.035
0.0521	T50	1.446		1.98	E162	0.918
0.104	T50D	1.327		3.57	E187	0.853
0.124	T68	1.298		10.2	E220	0.748
0.289	T80	1.168		156	E450	0.532

The optimization of N_s between these limits is like optimizing turns $N_b = N_{opt}$ in the magnetic design, between N_λ (core power loss) and N_i (saturation) limits. When the N_s range shrinks to a single value, an optimal N_s is achieved:

$$\frac{\tilde{i}_p}{(\tilde{J}/\tilde{J}_0)\cdot \tilde{i}_{\max}(r_{cw}^2)} = N_{si} = N_{sopt} = N_{sw} = \frac{k_{ww}(\max)\cdot A_w}{N_b \cdot A_{cwp}(r_{cw}^2)}$$

This optimization minimizes R_w under the condition that $N_s = N_{sw}$; the entire winding window is at maximum ampacity for maximum window utilization. The above equation can be rearranged and then interpreted as

$$\frac{\tilde{i}_p}{A_w/2} = \frac{(\tilde{J}/\tilde{J}_0)\cdot \tilde{i}_{\max}(r_{cw}^2)}{N_b \cdot A_{cwp}(r_{cw}^2)} \approx \frac{(\tilde{J}/\tilde{J}_0)}{N_b/k_p}\cdot(4.5\ \mathrm{A/mm^2})$$

$$\Updownarrow$$

current density in winding = current density in strands

For $N_s > N_{sw}$, $k_{ww} > 1$, and the winding will not fit the window. Then either N_s or wire size r_{cw} must be reduced. It is analogous to $N_b > N_i$; the window is "oversaturated" with winding. At the opposite extreme, $k_{ww} << k_{ww}(\max)$ (perhaps $k_{ww} < 0.5$), and the winding window is underutilized; a higher A_{ww} (for lower R_w) could be chosen subject to winding cost.

For $N_s < N_{si}$, the winding has inadequate ampacity and overheats just as the core overheats whenever $N_b < N_\lambda$. (See reference [6] for explanation of N_λ and N_i.) For minimum R_w, N_s can be set as close to N_{sw} as the winding-area tolerance will allow. (Adequate margins for manual construction per winding are typically $k_{ww}(\max) \approx 0.475$ for linear windows and $k_{ww}(\max) \approx 0.375$ for toroids.)

A minimum-A_{ww}, min-k_{ww} solution sets $N_s = N_{si}$. There is nothing advantageous about this extreme except to minimize wire and bundle cost and simplify construction. For $N_s = N_{si}$, R_w is highest and transfer efficiency is thereby reduced, though possibly acceptable, and over an output current range, possibly maximized.

An optimal N_s to achieve both minimal R_w and cost (continuing the N_{opt} analogy from magnetic design) is to set N_s to the geometric mean between the two limits:

$$N_{sopt} = \sqrt{N_{si}\cdot N_{sw}}$$

This puts N_s maximally far from both limits simultaneously, providing design margin for both F_r and A_w. For minimum R_w, set N_s close to N_{sw}; to ensure adequate winding area in the window and lower wire cost, set N_s closer to N_{si}. If the limits are too far apart, iterate the magnetic design by reducing core size.

The practical range of wire size also limits design. As size decreases, k_{pw} decreases, and less of the window is used for conducting current and is suboptimal. For high-ξ_r solutions, wire can be too large and too hard to bend around small core-window shapes.

An empirical rule for acceptable wire range for manual prototype construction is #40 AWG to #16 AWG, or if you have sharp vision and are agile, #43 to #10. Below #16, consider using foil. Above #43, consider commercially-woven (bunched or cabled) Litz wire. Mainstream transductor design (that is, transformer or coupled inductor design) falls typically within the #40 to #16 range for round wire. For an initial choice of N_s, choose $N_s = N_{sopt}$. Unless $N_{si} = N_{sw}$, this N_{sopt} value neither minimizes winding resistance nor fills the window; it only centers N_s between its limits.

Geometric Winding Parameters

From magnetic design we have the value of both N_s (total winding strands) and A_w; k_{ww}(max) is determined for each winding by the allotment of window area given it. Windings of opposite currents and square-wave waveforms have equal areas because they handle equal power. Primary and secondary windings are referred to the primary winding as one winding. Then $\tilde{i}_{\max}(r_{cw}^2)$ is the wire ampacity and $\tilde{i}_p$ is the maximum winding current. Before entering the procedure, we have an initial value of N_s, the two N_s limits, the F_r graph, which will be shown later in the article, and formulas for winding geometry and for calculating F_r, which is the constant-frequency eddy-current resistance ratio.

