

Resonant Current Source Powers Arbitrary Load

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There are many implementations that require a current-supply source for operation. These are mainly, but not limited to, components and devices having a diode-type characteristic that requires current limiting—diodes, LEDs, gas-filled bulbs etc. Generally, buck or boost converters are employed to power such devices. In place of an output voltage regulating circuit, these converters use a current-stabilizing loop to convert a buck or boost voltage source into a current source.

However, there is a little-known class of ac current sources whose operation is based on the effect of limiting ac load current in a series resonant circuit if the load is connected in parallel to the resonant capacitor (Fig. 1). These are not difficult to implement, mainly designers must attend to one safety consideration. Due to the series resonance condition in this circuit, very simple precautions should be considered to limit voltage across the resonant components once the load is disconnected. For lower-current applications, the needed voltage limiting can be accomplished with a transorb, while for higher current loads this can be accomplished with a regulating feedback loop.

Since this circuit can be used as a good ac current source, it is safe at the load short circuit and can be used as a step-up converter for supplying loads at voltages which are not physically practical using conventional converter circuits. Step-up coefficients in the resonant circuits can attain values of 15 to 20 and higher at an efficiency above 95%. Moreover, the circuit property of pushing constant ac current to the load *does not depend on the load type*: it works equally well into a reactive or a resistive load and even into a rectifier. This ability to supply unchangeable ac current into any type of load is a very valuable—and rarely considered—property of the resonant current source (RCS), and the goal of this publication is to prove that.

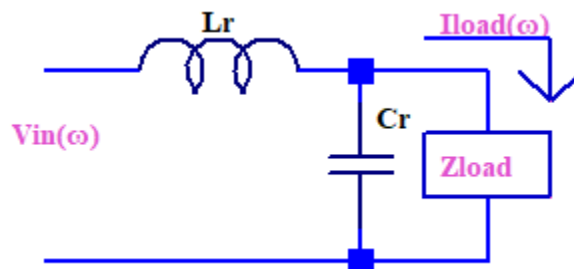


Fig. 1. A resonant current source is a series-resonant circuit having a load in parallel to the resonant capacitor. The circuit pushes constant current into the load regardless of its nature or configuration. However, if the load is a capacitor, it may not be connected directly across the resonant capacitor. In that case, an inductor is needed to separate the resonant capacitor from the load.

Analyzing The Resonant Current Source

To understand the operation of the resonant current source and to be able to design useful current sources based on it, we'll need to derive a transfer function for load current $I_{load}(\omega)$ as a function of input voltage $V_{in}(\omega)$. This will allow us to assess the sensitivity of the current source's operation to the load and other operating parameters, and to identify the key design equations.

We begin by defining the ac input voltage in terms of its amplitude, V_a , and its cyclic frequency, ω .

$$V_{in}(t) = V_a \cdot \sin(\omega \cdot t) \quad (1)$$

We have to assume that the input voltage is a rectangular pulse train, but the resonant circuit filters out all harmonics except for the first one (the fundamental). For the first harmonic, it holds true that if V_{pulse} is the input pulse train amplitude, then provided the pulse train has a duty-cycle of 0.5,

$$V_{\text{in}}(t) = \frac{2 \cdot V_{\text{pulse}} \cdot \sin(\omega \cdot t)}{\pi} \quad (2)$$

Hence for the input pulse train, the first harmonic RMS value is

$$V_{\text{in_rms}} = V_{\text{pulse}} \cdot \frac{\sqrt{2}}{\pi} \quad (3)$$

The derivation of equations 2 and 3 is shown in the appendix at the end of this article.

The next step in deriving the transfer function is to find an expression for the load impedance that includes both the impedance of the resonant capacitor and the load. Since the load $Z_{\text{load}}(s)$ is connected in parallel with the resonant capacitor C_r their total impedance would be

$$Z_r = \frac{1}{\frac{1}{Z_{C_r}} + \frac{1}{Z_{\text{load}}}}$$

At this point, the math is easier if we switch to the s-domain. Using the Laplace transform for the above expression yields:

$$Z_r(s) = \frac{1}{s \cdot C_r + \frac{1}{Z_{\text{load}}(s)}}$$

Simplifying we get:

$$Z_r(s) = \frac{1}{C_r \cdot s} - \frac{1}{C_r \cdot s + C_r^2 \cdot s^2 \cdot Z_{\text{load}}(s)} \quad (4)$$

