Calculating Minimum Magnetic Core Size For A Transformer

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My previous publications on defining magnetic core size[1, 2] were dedicated to magnetic cores designed for intentionally storing magnetic energy during power conversion. Those magnetic cores are ones used in inductors or coupled inductors in which the converters store energy during a portion of the switching cycle and then subsequently release that energy into the load. Those articles explained the relationships and derived the equations that determine the minimum core size (in volume) required for the inductors and coupled inductors to store a given amount of energy for a given core permeability and other circuit conditions.

On the other hand, transformers, whose operation is not based on intentional storing of magnetic energy, also have a magnetic core size limitation—a minimum volume based on certain operating conditions. This means that the transformer magnetic core would fall into saturation if its physical size was too small to meet the magnetic and electrical requirements of the transformer.

In inductors the load current is the core magnetizing current, while in transformers the core magnetizing current is separate from the load current. This is why in inductors (chokes) the size-defining parameter is operating power, while in transformers the magnetizing current ripple, which may constitute 0.5% to 1% of the input ac current, determines core saturation and thus the minimum size of the core. Nevertheless, the resulting magnetic core size in transformers usually must be adjusted (made larger) to accommodate the windings.

Therefore, a transformer design needs answers to more questions on what assumptions should be made to design a transformer properly. This article is going to clarify what these assumptions are, while deriving the equations needed to determine minimum core size in a power converter application.

Assumptions About Transformer Characteristics

In order to analyze the factors that determine minimum magnetic core size in transformers, we need to define certain characteristics of these components. Fig. 1 offers a simple electrical model of a transformer that accounts for its magnetizing inductances and leakage (or stray) inductances. While the stray inductances are not analyzed in this discussion, they will be discussed in a future article.

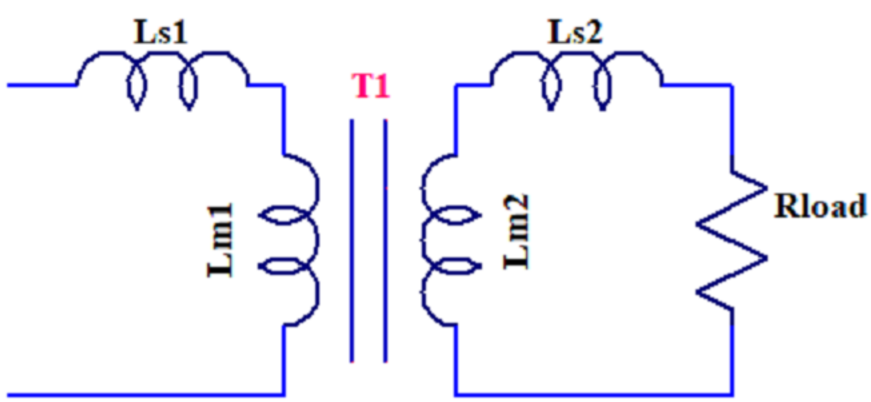

Fig. 1 Simplified electrical model of a transformer including leakage inductances L_{s1} and L_{s2}.

- Magnetizing inductances L_{m1} and L_{m2} do not participate in energy transfer.
- Stray or leakage inductances L_{s1} and L_{s2} are parasitic components that just create ringing during switching, which reduces the resulting transformer efficiency.
- Depending on the transformer function (buck or boost), the turns ratio secondary-to-primary may be below or above unity.
- Magnetic flux lines in the magnetic core are uninterruptible.
- Magnetic core shall not store any magnetic energy at power transfer.

Parameter Designations

In addition to the general transformer characteristics noted above, we also need to identify a series of transformer core and winding parameters, as well as circuit operating parameters that will be needed to analyze the relationships that determine core size.

- N_1 = primary winding number of turns
- N_2 = secondary winding number of turns
- Ψ = core magnetic flux linkage
- Φ = core magnetic flux created by one winding wire turn
- V_1 and V_2 = transformer input and output voltages
- t = current time
- S_m = magnetic core cross-sectional area
- l_m = magnetic core magnetic path average length
- B_m = operating magnetic flux density in the core
- T_{SW} = period of the switching voltage
- f_{SW} = frequency of the switching voltage
- I_m = ripple current amplitude
- H_m = magnetic field strength
- μ_0 = absolute material permeability
- μ_r = relative material permeability
- L_m = primary winding inductance
- A_L = inductance index
- Vol = magnetic core volume.

Neglect Stray Inductances

To begin this analysis, let's review the key equations that relate electrical and magnetic operation of a transformer. In the following equations, we assume leakage inductances are zero.