The winding geometry formulas are for iterating N_s and winding length l_w. For linear layers,

$$l_w = N_s \cdot [2 \cdot \pi \cdot (r_i + \tfrac{1}{2} \cdot r_{cw} \cdot (2 + k_{pl} \cdot (M-1))) + 2 \cdot r_{cw}],\ M = N_s \cdot (2 \cdot r_{cw}) / w_w \text{, round window}$$

$$\text{Average turn length} = \bar{l}_c = 8 \cdot a_i + 2 \cdot \pi \cdot [1 + k_{pl} \cdot (M-1)] \cdot r_{cw} = 8 \cdot a_i + 2 \cdot \pi \cdot (\Delta a - r_{cw}) \text{, square window}$$

$$l_w = N_s \cdot \bar{l}_c + M \cdot w_w = N_s \cdot (\bar{l}_c + 2 \cdot r_{cw}) \text{, square window}$$

where w_w = window width (into page), and the other window dimensions are shown in Fig. 1. The average of hexagonal and square strand packing in the winding is $k_{pl} = 1 + \sqrt{3}/2 \approx 1.866$.

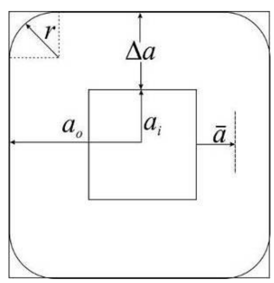

Fig. 1. Square winding-window dimensions. Window width w_w is into the page.

For toroids with outer and inner radii and ring width,

$$r_o = OD/2\ ;\ r_i = ID/2\ ;\ w = \frac{OD - ID}{2} = r_o - r_i$$

The radius from the center of the ring to midway within the ring is

$$\bar{r} = r_i + w/2 = \tfrac{1}{2} \cdot (r_o + r_i)$$

In order of calculation of l_w, the intermediate parameters are

$$\text{Max layers} = \hat{M} = \frac{r_i}{(1+\sqrt{3}/2) \cdot r_{cw}} \approx \frac{r_i}{(1.866) \cdot r_{cw}} \qquad \text{Max strands} = N_w = \pi \cdot \hat{M}^2$$

$$\text{Layers for } N \text{ strands} = M = \hat{M} \cdot \left(1 - \sqrt{1 - \frac{N_s}{N_w}}\right)$$

$$\text{Winding length} = l_w = 2\cdot\pi\cdot M\cdot[(2\cdot(h+w)+8\cdot r_{cw}\cdot M)\cdot(\hat{M}-\tfrac{1}{2}\cdot M)+\tfrac{4}{3}\cdot r_{cw}\cdot(1-M^2)+\bar{r}]$$

The additional geometric constraint that affects winding design is the choice of the function relating M and r_c to N_s. We will need the insulated wire radius r_{cw} from r_c. It is found from a wire table. The Innovatia wire table is given below in Table 2 for reference. The table has columns for AWG, r_c, and r_{cw}. Knowing AWG or r_c find either as the closest table row entry and read the r_{cw} value of that row.

Table 2. Innovatia wire table for finding the insulated wire radius r_{cw}. With heavy insulation r_c: 20 AWG/dec ≈ 6 AWG/oct and A_c:10 AWG/dec ≈ 3 AWG/oct at f_δ @ 80°C.

