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Subharmonic Behavior In Current-Mode Control (Part 2): The Causes And Cures At Low Duty Cycles

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Part 1 of this article series discussed the subharmonic behavior that occurs in power converters operating under current-mode control at high duty cycles and how this problem can be remedied with slope compensation. At high duty cycles, subharmonic behavior is not merely a possibility, but is expected unless we apply slope compensation. So we typically plan ahead for it. Including slope compensation is simply a necessary part of the design process whenever we're using current-mode control at high duty cycles.

On the other hand, at low duty cycles we often do not expect to see subharmonic behavior. So when it arises we may take the step of designing in slope compensation after the fact, which often resolves the problem, but not always. So now what do we do?

This second and final part of the article answers that question. The causes of subharmonic behavior at low duty cycles are explained along with techniques to counteract them. This discussion culminates in the description of circuits that generate a precise analog of the inductor current, which is then combined with slope compensation to eliminate the subharmonic behavior. Since this part 2 draws on the circuits and issues discussed in [part 1](#), it is suggested that the reader review that part before proceeding.

Typical Expected Behavior At Low Duty Cycle

At low duty cycles the nature of the subharmonic problem is different than the high duty cycle case. Fig. 1 illustrates a stable low duty cycle (20%) current analog wave form.



Fig. 1. A current analog waveform for 20% duty cycle operation.

At the low duty cycle of the Fig. 1 current analog, slope compensation is typically not needed. A current analog waveform illustrating the response of the 20% duty cycle system (without slope compensation) to a perturbation is illustrated in Fig. 2.



Fig. 2. The current analog of a 20% duty cycle system responding to a large initial perturbation.

In the Fig. 2 condition, there is a relatively large initial perturbation that fully recovers in just a few switching cycles without slope compensation. The response is even better if optimal slope compensation is used, as illustrated in Fig. 3.

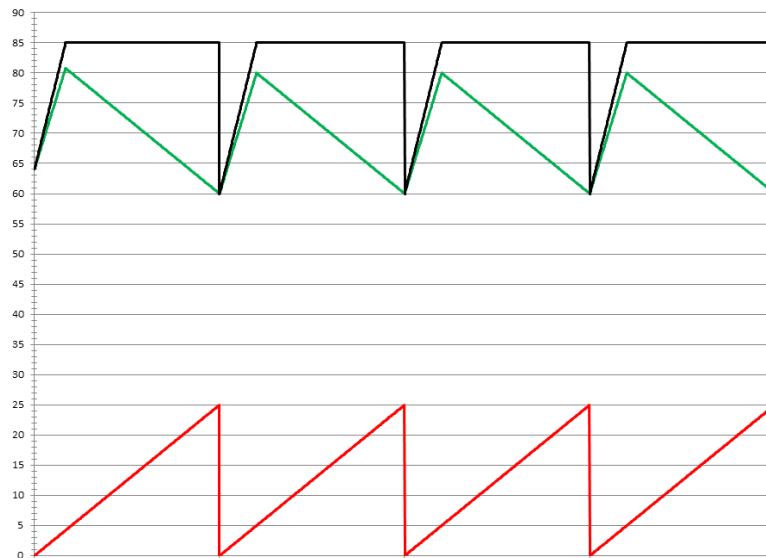


Fig. 3. A low duty cycle system with slope compensation responding to a large initial perturbation. The green line represents the inductor current analog. The red line is the slope compensation. The black line is the composite current analog signal consisting of the combination of slope compensation (red line) and inductor current analog (green line).

In the Fig. 3 system, the perturbation, with optimal slope compensation, is corrected in a single switching cycle. The composite signal combining slope compensation with inductor current analog forms the inverting input to the modulator comparator. Based on the theory and what the author has presented thus far, it would appear that subharmonic instabilities are a problem mainly for high duty cycle operation, but not for low duty cycles. But as we are about to see, that is not always the case.

Subharmonic Behavior At Duty Cycles Below 50%

In real world circuits, the current analog does not appear as a perfect triangle waveform as the current analog waveforms shown above. In real world systems there are propagation delays and distortion that result from filtering and other bandwidth limitations.

A more realistic picture of the current analog and composite waveforms, considering real world delays and bandwidth limitations, is illustrated in Fig. 4.