Input voltage deviation creates magnetic core flux linkage change:

$$\frac{d}{dt}\Psi_1 = V_1 = N_1 \cdot \left(\frac{d}{d}\Phi_1\right) \tag{1}$$

Voltage induced in the secondary winding by the same flux linkage:

$$V_2 = \frac{d}{dt}\Psi_1 = N_2 \cdot \left(\frac{d}{d}\Phi_1\right) \tag{2}$$

Dividing (1) by (2), we obtain the familiar expression relating voltage and turns ratios:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \tag{3}$$

Calculation Of The Primary Winding Number Of Turns

From equation (1) we have:

$$V_1 = \frac{d}{dt}\Psi_1 = N_1 \cdot \frac{d}{d}\Phi_1$$

From equations (1) and (2), and from the definition of flux density B, we know that

$$\frac{d}{dt}\Phi_1 = S_m \cdot \left(\frac{d}{dt}B_m\right)$$

which when substituted into (1) gives us

$$V_1 = \frac{d}{dt}\Psi_1 = N_1 \cdot \frac{d}{d}\Phi_1 = N_1 \cdot S_m \cdot \left(\frac{d}{dt}B_m\right) \qquad (4)$$

Since we are dealing with parameters swinging under the influence of rectangular pulses,

$$V_1 = N_1 \cdot S_m \cdot \left(\frac{d}{dt}B_m\right) = N_1 \cdot S_m \cdot \frac{2\Delta B_m}{\Delta t} \qquad (5)$$

Since

$$\Delta t = \frac{T_{SW}}{2} \qquad (6)$$

in equation (5) ΔB_m is multiplied by the factor of 2 since the value for ΔB_m is taken for just one branch of the hysteresis loop, and we are calculating for the whole swing during T_{SW} which is 2*Δt. We also note that ΔB_m is specified as B_m in data books.

Therefore

$$V_1 = 4 \cdot f_{SW} \cdot N_1 \cdot S_m \cdot B_m \qquad (7)$$

Hence we get a well-known formula:

$$N_1 = \frac{V_1}{4 \cdot f_{SW} \cdot B_m \cdot S_m} \qquad (8)$$

Since S_m is a factor in the equation for core volume, this expression for N1 will be useful in deriving an expression for core volume in terms of the transformer and circuit operating parameters.

Calculation Of The Required Core Volume

Based on the formula for the voltage that appears across an inductor when changing current is induced in its winding, we can derive the following:

$$V_1 = L_m \cdot \frac{\Delta I_m}{\Delta t} \qquad (9)$$

where ΔI_m is the primary winding current ripple.

Then, from the Full Current law applied to the rectangular voltage pulses acting across an inductance, we can write

$$\Delta I_m \cdot N_1 = \Delta H_m \cdot l_m$$

This gives us an expression for the current ripple:

$$\Delta I_m = \frac{\Delta H_m \cdot l_m}{N_1}$$

which can be plugged into (9) to yield

$$V_1 = \frac{L_m \cdot l_m \cdot \Delta H_m}{N_1 \cdot \Delta t} \qquad (10)$$

We can develop this equation further if we recall the basic formula for flux density:

$$\Delta B = \mu_0 \cdot \mu_r \cdot \Delta H$$

which then gives us an expression for ΔH:

$$\Delta H_m = 2 \cdot H_m = \frac{2B_m}{(\mu_0 \cdot \mu_r)} \tag{11}$$

where ΔH_m is a magnetic field full swing and B_m is the maximum operating flux density as depicted in Fig. 2.

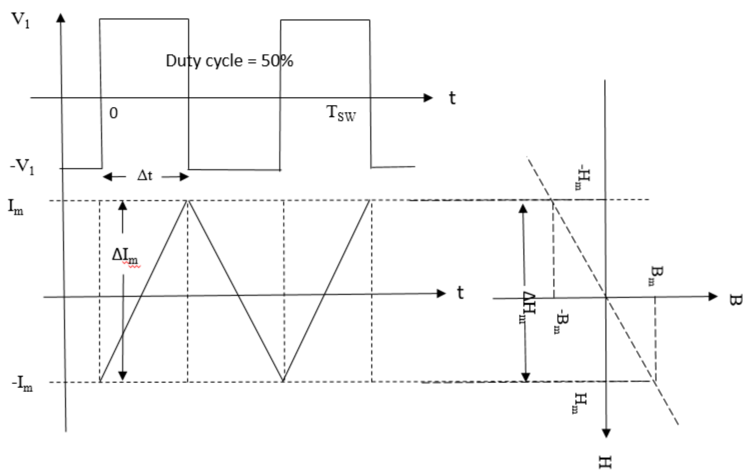

Fig. 2. ΔHm is the magnetic field full swing and Bm is the maximum operating flux density produced by the primary winding current ripple ΔIm, which results from the pulse train driving the transformer.