Gage, AWG	r_c, mm	r_{cw}, mm	A_c, mm^2	A_{cwp}, mm^2	k_p	I_{max}, A	$f_{\delta Cu}$, kHz	$f_{\delta Al}$, kHz
0	4.126	4.251	53.482	71.552	0.747	240.67	0.317	0.519
1	3.676	3.794	42.449	56.987	0.745	191.019	0.400	0.655
2	3.275	3.023	33.692	45.395	0.742	151.612	0.505	0.826
3	2.918	2.986	26.741	36.169	0.739	120.335	0.636	1.041
4	2.599	2.698	21.224	28.824	0.736	95.510	0.802	1.312
5	2.316	2.409	16.846	22.976	0.733	75.806	1.012	1.656
6	2.063	2.151	13.371	18.318	0.730	60.167	1.276	2.086
7	1.838	1.921	10.612	14.608	0.726	47.755	1.608	2.630
8	1.637	1.716	8.423	11.653	0.723	37.903	2.028	3.318
9	1.459	1.533	6.685	9.298	0.719	30.084	2.559	4.185
10	1.300	1.369	5.306	7.421	0.715	23.877	3.226	5.277
11	1.158	1.223	4.211	5.925	0.711	18.952	4.071	6.658
12	1.032	1.093	3.343	4.732	0.706	15.042	5.132	8.394
13	0.919	0.977	2.653	3.780	0.702	11.939	6.453	10.554
14	0.819	0.874	2.106	3.021	0.697	9.476	8.153	13.335
15	0.729	0.781	1.671	2.415	0.692	7.521	10.278	16.810
16	0.650	0.698	1.327	1.931	0.687	5.969	12.985	21.239
17	0.579	0.625	1.053	1.545	0.681	4.738	16.340	26.725
18	0.516	0.559	0.836	1.237	0.670	3.760	20.608	29.668
19	0.459	0.500	0.663	0.990	0.671	2.985	25.980	42.494
20	0.409	0.448	0.526	0.793	0.664	2.369	32.613	53.342
21	0.365	0.401	0.418	0.636	0.657	1.880	41.225	67.428
22	0.325	0.359	0.332	0.510	0.651	1.492	52.103	85.220
23	0.289	0.321	0.263	0.409	0.644	1.184	65.586	107.27
24	0.258	0.288	0.288	0.328	0.637	0.940	82.432	134.83
25	0.230	0.258	0.166	0.264	0.629	0.746	103.92	169.98
26	0.205	0.231	0.132	0.212	0.622	0.592	132.40	216.55
27	0.182	0.207	0.104	0.170	0.614	0.470	164.90	269.71
28	0.162	0.186	0.083	0.137	0.605	0.373	211.03	345.16
29	0.145	0.167	0.066	0.110	0.597	0.296	260.53	426.12
30	0.129	0.150	0.052	0.089	0.588	0.235	334.94	547.83
31	0.115	0.134	0.041	0.072	0.579	0.187	423.08	691.99
32	0.102	0.121	0.033	0.058	0.570	0.148	519.25	849.29
33	0.091	0.108	0.026	0.047	0.561	0.118	666.94	1091
34	0.081	0.097	0.021	0.038	0.551	0.093	844.10	1381
35	0.072	0.088	0.016	0.030	0.541	0.074	1072	1753
36	0.064	0.079	0.013	0.025	0.531	0.059	1148	2157
37	0.057	0.071	0.010	0.020	0.521	0.047	1663	2720
38	0.051	0.064	0.00823	0.016	0.511	0.037	2077	3397
39	0.046	0.057	0.00653	0.013	0.500	0.029	2668	4364
40	0.041	0.052	0.00518	0.011	0.489	0.023	3376	5523
41	0.036	0.047	0.00411	0.0086	0.479	0.019	4168	6818
42	0.032	0.042	0.00326	0.0070	0.467	0.015	5276	8629
	0.892		2.5	3-wire cable		11.25	6.79	
	1.128		4.0			18.0	4.24	
12.7 mm × 34 µm Cu foil			0.452	1.03	0.42	1.53/cm	4272	6988

Notes: $f_\delta = f\,(r_c = \delta) = f\,(\xi_r = 1)$; I_{max} = ampacity; k_p = round-wire packing factor; A_c = conductor area; $A_{cwp} = A_c/k_p$; r_{cw} = insulated wire radius; r_c = conductor radius; AWG = wire size (gauge).

N_s varies linearly with M/r_{cw} for window shapes with constant turns per layer;

$$N_s = \frac{M \cdot w_w}{2 \cdot r_{cw}} = \text{constant} \cdot \frac{M}{r_{cw}} = \text{constant} \cdot \frac{M}{\sqrt{A_{cw}}}$$

where w_w and N_s are from magnetic design, and M and r_c are read from the F_r graph. Window winding occupancy,

$$k_{ww} = \frac{A_{ww}}{A_w} = \frac{N_s \cdot A_{cwp}}{A_w} = \frac{N_s \cdot (A_{cw}/k_p)}{A_w} = \frac{N_s \cdot (\pi \cdot r_c^2)}{k_p \cdot A_w} = N_s \cdot \frac{k_{pw} \cdot \pi}{k_p \cdot A_w} \cdot r_{cw}^2$$

k_{ww} varies linearly with N_s and for nearly-constant strand porosity (from wire insulation separating conductors) k_{pw}, then the insulated strand area is $A_{cw} = A_c/k_{pw} = \pi \cdot r_{cw}^2$. Cross-sectional winding area is

$$A_{ww} = N_s \cdot A_{cwp} = N_s \cdot (A_c/k_p) = N_s \cdot (A_{cw}/k_{pf}) = N_s \cdot (\pi \cdot r_{cw}^2)/k_{pf}$$

