Analysis Of Core Hysteresis Loss Underscores Transformer Efficiency Challenges

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Rapid development of very fast and efficient SiC and GaN semiconductors allows for creating highly efficient power converters operating in the megahertz range. Some designers and manufacturers boast product efficiencies above 99%. To achieve this high total converter efficiency requires not only that the semiconductor portion of the converter be extremely efficient, but also that the power transformer must have record-high efficiency.

Although modern magnetic materials have many outstanding parameters, the laws of physics cannot be violated, and therefore, the transformer's efficiency is limited by a few factors, which describe power loss in the magnetic core.

- Hysteresis loss, which occurs due to magnetic material domains changing their magnetization and then retaining their magnetization after the magnetizing magnetic field is gone. This loss does not directly depend on magnetic field frequency but rather on the core volume, which in turn, depends on frequency inversely.
- Magnetic core loss, which results from inertial mechanical effects in the core (domain friction and inertia).
- Eddy current loss, which occurs in cores (even ferrites are subject to them).

In addition to these core losses, there are eddy current losses in clamps and winding wires (due to proximity and skin effects), plus active losses in the windings and PCB traces due to their resistivity. All these losses reduce the actual efficiency of the transformer significantly, and even the most "mild" hysteresis loss is capable of botching the transformer performance.

In this article we will analyze how the hysteresis loss affects the magnetic core loss, and what core material properties affect the hysteresis loss most of all. Analyzing this loss underscores the difficulty of achieving extremely high efficiency in transformers because if hysteresis loss is significant, the other losses will be even more so. Moreover, determining the other losses would require a more lengthy analysis.

In reference 1, I described how to build a hysteresis loop using conventional magnetic core parameters such as B_{sat}, μ_r and B_r for a real core. We'll repeat those steps here to better understand the material. Then, we'll extend the analysis to determine the efficiency and power loss for this core as a function of hysteresis loss, given a selected core volume and operating frequency.

Building A Hysteresis Loop

To begin, let's designate B_{sat} the core saturation flux density and H_C the core coercive force. Then H_0 is a virtual magnetic field strength, for which we will derive an expression. H_X is the current magnetic field strength. $B_1(H_X)$ and $B_2(H_X)$ are values for the present flux densities for both branches of the hysteresis loop. These are the values we'll be looking to plot for our hysteresis loop.

It is also helpful to find a derivative of one of the above expressions (either $B_1(H_X)$ or $B_2(H_X)$ because the derivative is the same for both), as this derivative represents permeability of the core material in the vicinity of the coordinates crossing:

From reference 1 and a prior source, we know that

$$B_1(H_x) = B_{sat}\left[\frac{2}{1+e^{\frac{-(H_x-H_c)}{H_0}}} - 1\right] \qquad (1)$$

$$B_2(H_x) = B_{sat}\left[\frac{2}{1 + e^{\frac{-(H_x+H_c)}{H_0}}} - 1\right] \quad (2)$$

Taking the derivative of either expression we obtain

$$\frac{d}{dH_x}\left[B_{sat}\frac{2}{1 + e^{\frac{-(H_x+H_c)}{H_0}}} - 1\right] \rightarrow \frac{2 \cdot B_{sat} \cdot e^{-\frac{H_c+H_x}{H_0}}}{H_0 \cdot \left(e^{-\frac{H_c+H_x}{H_0}} + 1\right)^2} \quad (3)$$

After simplifying (3), it can be rewritten to obtain an expression for core permeability,

$$\mu(H_x) = \frac{B_{sat}}{2 \cdot H_0 \cdot \cosh\left(\frac{H_c + H_x}{2 \cdot H_0}\right)^2} \quad (4)$$

At the beginning of the coordinates where $H_X = 0$ and $H_C = 0$

$$\mu(0) = \frac{B_{sat}}{2H_0} = \mu \quad (5)$$

At the same time

$$\mu = \frac{B_{sat}}{H_{sat}} \quad (6)$$

Hence

$$\frac{B_{sat}}{2 \cdot H_0} = \frac{B_{sat}}{H_{sat}}$$

Therefore

$$\left.\begin{aligned} H_0 &= \frac{H_{sat}}{2} \\ H_{sat} &= \frac{B_{sat}}{\mu} \end{aligned}\right\} \qquad (7)$$

