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Measuring The Negative Input Resistance Of DC-DC Converters

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It seems that most power integrity engineers spend time focused on the output impedance of their board-level dc-dc converters (often referred to as voltage regulator modules or VRMs). That's understandable because output impedance provides a metric for quantifying and controlling the voltage excursions that result from dynamic load currents drawn by high-speed digital loads. At the system-level, there's even a specification for the maximum power rail impedance and this requirement is generally referred to as the "target impedance."

Many power integrity engineers argue that we don't even need to include the switching dc-dc converter in our simulations, we can simply use a passive network to represent the output impedance.[1,2] I've generally opposed that approach, arguing that the power supply has many noise sources and sensitivities at both its input and the output and to ignore them causes the output impedance to be inaccurate.[3] An obvious noise source is the switching ripple voltage, which would not be represented using a passive network. This is just one example of why a passive network isn't sufficient.

In this article I'll discuss another aspect of the dc-dc converter that is not modeled by a passive network and that's its negative input resistance—an issue that was long ago discovered but one that many engineers seem to have forgotten. I'll also describe a way to measure this negative input resistance using a new device (Picotest's J2121A line injector) which makes this measurement easier. This measurement can then be used to verify (or design) a stable input filter for the converter. (The details of filter design are beyond the scope of this article, but are well documented elsewhere.)

Negative Input Resistance: What Is It And Why Does It Matter?

At low frequency the dc-dc converter's input impedance is negative. That's important because negative resistance is the "heart" of an oscillator and we certainly don't want the dc-dc converter input oscillating. This would generate additional noise and could also cause EMI issues. Of course, this negative input resistance is not news. Albert Hull was writing about negative resistance for Dynatron oscillators in the 1920s, Nathan Sokal applied this to power supplies in the 1970s and R.D. Middlebrook made this concept popular with his extra element theorem shortly thereafter.

So, what exactly is a negative resistor? In a positive resistor, applying an increasing voltage results in an increasing current. In a negative resistor, applying an increasing voltage results in a *decreasing* current.

The input power to a switching regulator is relatively constant, thanks to the efficient power conversion. This can be represented as:

$$P_{input} = V_{input} \cdot I_{input}$$

Rearranging the equation to solve for input current results in:

$$I_{input} = \frac{P_{input}}{V_{input}}$$

You might already see that, with power held constant, increasing the input voltage results in a decreasing current, which is how we defined a negative resistance. But in case this is not obvious we can show it mathematically by assessing the partial derivative of the input current with respect to input voltage to obtain:

$$\frac{\partial I_{input}}{\partial V_{input}} = -\frac{P_{input}}{{V_{input}}^2} = \frac{1}{R_{input}}$$

Leading to the negative input resistance equation:

$$R_{input} = -\frac{V_{input}^2}{P_{input}}$$

Large-signal VRM models, such as the one that I frequently publish, and lecture about will properly represent this ac negative resistance.[4] This is another reason to include a full, large-signal VRM model in simulations.

As Albert Hull, Nathan Sokal, R.D. Middlebrook and many others since have published, this negative resistance can overcome the positive resistance of the input filter, creating an oscillator. For those interested, this is a heavily researched topic and the relevant literature can be founding using a few good IEEE search terms such as "impedance based stability," "minor loop gain" and "forbidden region".

Given the impact of negative input resistance on the design of the input filter, it is, therefore, critical to measure the input impedance of your converter.

Measuring Negative Resistance

Measuring the negative resistance of a dc-dc converter input requires introducing a perturbation of the dc-dc converter input voltage while measuring the input voltage and the input current. Using a frequency response analyzer (FRA), the modulated voltage divided by the resulting modulated current provides a direct measurement of input impedance.

The Picotest J2121A line injector simplifies this measurement by integrating a modulator and a hall current monitor into a single device.[5] For converter input voltages up to 400 V and operating currents up to 20 A this single injector provides everything required other than the voltage probe and the FRA. The typical setup is shown below in Fig. 1.