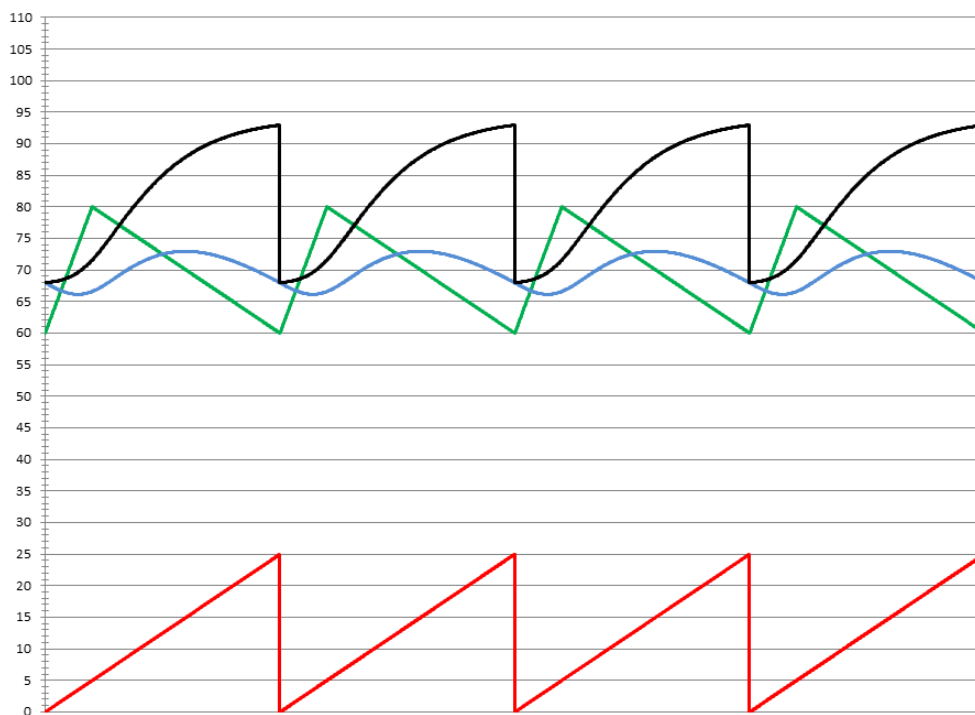


Fig. 4. Low duty cycle waveforms illustrating the effects of circuit delays and bandwidth limitations. The green line represents the actual inductor current in the ideal case. The blue line is a representation of what the inductor current analog may actually be considering real world delays and bandwidth limitations. The red line is slope compensation. The black line is the composite current analog signal, which is a combination of slope compensation (red line) and the delayed and bandwidth-limited inductor current analog (blue line).

The sensed current is often the main switch current. The sensing mechanism is usually a current-sense resistor in series with the MOSFET main switch. In this case, the sensed current signal is typically filtered to eliminate current spikes due to charging of gate-source capacitance in the MOSFET main switch during the turn-on instant. The filter used to remove these current spike results in a current-sense signal that is delayed and bandwidth limited.

Alternately, when the inductor current is sensed, usually some signal processing in the form of level shifting is required to create the current analog at the current-sense input to the control circuit. The signal processing of

the current-sense signal results in delays and bandwidth limitations similar to those caused by filtering the current-sense signal derived from a current-sense resistor used to sense current in the MOSFET main switch. The effect of the delay and bandwidth limitation is seen in the blue line of Fig. 4.

At the instant just prior to the turn-on of the main switch, the sensed current is decreasing, i.e., the slope of the inductor current is negative. A delayed and bandwidth-limited analog of the inductor current will continue to have a negative slope for a time interval after the turn-on of the main switch. This effect is seen in the blue line of Fig. 4.

If the duty cycle is small, the negative slope of the current analog signal can create a problem for the modulator. If we were to attempt to use the blue line as a ramp signal for the modulator without slope compensation we can see that we would have a problem because the value of the current analog signal (blue line) at the instant that we want to turn off the main switch is less than the magnitude of the current analog signal at the beginning of the switching cycle. If the switching threshold of the modulator is set for the value of the current analog signal at the desired switching instant then the main switch would not turn on.

Clearly, with the delay and bandwidth limitation, the modulator will not work right without slope compensation. What will happen is that the circuit will skip cycles resulting in subharmonic behavior. Depending on the magnitude of the delay and bandwidth limitation, we may see subharmonic behavior at low duty cycles even if we add a large amount of slope compensation.

We can describe the subharmonic problem in terms of the loop gain of the inner current loop. Loop gain depends on the inverse of the slope of the composite inductor current analog signal. For a larger slope, the current loop gain is relatively small and for a smaller slope the current loop gain is relatively large. When the slope of the current analog varies, the relevant slope is the slope of the composite current analog (black line) at the instant that the composite current analog crosses the modulator comparator threshold, which, for the Fig. 4 example, is about 71, the value of the black composite signal at the switching instant.