Total packing factor $k_p = k_{pf} \cdot k_{pw}$ and the effective A_{cw} with packing factor is $A_{cwp} = A_{cw}/k_p$. The form of winding area

$$A_{ww} = \text{constant} \cdot N_s \cdot r_{cw}^2 = \text{constant} \cdot N_s \cdot A_{cw}$$

For constant k_p and A_{ww}, A_{cw} varies inversely with N_s.

Dowell Eddy-Current Graph

A graph at the chosen magnetic frequency f for the Dowell equation has the number of layers M and the horizontal axis in wire size, AWG, as shown in Fig. 2 for f = 100 kHz. For each frequency, a different graph is required. Alternatively, you can use one graph, given in Fig. 3, of $F_r(\xi_r, M)$, $\xi_r = r_c/\delta$, plotted against conductive wire radius r_c in units of skin depth for Cu or Al wire;

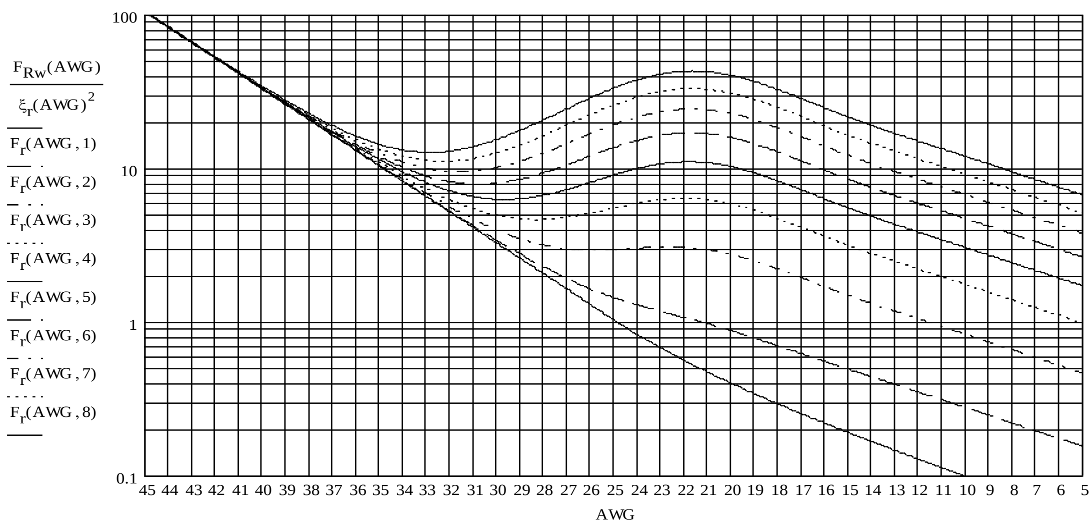

Fig. 2. The Dowell equation, graphed with wire size (horizontal axis) in AWG at f = 100 kHz; $R_{\delta r}/l_w$ = 125 mΩ/m, Cu, 80°C.