Resonant inductor L_r together with the $Z_r(s)$ composes an impedance of

$$Z_{\text{tot}}(s) = s \cdot L_r + \left(\frac{1}{C_r \cdot s} - \frac{1}{C_r \cdot s + C_r^2 \cdot s^2 \cdot Z_{\text{load}}(s)} \right) \text{ which simplifies to}$$

$$Z_{\text{tot}}(s) = \frac{Z_{\text{load}}(s) + L_r \cdot s + C_r \cdot L_r \cdot s^2 \cdot Z_{\text{load}}(s)}{C_r \cdot s \cdot Z_{\text{load}}(s) + 1} \quad (5)$$

This circuit gain is a ratio of the load voltage $V_{\text{load}}(s)$ to the input voltage $V_{\text{in}}(s)$,

$$G(s) = \frac{V_{\text{load}}(s)}{V_{\text{in}}(s)} = \frac{Z_r(s)}{Z_{\text{tot}}(s)}$$

which can be interpreted this way:

$$G(s) = \frac{\frac{1}{C_r \cdot s} - \frac{1}{C_r \cdot s + C_r^2 \cdot s^2 \cdot Z_{load}(s)}}{\frac{Z_{load}(s) + L_r \cdot s + C_r \cdot L_r \cdot s^2 \cdot Z_{load}(s)}{C_r \cdot s \cdot Z_{load}(s) + 1}}$$

which simplifies to

$$G(s) = \frac{Z_{load}(s)}{Z_{load}(s) + L_r \cdot s + C_r \cdot L_r \cdot s^2 \cdot Z_{load}(s)} \quad (6)$$

and the current to the load is

$$I_{load}(s) = V_{in}(s) \cdot \frac{\frac{Z_{load}(s)}{Z_{load}(s) + L_r \cdot s + C_r \cdot L_r \cdot s^2 \cdot Z_{load}(s)}}{Z_{load}(s) + L_r \cdot s + C_r \cdot L_r \cdot s^2 \cdot Z_{load}(s)}}$$

which simplifies to

$$I_{load}(s) = \frac{V_{in}(s)}{Z_{load}(s) + L_r \cdot s + C_r \cdot L_r \cdot s^2 \cdot Z_{load}(s)} \quad (7)$$

Using the complex form of representation, we obtain:

$$I_{load}(j \cdot \omega) = V_{in}(j \cdot \omega) \cdot \frac{1}{Z_{load}(j \cdot \omega) \cdot (1 - C_r \cdot L_r \cdot \omega^2) + j \cdot \omega \cdot L_r} \quad (8)$$

Assuming that at resonant frequency ω_0 , $1 - C_r \cdot L_r \cdot \omega_0^2 = 0$, then after getting rid of the complex values in the above expression, we obtain

$$I_{load}(\omega_0) = V_{in}(\omega_0) \cdot \frac{1}{\omega_0 \cdot L_r} \quad (9)$$

This means that any kind of load (rectifier as well) connected in parallel to the resonance capacitor in a series resonant schematic will be supplied by a current whose value is solely defined by the supplied ac voltage and resonant inductor reactance if the ac voltage is at the circuit resonant frequency.

In these considerations $I_{load}(\omega_0)$ and $V_{in}(\omega_0)$ are functions of time and cyclic frequency ω_0 but we are reviewing the circuit properties at resonance.

Therefore, ω_0 is the power source pulse train frequency, which should be equal to the resonant frequency of the resonant LC circuit.

What is the current through the resonant capacitor $I_c(\omega)$?

$$I_C(s) = \frac{V_{in}(s) \cdot \frac{Z_{load}(s)}{Z_{load}(s) + L_r \cdot s + C_r \cdot L_r \cdot s^2 \cdot Z_{load}(s)}}{\frac{1}{s \cdot C_r}}$$

which simplifies to

$$I_C(s) = \frac{C_r \cdot s \cdot V_{in}(s) \cdot Z_{load}(s)}{Z_{load}(s) + L_r \cdot s + C_r \cdot L_r \cdot s^2 \cdot Z_{load}(s)} \quad (10)$$

In the frequency domain this becomes

$$I_C(\omega) = \frac{C_r \cdot \omega \cdot V_{in}(\omega) \cdot Z_{load}(\omega)}{\omega \cdot L_r}$$

which simplifies to

$$I_C(\omega) = \frac{C_r \cdot V_{in}(\omega) \cdot Z_{load}(\omega)}{L_r}$$

which, at the resonant frequency, is simply

$$I_C(\omega_0) = \frac{C_r \cdot V_{in}(\omega_0) \cdot Z_{load}(\omega_0)}{L_r} \quad (11)$$

We cannot underestimate the importance of calculating this current to ensure the circuit is viable. The capacitor must be able to tolerate this high resonant current as it causes capacitor heating.