Because the slope varies one can see that the problem of too much current loop gain will be greatest for small duty cycles, where the slope of the current analog signal is small, due to the time delays and bandwidth limitations in the current analog. The effect of high loop gain is a high loop gain crossover frequency and increased likelihood that the loop phase will be greater than 360 degrees at the loop gain crossover frequency, which is the condition for oscillation.

Just prior to the end of the switching cycle, the current slope is negative. So, because of the delays and bandwidth limitations, the slope of the current analog at the beginning of the next cycle may be negative or near zero. The current loop gain relates to how much the inductor current changes for a given change in control voltage. The control voltage is the modulator threshold voltage, which is typically set by the outer voltage control loop. For an inductor current slope that is zero, the inner current loop gain is theoretically infinite. All of this suggests that without slope compensation the inner current loop gain is large and instability is likely.

But even with slope compensation, the slope of the composite signal with slope compensation (black line) is small at very low duty cycles. The gain of the inner current loop changes with duty cycle. The slope of the composite waveform that forms the inverting input to the modulator comparator changes dramatically over the switching cycle.

In a real world converter operating with a delayed and bandwidth-limited inductor current analog, one may see a stable subharmonic operating condition that is more stable than the condition illustrated in Fig. 4. The waveforms for a stable subharmonic operating condition are illustrated in Fig. 5.

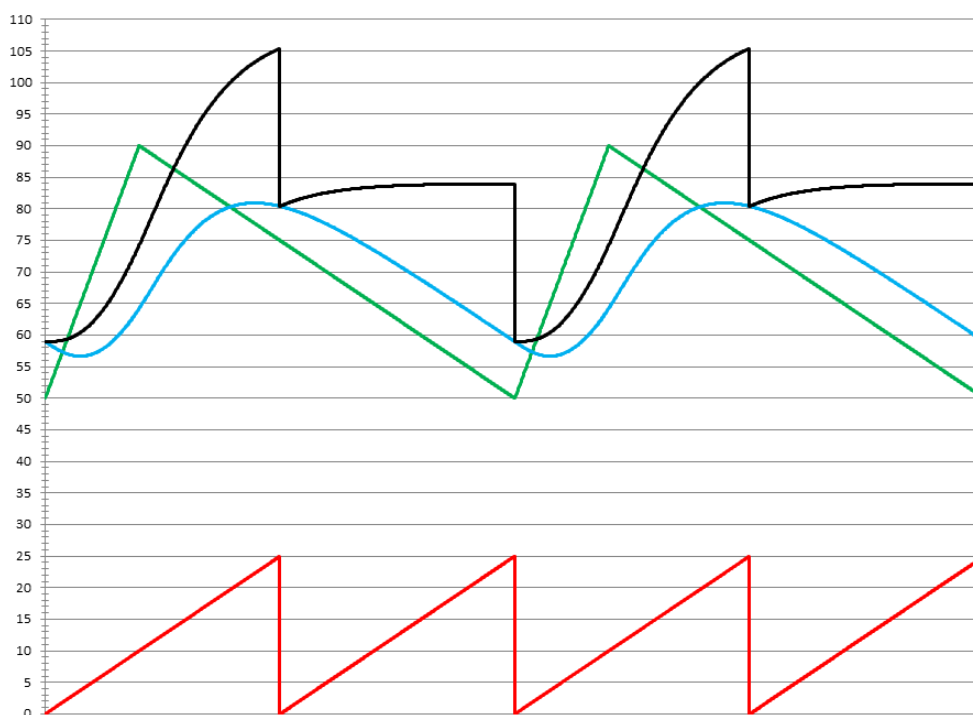


Fig. 5. Waveforms for a stable subharmonic operating condition at low (20%) duty cycle.

In the stable subharmonic operating condition illustrated in Fig. 5, we can see that the value of the composite (black) waveform at the instant that the turn-off transition of the main switch is initiated is about 75 units. At the beginning of the (intended) second switching cycle, the value of the composite signal is about 80 units. The composite signal is therefore higher than the turn-off threshold for the modulator comparator at the beginning of the (intended) second switching cycle. This causes the (intended) second switching cycle to be skipped. The high value signal at the inverting (current analog) input to the modulator comparator prevents the turn-on transition of the main switch at the beginning of the (intended) second switching cycle.

At the beginning of the (intended) third switching cycle, the value of the composite current analog signal is less than 60 units. The main switch is enabled at the beginning of the (intended) third switching cycle. The subharmonic waveforms repeat at half the intended switching frequency. Now we have two stable operating conditions. Which one will dominate? Typically, the lower-frequency subharmonic condition dominates.