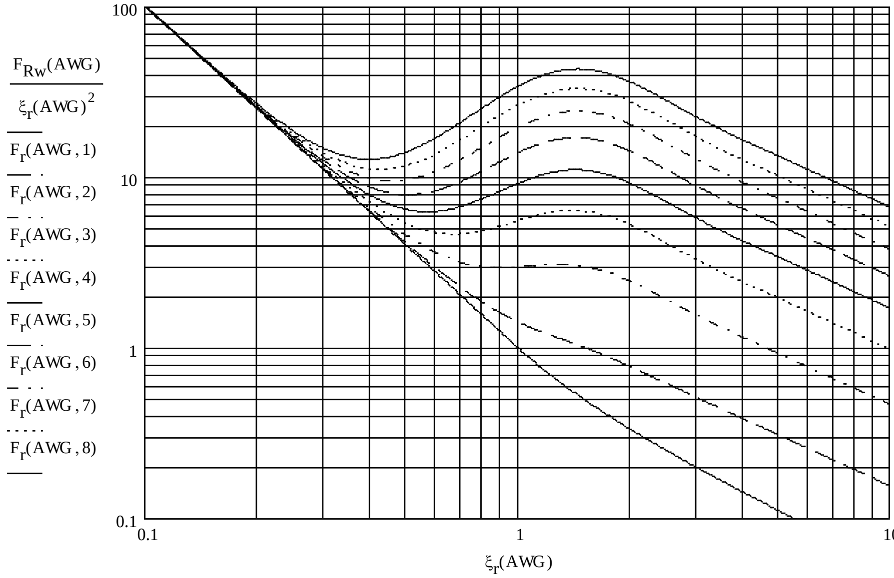

Fig. 3. Log-log graph of $F_r(\xi_r, M)$. The lowest plot is of a single isolated wire. Wire conductive radius $r_c = \xi_r \cdot \delta$.

To convert from AWG of Fig. 2 to conductive radius r_c, apply

$$r_c = r_0 \cdot (2^{\frac{1}{6}})^{-\mathrm{AWG}} \approx (4.126\ \mathrm{mm}) \cdot (1.123)^{-\mathrm{AWG}}$$

Design Procedure

The procedure forms a loop and is iterative. The core is determined from magnetic design. Hence, we have the value of both N_b (winding turns, hence N_s) and A_w; k_{ww}(max) is determined for each winding by the allotment of window area given it. Windings of opposite currents and square-wave waveforms have equal areas because they handle equal power, assuming no significant winding loss. For one primary and one secondary winding and linear (constant turns per layer) windings, set k_{ww}(max) = 0.45, somewhat less than ½. $\tilde{i}_{\max}(r_{cw}^2)$ is the wire ampacity and $\tilde{i}_p$ is the maximum winding current. Before starting the procedure, we have an initial value of N_s, the two N_s limits, the F_r graph, and formulas for winding geometry and for calculating F_r, included in the procedure.

Fig. 4 is a flowchart of the winding design procedure. It can be computerized but is not difficult to use manually with a calculator. The procedure can be entered anywhere in its iterative loop but starting from the top at the F_r graph, an initial guess at an operating-point on the graph can be made.

To minimize F_r, choose a valley point on one of the plots, making a guess about number of layers M. The ξ_r value is converted to r_c (with δ), and then the insulated radius r_{cw} in millimeters is found either from r_c or from the AWG value as read from the wire table columns for r_c or AWG.