A Design Example

To illustrate the use of the resonant current source, let's consider the case where a full-bridge converter configuration supplies the RCS with uniform 208-kHz rectangular pulses at 0.5 duty-cycle and 48-V amplitude. Operating with that input, we need the RCS to generate an output of 9 A at 800 V dc. So, we have to define parameters of the resonant circuit L_r and C_r as well as the current through the resonant capacitor to obtain the required output.

First of all, as noted above, the resonant circuit filters out all harmonics except for the first one. The input voltage is a uniform rectangular pulse train that has infinite spectrum. Therefore, we have to define the first harmonic amplitude and RMS value for further analysis and calculations.

Per (1)

$$V_{in}(t) = V_a \cdot \sin(\omega \cdot t)$$

Per (2)

$$V_{in}(t) = \frac{2 \cdot V_{pulse} \cdot \sin(\omega \cdot t)}{\pi}$$

And per (3)

$$V_{in_rms} = V_{pulse} \cdot \frac{\sqrt{2}}{\pi}$$

Furthermore, from equations (9) and (2)

$$I_{load_rms} = V_{in_rms} \cdot \frac{1}{\omega_0 \cdot L_r}$$

Substituting the expression for V_{in_RMS} from equation (3),

$$I_{load_rms}(\omega_0) = V_{pulse} \cdot \frac{\sqrt{2}}{\pi} \cdot \frac{1}{\omega_0 \cdot L_r} \quad (12)$$

We can then define L_r from (12):

$$L_r = \frac{\sqrt{2} \cdot V_{pulse}}{\pi \cdot \omega_0 \cdot I_{load_rms}} \quad (13)$$

Plugging in this expression into the well-known formula for the resonant frequency,

$$\omega_0 = \frac{1}{\sqrt{C_r \cdot L_r}} \quad (14)$$

we can obtain the value for the resonant capacitor value:

$$C_r = \frac{1}{\frac{\sqrt{2} \cdot V_{pulse}}{\pi \cdot \omega_0 \cdot I_{load_rms}} \cdot \omega_0^2} \quad (15)$$

For a resistive load:

$$V_{out} = I_{load_rms} \cdot R_{load} \quad (16)$$

and

$$R_{load} = \frac{V_{out}}{I_{load_rms}} \quad (17)$$

Going back to our earlier description of the design example, we can assign the following values to the circuit parameters in the above equations:

$$V_{pulse} = 48 \text{ V}$$

$$I_{load_rms} = 9 \text{ A}$$

$$V_{out} = 800 \text{ V}$$

$$\omega_0 = 208 \text{ kHz} \cdot 2 \cdot \pi$$

This leads to the following values for our RCS components:

$$L_r = \frac{\sqrt{2} \cdot V_{\text{pulse}}}{\pi \cdot \omega_0 \cdot I_{\text{load_rms}}} = 1.837 \times 10^{-6} \text{ H}$$

$$C_r = \frac{1}{\frac{\sqrt{2} \cdot V_{\text{pulse}}}{\pi \cdot \omega_0 \cdot I_{\text{load_rms}}} \cdot \omega_0^2} = 318.708 \times 10^{-9} \text{ F}$$

$$R_{\text{load}} = \frac{V_{\text{out}}}{I_{\text{load_rms}}} = 88.889 \text{ } \Omega$$

$$V_{\text{in_rms}} = V_{\text{pulse}} \cdot \frac{\sqrt{2}}{\pi} = 21.608 \text{ V}$$

$$I_{C_rms} = \frac{C_r \cdot V_{\text{in_rms}} \cdot R_{\text{load}}}{L_r} = 333.216 \text{ A}$$

The above example corresponds to a link capacitor charger (the RCS) that can be used to charge an industrial 7.2-kW, 800-V Li-Ion battery pack. The huge step-up ratio of $800 \text{ V}/21.6 \text{ V} = 37$ explains the high resonant and primary currents, but the resonant current dissipates little power while the primary current may cause the bridge switches overheat.