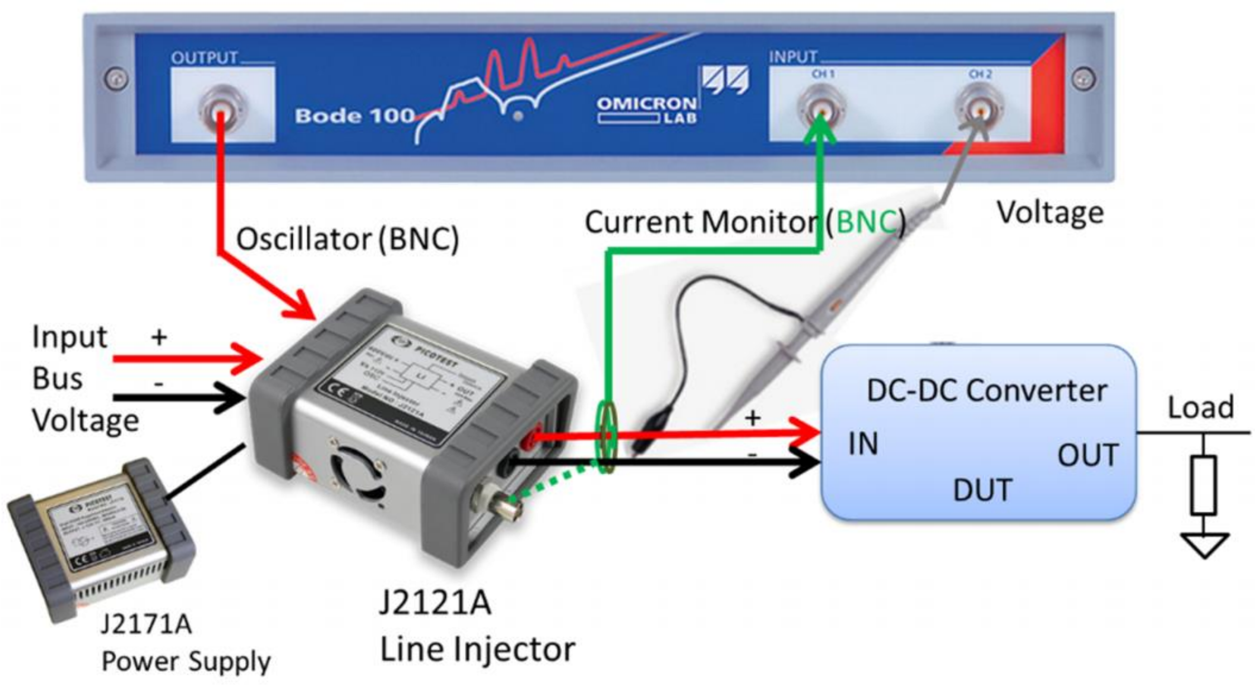

Fig. 1. Test setup for measuring the input impedance of a dc-dc converter using the J2121A line injector and the Bode 100 VNA.

The J2121A includes a THRU impedance calibrator, to correct for any probe and cable fluctuations. Once the THRU calibration is performed, the impedance measurement can be performed and displayed in any format supported by the FRA. Choosing the REAL format on the FRA conveniently includes the negative sign, confirming that the resistance is negative, as shown using the Bode 100 VNA in Fig. 2.

As predicted by the simple equation for negative input resistance ($R_{input} = V_{input}^2/P_{input}$), the resistance is related to input power. In this instrument, the measurement is repeated at several operating power levels and saved to individual memories so that they can all be displayed together.