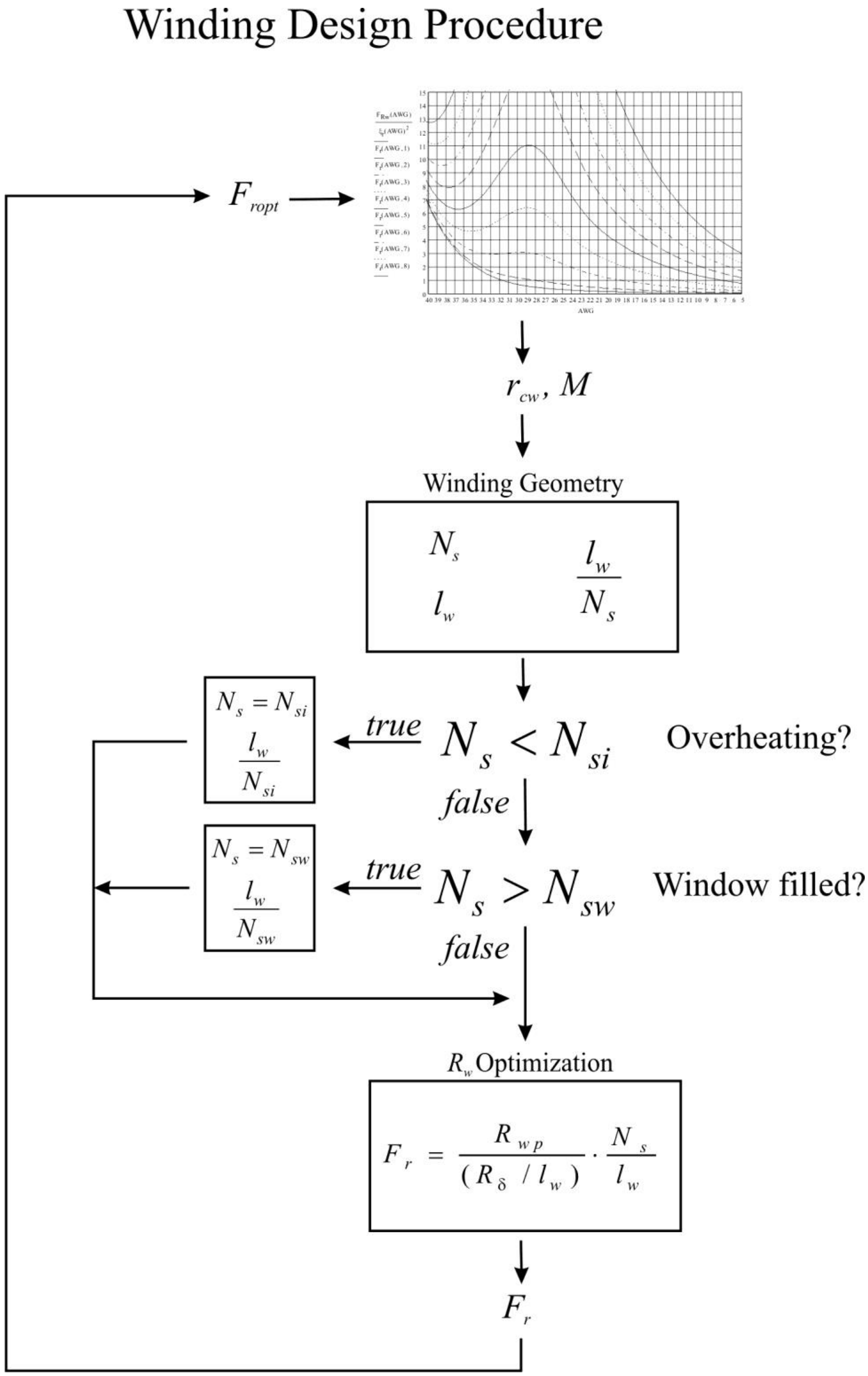

Fig. 4. The winding design procedure. The graph at the top is the constant-frequency resistance-ratio graph of F_r(AWG, M) for the chosen magnetic frequency. (The F_r graph, as shown, is semi-log.)

F_r *Calculation*

At the bottom of the loop in Fig. 4, R_w has been calculated and F_r is needed from it as the F_r graph entry. F_r is calculated from R_w for round Cu wire from

$$\frac{R_{\delta r}}{l_w} = \frac{\rho_{\text{Cu}}}{\pi \cdot \delta^2} \approx \left(1.256 \frac{\mu\Omega}{\text{m} \cdot \text{Hz}}\right) \cdot f \text{ , round-wire, Cu, 80°C} \Rightarrow$$

$$R_w(\text{AWG}, M, f) = F_r(\text{AWG}, M) \cdot (R_{\delta r}(f)/l_w) \cdot l_w$$

$R_{\delta r}(f)$ is the wire resistance at f_δ. $R_{\delta r}(f)/l_w$ is tabulated below for some frequencies.

Table 3. Values of $R_{\delta r}(f)/l_w$ over a practical range of frequencies.

f, kHz	$R_{\delta r}/l_w$, mΩ/m ; Cu, 80°C
100	125
150	188
200	251
250	314
300	377
350	440
400	502
450	565
500	628

Looking at the flowchart, the l_w and N_s formulas are applied and N_s compared to the two limits. If N_s is outside the range of $N_{si} < N_s < N_{sw}$, then it is "clamped" to the limiting value. Then N_s/l_w is calculated and substituted into the F_r formula. R_{wp} is the effective single winding resistance when all other windings are referred to the primary winding. ($R_{\delta r}/l_w$) for the given frequency is known and can be taken from the $R_{\delta r}/l_w$ table. R_{wp} is the iterated winding resistance and from these quantities,

$$F_r = \frac{R_{wp}}{R_{\delta r}/l_w} \cdot \frac{N_s}{l_w}$$

The F_r value is then compared to the previous value and if it is close enough to it, the winding design has a solution. If R_{wp} is chosen too low then no F_r minimum will be realized for the given core parameters. Either increase R_{wp} or return to magnetic design and choose a different core and turns N_b.

Closure

This design procedure trades off refinement to achieve procedural simplicity. It does not take into account bundled turns of strands; each "bundle" is a single strand. Eddy-current effects within bundles are ignored. Neither does it seek minimum F_r for a given N_s as do references [1 - 5]. Yet it is simple enough to avoid either excessive calculator button-pushing or having to write a computer math program.

References

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About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

For more on magnetics design, see these How2Power Design Guide search results.