Plugging equation (7) into (1) and (2), and simplifying, one can get:

$$B_1(H_x) = -B_{sat} \cdot \tanh\left[\frac{\mu \cdot (H_c - H_x)}{B_{sat}}\right] \qquad (8)$$

$$B_2(H_x) = B_{sat} \cdot \tanh\left[\frac{\mu \cdot (H_c + H_x)}{B_{sat}}\right] \qquad (9)$$

Recalling that $\mu = \mu_r \bullet \mu_0$, we note that the expressions in (8) and (9) now contain two parameters, μ and B_{sat}, that are obtainable from a core data sheet. However, some additional steps will be required to determine a value for H_C that can be calculated from data sheet information.

To find H_C we observe that the remanence B_r value can be obtained from equation (8) or (9) by assuming H_X = 0:

$$\left.\begin{aligned} B_{r1} &= -B_{sat} \cdot \tanh\left(\frac{\mu \cdot H_c}{B_{sat}}\right) \qquad \text{and} \\ B_{r2} &= B_{sat} \cdot \tanh\left(\frac{\mu \cdot H_c}{B_{sat}}\right) \end{aligned}\right\} \qquad (10)$$

The equations in (10) allow us to define the values of coercive force for both hysteresis loop branches:

$$\left.\begin{aligned} H_{c1} &= -\frac{B_{sat} \cdot \ln\left(-\dfrac{B_{r1} + B_{sat}}{B_{r1} - B_{sat}}\right)}{2 \cdot \mu} \\ H_{c2} &= -\frac{B_{sat} \cdot \ln\left(-\dfrac{B_{r2} - B_{sat}}{B_{r2} + B_{sat}}\right)}{2 \cdot \mu} \end{aligned}\right\} \qquad (11)$$

H_{C1} and H_{C2} as well as B_{r1} and B_{r2} are symmetrical loop values with respect to the hysteresis loop coordinates if the magnetic core material is homogenous. Therefore $H_C = H_{C1} = H_{C2}$ and $B_r = B_{r1} = B_{r2}$.

Now we can plot a hysteresis loop using datasheet parameters per the example in the appendix where the following datasheet parameters are obtained for an R34 toroid core made of N88 ferrite material:

$B_{sat} = 0.37$ T

$\mu_r = 6000$

$\mu = \mu_r \bullet \mu_0 = 7.54 \times 10^{-3}$ m•kg/A^2•s^2

$B_r = 0.02$ T

$H_{sat} = B_{sat}/\mu = 49.073$ A/m

Now we'll check if the coercive force H_C value defined by the two formulas is correct. From (11) we find

$$H_c := -\frac{B_{sat} \cdot \ln\left(-\frac{B_r - B_{sat}}{B_r + B_{sat}}\right)}{2 \cdot \mu} = 2.655 \frac{A}{m}$$

and we can check this calculation by using the simpler equation

$$H_c := \left(\frac{B_r}{\mu}\right) = 2.653 \frac{A}{m}$$

The values are very close, indicating that the relative permeability of this particular core material does not change much within the loop span from $-B_{sat}$ to $+B_{sat}$. This would not be the case if the loop shape was more curvy or "thicker" and then we would have to rely on equation (11) to determine H_C. However, in practice, we will want to consider only the "slim" loop materials—as we will see.

Next, we'll use equations (8) and (9) to plot the graph in the figure.

$$B_1(H_x) := -B_{sat} \cdot \tanh\left[\frac{\mu \cdot (H_c - H_x)}{B_{sat}}\right]$$

$$B_2(H_x) := B_{sat} \cdot \tanh\left[\frac{\mu \cdot (H_c + H_x)}{B_{sat}}\right]$$

