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Determining Maximum Usable Switching Frequency For Magnetics In CCM-Operated Converters

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Power converter size limitations require that magnetic components be made as small as possible while delivering the specified output power. The goal of this article is to develop design constraints for the optimum switching frequency, f_s, that produces maximum core power transfer within acceptable power loss for converters operating deep in continuous conduction mode (CCM.) These include buck-boost (common inductor), flyback and Cuk-derived circuits.

The basic equation for average power transfer through a linear transductor is

$$\overline{P} = \Delta W_L \cdot f_s = [(2 \cdot \hat{B}_{\sim}) \cdot \overline{H} \cdot V] \cdot f_s$$

where the magnetic field density ripple amplitude = $\hat{B}_{\sim}$ = $\Delta B/2$, the average (operating-point) field intensity = $\overline{H}$, core volume = V, and switching frequency = f_s. For linear magnetics, the per-cycle energy transfer through the core is ΔW_L.

$\hat{H}_{\sim}$ corresponds to input current ripple in the circuit and is kept constant by controlling the duty ratio. Then *incremental permeability*, μ at the operating-point $\overline{H}$, is

$$\mu(\overline{H}) = \frac{dB}{dH}$$

and $dB = d\mu$. Under the small-ripple assumption ($\Delta B << B$), which applies to converters operating deep into CCM, $dB \approx \Delta B$.

Per-cycle transfer occurs at a rate of f_s and output power increases proportional to frequency for constant $\hat{B}_{\sim}$. When $\hat{H}_{\sim}$ is held constant, then ΔW_L decreases with frequency because $\mu(f_s)$ decreases with f_s. With a constant H waveform, $\hat{B}_{\sim} = \mu(f_s) \cdot \hat{H}_{\sim}$ and also decreases with f_s. The operating-point current corresponds to $\overline{H}$, which is constant along with core volume.

The transfer-power equation can be regrouped into constant and frequency-dependent factors:

$$\overline{P} = \Delta W_L \cdot f_s = [2 \cdot \overline{H} \cdot V] \cdot [\hat{B}_{\sim}(f_s) \cdot f_s] = \text{constant} \cdot [\hat{B}_{\sim} \cdot f_s].$$

As f_s increases and $\hat{B}_{\sim}(f_s)$ decreases, constant $\overline{P}(f_s)$ is found by setting the derivative of the transfer power to zero and solving;

$$\frac{d\overline{P}(f_s)}{df_s} = \text{constant} \cdot \frac{d}{df_s}[\hat{B}_{\sim} \cdot f_s] = 0 \Rightarrow \frac{d}{df_s}[\hat{B}_{\sim} \cdot f_s] = 0 \text{ , constant} \neq 0.$$

Then differentiating, constant power occurs under the condition that

$$\hat{B}_{\sim} + f_s \cdot \frac{d\hat{B}_{\sim}}{df_s} = 0 \Rightarrow \frac{d\hat{B}_{\sim}}{\hat{B}_{\sim}} = -\frac{df_s}{f_s} \Rightarrow \frac{\dfrac{d\hat{B}_{\sim}}{\hat{B}_{\sim}}}{\dfrac{df_s}{f_s}} = -1 .$$

Integrate both sides of the above differential equation. The result is

$$\ln \hat{B}_{\sim} = -\ln f_s + C$$

where C is the arbitrary constant of integration. Choose the point (f_{s0}, $\hat{B}_{\sim 0}$) to determine C. Then

$$C = \ln \hat{B}_{\sim 0} + \ln f_{s0} .$$

Substituting and rearranging,

$$\ln \frac{\hat{B}_{\sim}}{\hat{B}_{\sim 0}} = -\ln \frac{f_s}{f_{s0}} = \ln \frac{f_{s0}}{f_s} \Rightarrow \frac{\hat{B}_{\sim}}{\hat{B}_{\sim 0}} = \frac{f_{s0}}{f_s} .$$

When this *constant-transfer-power* condition is substituted back into the transfer-power equation, $\overline{P} / \overline{P}_0 = 1$, the same value as at ($f_{s0}$, $\hat{B}_{\sim 0}$)—a constant value of 1.