Since we are applying a symmetrical pulse train we can also state that

$$\Delta t = \frac{T_{SW}}{2} \tag{12}$$

So, plugging in (11) and (12) into (10) and simplify to get:

$$V_1 = \frac{4 \cdot L_m \cdot l_m \cdot B_m}{N_1 \cdot T_{SW} \cdot \mu_0 \cdot \mu_r}$$

Then assuming

$$T_{SW} = \frac{1}{f_{SW}}$$

we get

$$V_1 = 4 \cdot \frac{L_m \cdot B_m \cdot l_m \cdot f_{SW}}{\mu_0 \cdot \mu_r \cdot N_1} \tag{13}$$

Hence

$$V_1 \cdot \mu_0 \cdot \mu_r \cdot N_1 = 4 \cdot L_m \cdot B_m \cdot l_m \cdot f_{SW} \tag{14}$$

and

$$N_1 = \frac{4 \cdot B_m \cdot L_m \cdot f_{SW} \cdot l_m}{V_1 \cdot \mu_0 \cdot \mu_r} \tag{15}$$

Note that this last equation gives N1 in terms of l_m, which is another factor in the expression for core volume.

So, equating (8) and (15) we obtain:

$$\frac{4\cdot B_m\cdot L_m\cdot f_{SW}\cdot l_m}{V_1\cdot\mu_0\cdot\mu_r}=\frac{V_1}{4\cdot f_{SW}\cdot B_m\cdot S_m} \qquad (16)$$

Solving (16) with respect to L_m, we get:

$$L_m=\frac{{V_1}^2\cdot\mu_0\cdot\mu_r}{16\cdot {B_m}^2\cdot S_m\cdot {f_{SW}}^2\cdot l_m} \qquad (17)$$

Recollecting that

$$S_m\cdot l_m=Vol$$

we can re-write (17) as:

$$L_m=\frac{{V_1}^2\cdot\mu_0\cdot\mu_r}{16\cdot {B_m}^2\cdot {f_{SW}}^2\cdot Vol} \qquad (18)$$

Now we have an expression that relates magnetizing inductance to core volume.

However, magnetizing inductance can also be expressed in terms of the required current ripple. Designating I_m as the primary winding ripple current amplitude, we get the following equation for magnetizing inductance:

$$L_m=V_1\cdot\frac{\Delta t}{\Delta I_m}=\frac{V_1}{4\cdot I_m\cdot f_{SW}} \qquad (19)$$

So, from (18) and (19) we get:

$$Vol=\frac{I_m\cdot V_1\cdot\mu_0\cdot\mu_r}{4\cdot {B_m}^2\cdot f_{SW}} \qquad (20)$$

which gives us core volume in terms of I_m. While this is the key equation we'll need to determine minimum core volume, we also need to consider two more relationships.

From (19) and the following formula for the winding inductance,

$$L_m=\mu_0\cdot\mu_r\cdot\frac{S_m}{l_m}\cdot {N_1}^2 \qquad (21)$$

we obtain another expression for magnetizing current:

$$I_m=\frac{V_1}{4\cdot\left(\mu_0\cdot\mu_r\cdot\frac{S_m}{l_m}\cdot {N_1}^2\right)\cdot f_{SW}} \qquad (22)$$

From equations (21) and (22), we can conclude that to keep the magnetization current I_m lower, it is reasonable to have magnetizing inductance L_m higher as that equates to having a higher S_m/l_m ratio of the magnetic core.

As this analysis has shown, a transformer's physical volume does not depend on the power the device handles, but rather on the permissible magnetizing current ripple, core material, input voltage, permissible operating magnetic flux density and operating frequency.

Transformer Electrical Design Example

To illustrate how we can apply the above equations in practice, let's consider a transformer example with the following parameters.

V_1 = 800 V

μ_r = 2500

B_m = 0.26 T

f_{SW} = 200 kHz

I_m = 600 mA

Recalling equation (20) above:

$$\mathrm{Vol} = \frac{I_m \cdot V_1 \cdot \mu_0 \cdot \mu_r}{4 \cdot {B_m}^2 \cdot f_{SW}} = 0.03\ \mathrm{L}$$

If we approximate the core as a cube, then the calculated volume corresponds to a cube with a side measuring

$$\sqrt[3]{\mathrm{Vol}} = 0.03\ \mathrm{m}$$

The transformer magnetic core should have physical volume no less than calculated above.

Recalling equation (19), the necessary inductance value that provides the selected I_m:

$$L_m = \frac{V_1}{4 \cdot I_m \cdot f_{SW}} = 1.67\ \mathrm{x}\ 10^{-3}\mathrm{H}$$

This inductance value defines the amplitude of the magnetizing current that would control the core state, and can be increased if cost permits. In fact, it will be increased in a real transformer with the windings made for a specific power on a bigger core.

References

1. "Analysis Of Energy Storage Inductor Eases Converter Design" by Gregory Mirsky, How2Power Today, March 2022.
2. "Designing Energy Storing Inductors Properly" by Gregory Mirsky, How2Power Today, January 2019.

About The Author

Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

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