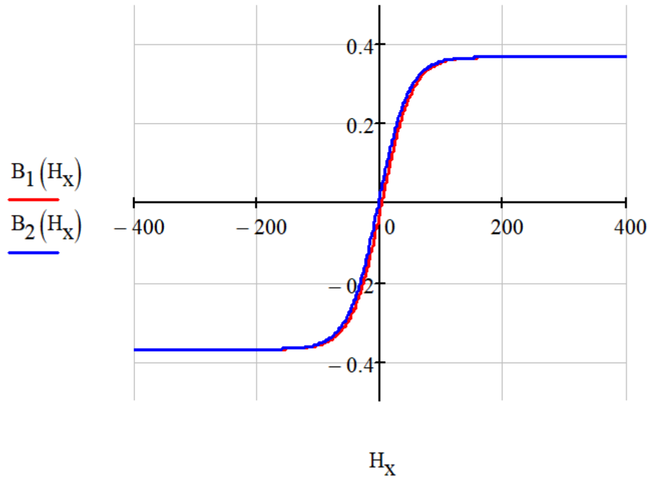


Figure. Hysteresis loop model for a real core (an R34 toroid core made of N88 ferrite material.)

The expressions derived above to plot the hysteresis model for the example core will be applied in the next section to determine efficiency due to hysteresis for a given core volume and power loss due to hysteresis for the core in a given core volume and a chosen switching frequency.

Transformer Power Loss And Efficiency As A Function Of Core Hysteresis

Transformer magnetic core volume necessary for transferring power P_1 can be defined[2] as

$$Vol_{mag} = \left(\frac{P_1 \cdot \mu_0 \cdot \mu_r}{4 \cdot B_{sat}{}^2 \cdot f_{sw}} \right) \tag{12}$$

Hysteresis loss is defined by the hysteresis loop area:[3]

$$P_{hyst\_loss} = \left[\int_{-H_{sat}}^{H_{sat}} f_{sw} \cdot Vol_{mag} \cdot \left(B_2(H_x) - B_1(H_x) \right) dH_x \right] \tag{13}$$

If we designate

$$\Delta B_{\ hyst} = B_2(H_x) - B_1(H_x) \tag{14}$$

and plug expressions (8) and (9) into (14) we get

$$\Delta B_{\ hyst} = \left[B_{sat} \cdot \tanh\left[\frac{\mu \cdot (H_c + H_x)}{B_{sat}} \right] + B_{sat} \cdot \tanh\left[\frac{\mu \cdot (H_c - H_x)}{B_{sat}} \right] \right] \tag{15}$$

We can then obtain the hysteresis loss as

$$P_{hyst\_loss} = \int_{-H_{sat}}^{H_{sat}} \frac{P_1 \cdot \mu}{4 \cdot B_{sat}^2} \cdot \left[B_{sat} \cdot \tanh\left[\frac{\mu \cdot (H_c + H_x)}{B_{sat}}\right] + B_{sat} \cdot \tanh\left[\frac{\mu \cdot (H_c - H_x)}{B_{sat}}\right]\right] dH_x \qquad (16)$$

From (16) we can determine the efficiency of the transformer due to the hysteresis loss only:

$$\eta_{hyst} = \frac{P_1 - P_{hyst\_loss}}{P_1} = 1 - \frac{\mu}{\left(4 \cdot B_{sat}^2\right)} \cdot \int_{-H_{sat}}^{H_{sat}} \left[B_{sat} \cdot \tanh\left[\frac{\mu \cdot (H_c + H_x)}{B_{sat}}\right] + B_{sat} \cdot \tanh\left[\frac{\mu \cdot (H_c - H_x)}{B_{sat}}\right]\right] dH_x \qquad (17)$$

As (17) indicates, we do not need P1 to determine transformer efficiency due to hysteresis. If we were to look at a transformer model, we would see that the magnetizing inductance exists in parallel with the input, so only magnetizing current, and not load current, is flowing through it. The magnetizing current depends solely on the core material properties, loop configuration (thicker or thinner) and core size.