Magnetic Power Loss Constraint

Core power-loss density also imposes a limit on f_s. The *generalized Steinmetz equation*, normalized to an operating-point at $\overline{p}_{c0}$ (f_0, B_0) (which eliminates a constant by using unitless ratios) is

$$\frac{\overline{p}_c}{\overline{p}_{c0}} = \left(\frac{f_s}{f_{s0}} \right)^{\alpha} \cdot \left(\frac{\hat{B}_{\sim}}{\hat{B}_{\sim 0}} \right)^{\beta}$$

where α and β depend on the material and are empirically determined. The "classical" values for them are $\alpha = 2$ and $\beta = 2$, but they vary with material. For typical ferrites, $\alpha \approx 1.25$ and $\beta \approx 2.5$, though both exponents vary with frequency. Micrometals 26 material has exponents $\alpha = 1.36$, $\beta = 2.03$. The equation as expressed around an operating-point (from values on its power-loss graph): $\overline{p}_c$ = 100 mW/cm^3, at $\hat{B}_{\sim}$ = 15 mT, f_s = 100 kHz, is

$$\frac{\overline{p}_c}{(100\,\mathrm{mW/cm^3})} = \left(\frac{f_s}{100\,\mathrm{kHz}} \right)^{1.36} \cdot \left(\frac{\hat{B}_{\sim}}{15\,\mathrm{mT}} \right)^{2.03} .$$

Constant power loss with f_s is derived by setting the differentiated Steinmetz equation to zero. The differentiated equation is

$$\frac{d}{df}\left(\frac{\overline{p}_c}{\overline{p}_{c0}}\right)=\frac{1}{f_{s0}^{\alpha}\cdot\hat{B}_{\sim0}^{\beta}}\cdot\left[\alpha\cdot f_s^{\alpha-1}\cdot\hat{B}_{\sim}^{\beta}+f_s^{\alpha}\cdot\beta\cdot\hat{B}_{\sim}^{\beta-1}\cdot\frac{d\hat{B}_{\sim}}{df_s}\right]=\frac{f_s^{\alpha-1}\cdot\hat{B}_{\sim}^{\beta-1}}{f_{s0}^{\alpha}\cdot\hat{B}_{\sim0}^{\beta}}\cdot\left[\alpha\cdot\hat{B}_{\sim}+f_s\cdot\beta\cdot\frac{d\hat{B}_{\sim}}{df_s}\right]=0\,.$$

Solving, the condition for constant loss is

$$\frac{d\hat{B}_{\sim}/\hat{B}_{\sim}}{df_s/f_s}=-\frac{\alpha}{\beta}\,.$$

For the classic values of $\alpha = 2$, $\beta = 2$, then

$$\frac{d\hat{B}_{\sim}/\hat{B}_{\sim}}{df_s/f_s}=-1\,.$$

Constant power loss occurs under the same condition as constant transfer power. Consequently, they are independent of f_s whenever $\alpha/\beta = 1$.

When the constant-loss equation is solved, the constraint on constant power loss is that

$$\frac{\hat{B}_{\sim}}{\hat{B}_{\sim0}}=\left(\frac{f_s}{f_{s0}}\right)^{-\alpha/\beta}\,.$$

When substituted into the Steinmetz equation, the result is a ratio of one.

The constant-power-loss constraint can be substituted into the transfer-power equation normalized to $\overline{P}_0(\hat{B}_{\sim0}, f_{s0})$:

$$\frac{\overline{P}}{\overline{P}_0}=\left(\frac{\hat{B}_{\sim}}{\hat{B}_{\sim0}}\right)\cdot\left(\frac{f_s}{f_{s0}}\right)=\left(\frac{f_s}{f_{s0}}\right)^{-\alpha/\beta}\cdot\left(\frac{f_s}{f_{s0}}\right)\,.$$

Then

$$\boxed{\frac{\overline{P}}{\overline{P}_0}=\left(\frac{f_s}{f_{s0}}\right)^{1-\frac{\alpha}{\beta}}}\,,\ \frac{\overline{p}_c}{\overline{p}_{c0}}\ \text{constant.}$$

This is the transfer power as a function of f_s with constant magnetic power loss. The closer α/β is to 1, the less dependent the transfer power is on frequency. For materials with $\alpha/\beta < 1$, transfer power rises with frequency with constant power loss.

Similarly, if the constant-transfer-power condition is substituted into the power-loss equation, then

$$\boxed{\frac{\bar{p}_c}{\bar{p}_{c0}} = \left(\frac{f_s}{f_{s0}}\right)^{\alpha-\beta}}, \quad \frac{\bar{P}}{\bar{P}_0} \text{ constant.}$$

For $\alpha < \beta$, the exponent is negative and power loss (along with $\hat{B}_{\sim}$) decreases with f_s under constant transfer power. For $\alpha = \beta$, power loss is independent of (and constant with) frequency.