One of the important requirements for this current source is holding expression $1 - C_r \cdot L_r \cdot \omega_0^2 = 0$ true within 1% or 2% at all possible circuit parameters deviations and switching frequency instability. The latter error source can be minimized by using the now very popular digital, crystal-stabilized controllers, while the former error source is the resonant inductance and resonant capacitors instability.

When using a composite load, one should know that there should be no capacitor connected directly to the resonant capacitor. Otherwise such a load capacitance would be accrued by the resonant capacitor, and the circuit would stop working. In this case an inductor is recommended in series with the load as shown in Fig. 2, which is a section of the circuit that we will use to simulate performance of the RCS.

It is interesting to know how the load and resonant capacitor C_r share the current.

From the equations above:

$$\frac{I_{\text{load_rms}}}{I_{C_rms}} = \frac{V_{\text{pulse}} \cdot \frac{\sqrt{2}}{\pi} \cdot \frac{1}{\omega_0 \cdot L_r}}{\frac{C_r \cdot V_{\text{in_rms}} \cdot R_{\text{load}}}{L_r}}$$

Assuming

$$V_{\text{in_rms}} = V_{\text{pulse}} \cdot \frac{\sqrt{2}}{\pi}$$

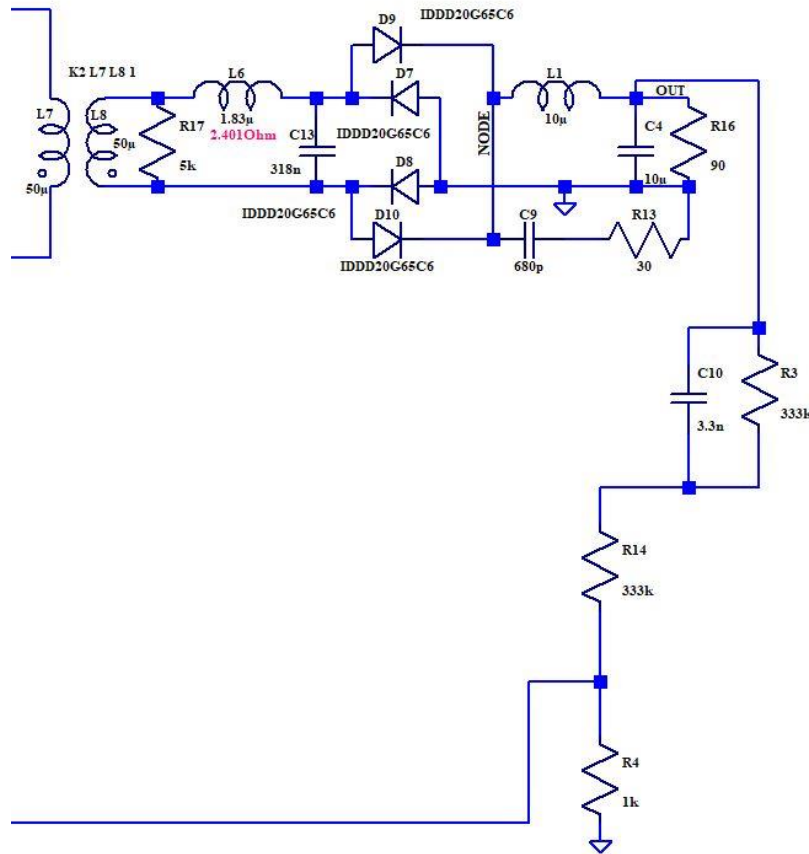


Fig. 2. A section of the RCS circuit that will be used to simulate performance of the design example. L6 and C13 are the resonant components. Note the use of the inductor L1 to "isolate" capacitor C4 from the resonant capacitor C13.