Recalling the datasheet parameters given above—B_{sat} = 0.37 T, μ_r = 6000, μ = 7.54 x 10^{-3} m•kg/A^2•s^2 and the value of H_C =2.655 A/m calculated above—we can determine the efficiency of the transformer due to hysteresis for the example core:

$$\eta_{hyst} := \left[1 - \frac{\mu}{\left(4 \cdot B_{sat}^2\right)} \cdot \int_{-H_{sat}}^{H_{sat}} \left[B_{sat} \cdot \tanh\left[\frac{\mu \cdot (H_c + H_x)}{B_{sat}}\right] + B_{sat} \cdot \tanh\left[\frac{\mu \cdot (H_c - H_x)}{B_{sat}}\right]\right] dH_x \right] = 0.959$$

Considering this result, we can say that while the core hysteresis loss is independent of the transformers' power transfer, its effect on transformer efficiency is nevertheless significant.

Now if we go a step further and factor in core size and frequency, we can determine the actual loss for a specific core. For a core having the size specified below (MAGNETICS P-type #2915),

l_{mag} = 73.2 mm and S_{mag} = 74.9 mm^2

and operating at a frequency of f = 250 kHz.

$$Vol_{mag} := S_{mag} \cdot l_{mag} = 5.483 \times 10^{-3} \, L$$

In this case, the hysteresis loop power loss is

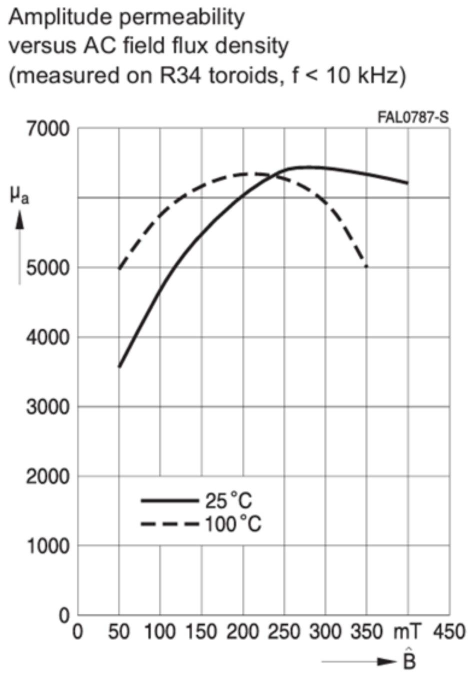
$$P_{hyst\_loss} := \int_{-H_{sat}}^{H_{sat}} f \cdot Vol_{mag} \cdot \left(B_2(H_x) - B_1(H_x)\right) dH_x = 4.096 \, W$$

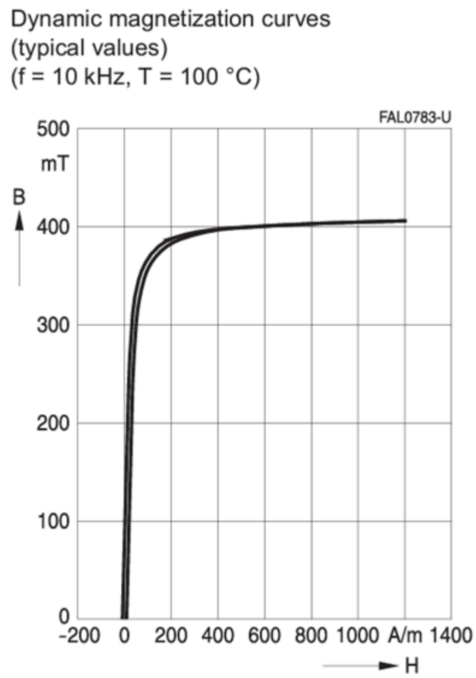
From this analysis we see that even such a low loss contributor as the hysteresis effect injects noticeable loss into the power transformer operation. Therefore for any application seeking extreme efficiency the transformer efficiency should be calculated thoroughly, and the converter efficiency measured accurately since magnetics are significant loss contributors.

To reduce magnetic core loss it is necessary to use adequate cores having the slimmest hysteresis loop, i.e. a loop having minimum area. In the example presented, we selected a ferrite material that has minimum hysteresis loop area for the transformer being designed. Remanence and coercive force for this material should be as close as possible to zero not to waste energy on re-magnetizing of the core.

Appendix

Analysis of a real core per reference [4]:





References

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2. "Calculating Minimum Magnetic Core Size For A Transformer" by Gregory Mirsky, How2Power Today, May 2022.
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About The Author

Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

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