Consequently, choice of core material is optimized whenever transfer power relative to power loss is maximized, and this occurs for a minimum α/β. Some core materials and their parameters are shown in the following table.

Table. Constants determining core power-loss density for various core materials.

Core Material	α	β	α/β
Fe-pwd Micrometals 26	1.36	2.03	0.68
Fe-pwd Micrometals 52	1.26	2.11	0.60
Fe-pwd Micrometals 18	1.18	2.27	0.52
FeSiAl (MagInc Kool-μ) 90μ, 125μ	1.29, 1.63	2.01, 2.2	0.64, 0.74
NiFeMo (MagInc MPP) 125μ, 550μ	1.40, 1.59	2.31, 2.36	0.61, 0.67
MnZn Ferrite MagInc P	[1.36, 3.47], [100, 500] kHz	[2.62, 2.54], [100, 500] kHz	[0.52, 1.37] α/β = 1 at 371 kHz
MnZn Ferrite MagInc K	[2.19, 4.13], [0.5, 1] MHz	[3.10, 2.98], [0.5, 1] MHz	α/β = 1 at 721 kHz

For ferrites, α and β vary significantly with f_s. The frequency at which f_s independence for both transfer power and power loss occurs is given in the table. At this frequency, the two power quantities are relatively constant with f_s, being at an extremum of $\alpha(f_s)/\beta(f_s)$. For $\alpha/\beta > 1$, power loss increases while transfer power decreases; thus, the frequency at which $\alpha/\beta = 1$ is a maximum usable frequency, f_{MAX}, for power transfer through the core.

Manufacturers use $\hat{B}_{\sim} \cdot f_s$ as a material performance parameter. Ferroxcube offers the curves shown below in the figure for MnZn (3-series) and NiZn (4-series) ferrites. The MnZn materials have an optimal f_s at maximum $\hat{B}_{\sim} \cdot f_s$—a maximum that results from an increasing $\alpha(f_s)/\beta(f_s)$. Ferrite 3C90 has a nearly linear increase with f_s, a result of near-constant $\alpha(f_s)/\beta(f_s)$.

Other categories of materials such as Fe-pwd, NiFeMo, and FeSiAl are specified with single values of α and β, presumably given as relatively constant with frequency. In this case, like 3C90 material and for $\alpha/\beta < 1$, power transfer increases with frequency and f_s is limited by circuit aspects other than magnetics.

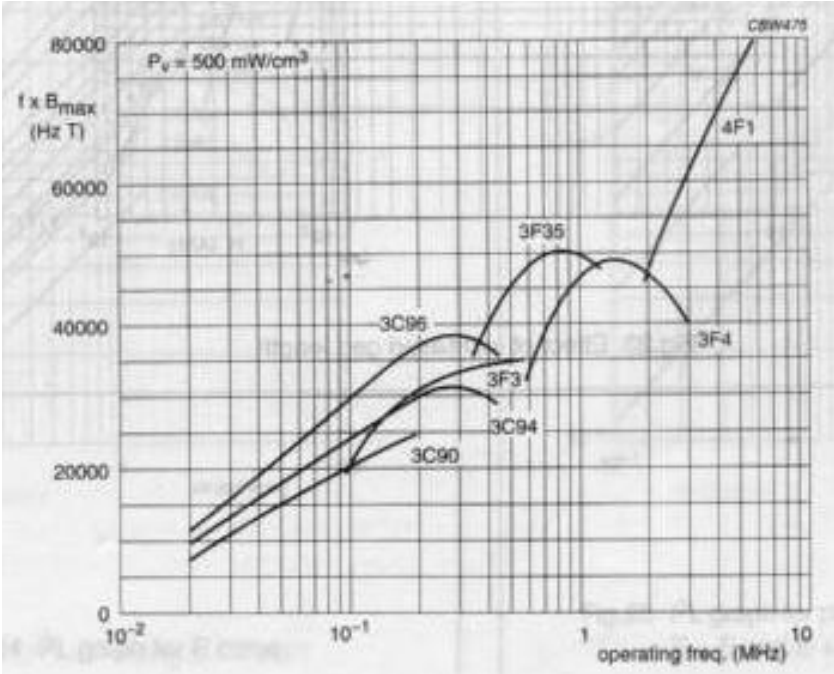

Figure. Frequency-dependent performance of Ferroxcube's MnZn (3-series) and NiZn (4-series) ferrites.

About The Author

Dennis Feucht has been involved in power electronics for 25 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been doing current-loop converter modeling and converter optimization.

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