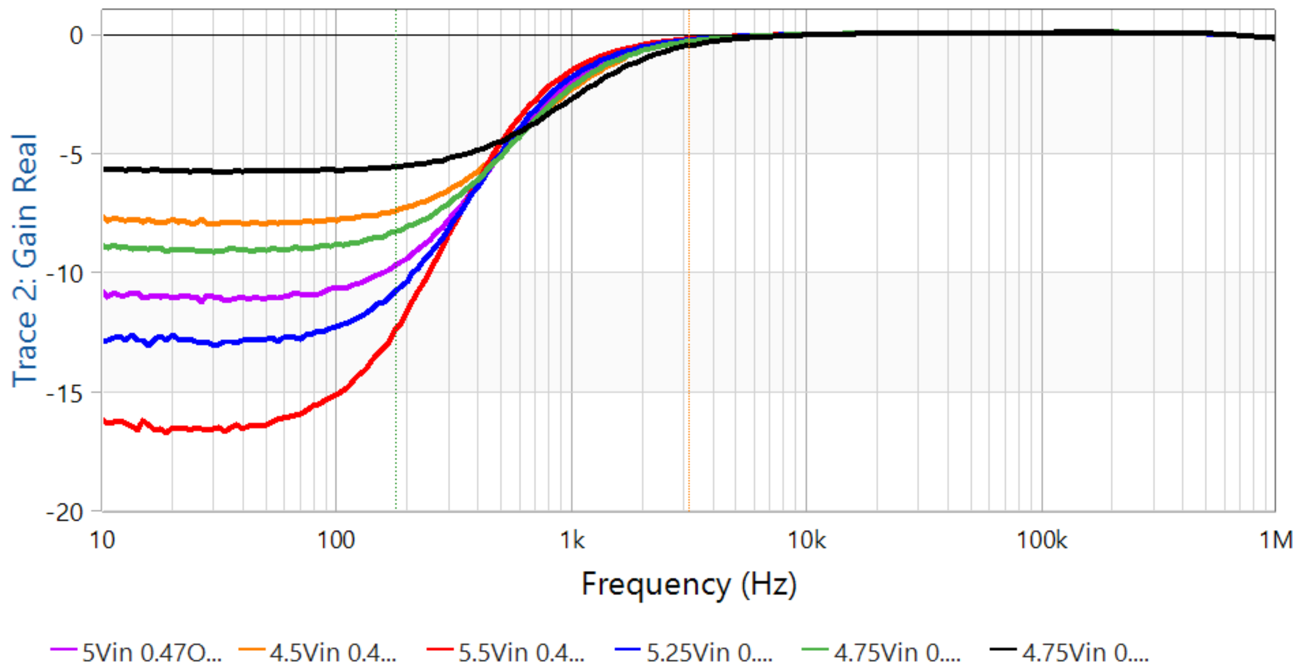

Fig .2. Measurement of dc-dc converter input impedance demonstrating negative input resistance. Results shown here are for a small dc-dc converter from VPT, configured with a 0.47-Ω load resistor for 1.2-V, 10-A output with input voltage varied from 4.5 V to 5.5 V (red, blue, purple, green and orange traces). An additional measurement (black trace) was taken with 4.75-V input and the load resistance reduced to 0.3 Ω, showing that the input resistance is both voltage and current dependent.

The measured impedance can be used directly, in a small-signal stability analysis, by exporting the data in either s-parameter or z-parameter format. Both formats are supported by most analog and system-level simulators, including Keysight ADS as shown in Fig. 3. The measured data can also be used for correlation with a large-signal model to verify the model accuracy.

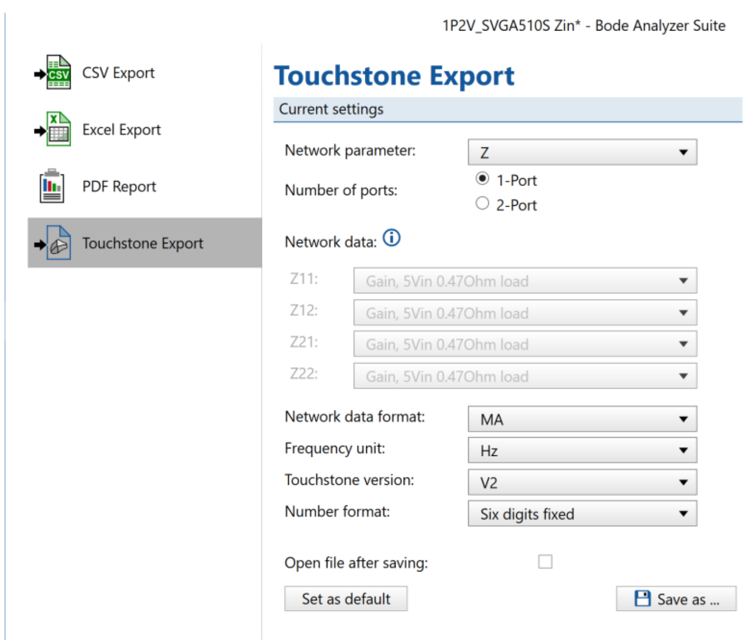

Fig. 3. The Keysight ADS simulator allows dc-dc converter input impedance measurements to be imported either as Z or S parameters.

The J2121A line injector is particularly suited to higher power measurements at input voltages up to 400 V and currents up to 20 A. For lower modulation and power levels, up to 50 V and 5 A, Picotest makes the J2120A line injector, which greatly simplifies PSRR and PSMR (power supply modulation ratio) testing. PSMR offers a measure of how noise on a power supply rail converts to phase noise or jitter in high-speed digital logic designs.

The measurement of dc-dc converter input impedance is an essential part of any system-level stability analysis and as I have hopefully shown here, is not difficult to perform.

References

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About The Author

Steven Sandler is the managing director of Picotest, a company specializing in precision test and measurement equipment. Sandler is also the founder and chief engineer of AEi Systems, where he leads development of high-fidelity simulation models for all types of simulators as well as the design and analysis of both power and RF systems.

Sandler has over 30 years of experience in engineering and is a recognized author, educator and entrepreneur in the areas of power, RF and instrumentation. His latest book, "Power Integrity: Measuring, Optimizing and Troubleshooting Power Related Parameters in Electronics Systems," was recently published by McGraw-Hill Education.

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