Designating

$$\rho = \sqrt{\frac{L_r}{C_r}} \tag{18}$$

we get

$$\frac{I_{load_rms}}{I_{C_rms}} = \frac{\sqrt{L_r}}{\sqrt{C_r \cdot R_{load}}} = \frac{\rho}{R_{load}} \tag{19}$$

This tells us that the ratio of the load current to the resonant capacitor current is equal to ratio of the resonant circuit characteristic resistance ρ to the load resistance. In this case

$$\text{ratio} = \frac{\sqrt{L_r}}{\sqrt{C_r \cdot R_{load}}} = 0.027$$

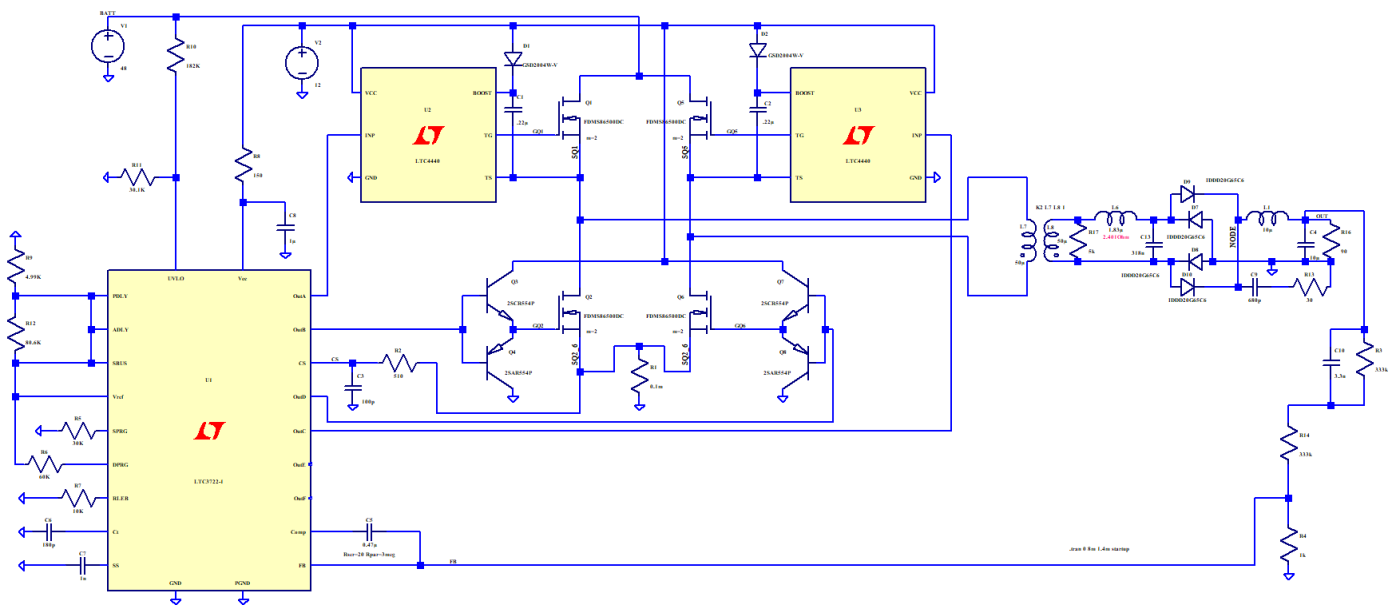
$$\frac{I_{load_rms}}{I_{C_rms}} = 0.027$$

These relationships allow us to quickly assess where we are at in the RCS design. Ultimately, this ratio should be in the range of 0.03 to 0.2.

To summarize, the RCS design process is as follows:

1. Define the resonant circuit inductance from equation (13). At this point we know the load current, operating frequency and input voltage produced by the bridge.
2. From equation (15), define the resonant capacitance.
3. Check I_c from equation (11).
4. If I_c is too high, we need to adjust V_{in} in (11). In our design example, this would mean adjusting the transformer turns ratio to reduce the resonant capacitor current to meet datasheet spec.
5. Keep I_{load_rms}/I_{C_rms} ratio = 0.03 to 0.2.

Fig. 3 presents the full schematic of the circuit used to simulate the RCS design example described above. Simulation results for this circuit are shown in Fig. 4. The green curve is the output current, while the blue curve is the output voltage. In this case, the output current reaches the target value in 5 ms. Although not stated previously, the requirement for this circuit to achieve current regulation was 200 ms. So the RCS circuit meets that requirement easily.



(a)

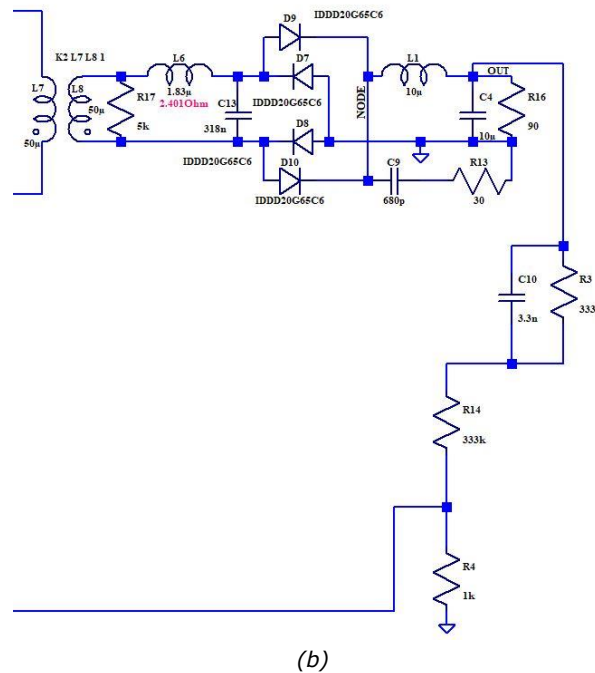


Fig. 3. LTSPICE model of the resonant current source described in the design example (a). The resonant current source is built around the resonant inductor L6 and the resonant capacitor C13. This resonant circuit feeds a diode-bridge rectifier, which in turn feeds the load R16 representing the battery and C4 representing the link capacitor. The voltage feedback loop then regulates the output current to the load R16. Part (b) zooms in on these components. Note: Components models and values beyond the resonant current source are not recommended parts and values. These were selected solely for the purpose of running the simulation only. (Simulation results are shown in Fig. 4.) A good result can be obtained when using a digital controller in this schematic. C9 and R13 compose a snubber, which can be optional.

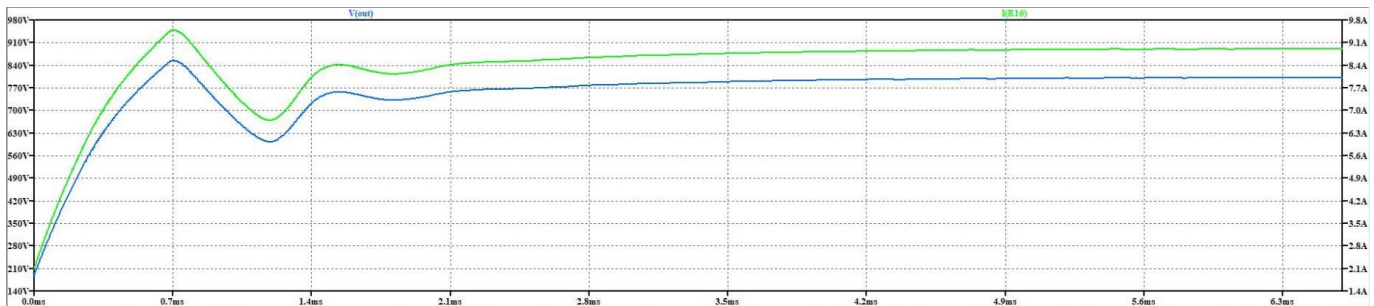


Fig. 4. Simulation results for Fig 3. The green curve represents the load current, while the blue curve represents the output voltage. Note that due to the use of RCS current-source topology, the output voltage ripple is much lower than it would be with a voltage-source topology, while also requiring a lower capacitance value for the link capacitor.

Appendix: Finding The First Harmonic Of A Rectangular Pulse Train

According to Fourier analysis, the first harmonic $F_1(t)$ of the pulse train having 0.5 duty-cycle is

$$F_1(t) = \frac{2 \cdot V_{in} \cdot \sin(\omega \cdot t) \cdot \sin(\pi \cdot 0.5)^2}{\pi} + \frac{V_{in} \cdot \sin(2 \cdot \pi \cdot 0.5) \cdot \cos(\omega \cdot t)}{\pi} \quad (1)$$

Simplifying, we obtain

$$F_1(t) = \frac{2 \cdot V_{in} \cdot \sin(\omega \cdot t)}{\pi}$$

The RMS value of this sinusoid is

$$V_{in_rms} = V_{in} \cdot \frac{2}{\pi} \cdot \frac{\sqrt{2}}{2} = V_{in} \cdot \frac{\sqrt{2}}{\pi}$$

About The Author



Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an

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Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels [online](#).