A Better Cure For Low Duty Cycle Subharmonic Behavior

Are there solutions to this problem? The remedies that may work include: (1) increase the slope compensation, (2) reduce the delays and use components with higher bandwidth, and (3) sense the switch current instead of the inductor current. Some of the aforementioned solutions may not be practical.

Increasing slope compensation reduces current-loop gain and reduces some of the advantages of current-mode control such as line regulation. Additionally, increasing slope compensation changes the nature of the voltage loop affecting frequency compensation, since the output filter resonance will begin to reappear. Reducing the delays and using higher-bandwidth components may be cost and parts count prohibitive. Sensing switch current may also be cost and parts count prohibitive if the switch is a high-side switch. Another problem with sensing switch current is that you are also sensing gate-drive current, which may create additional problems.

However, there is a fourth solution that provides very good performance. This remedy is illustrated in Fig. 6.

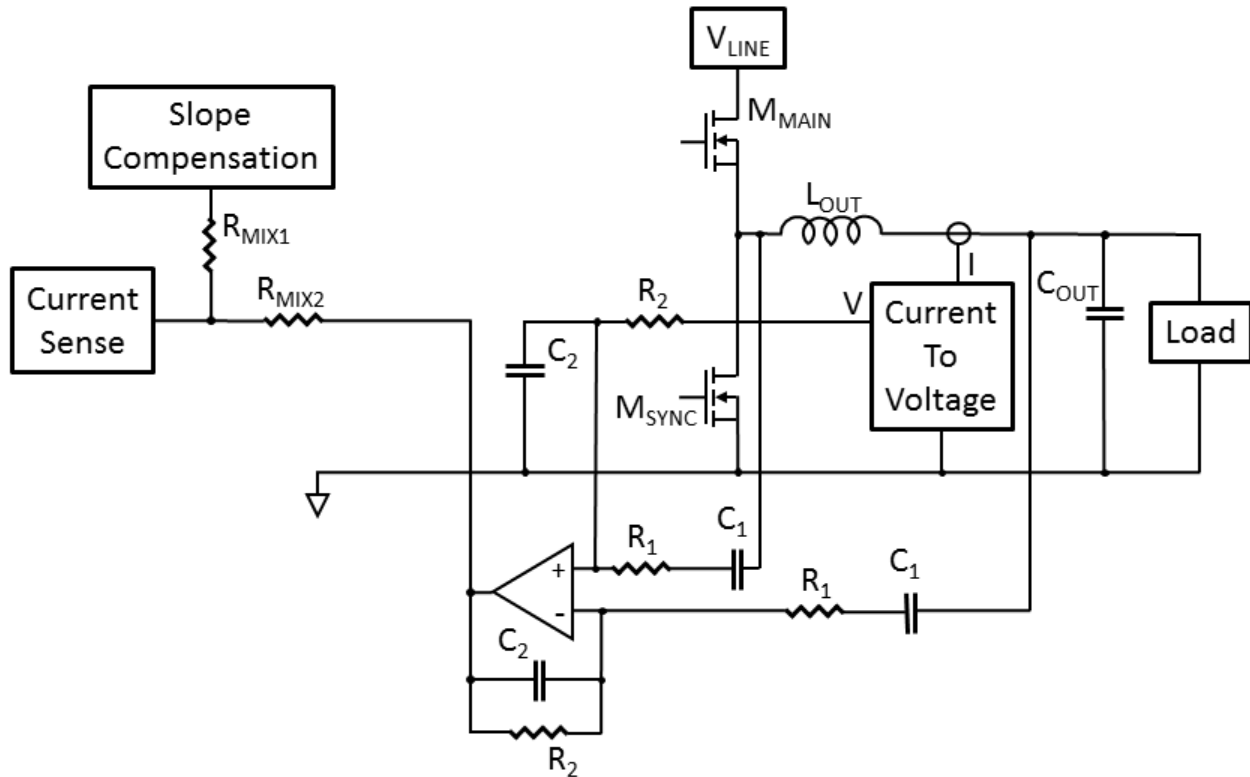


Fig. 6. Example of a composite inductor current analog formed by the combination of a low-frequency inductor current analog and a high-frequency inductor current analog.

In Fig. 6, an ac-coupled differential integrator, realized using an operational amplifier, integrates the ac inductor voltage. The integrated ac inductor voltage is a high-frequency replica or analog of the ac inductor current. This replica of the ac inductor current has very small, usually negligible, delay and bandwidth limitations given the use of a high-bandwidth operational amplifier. Meanwhile, a replica of the low-frequency and dc inductor current is formed by the current-to-voltage converter, which can be realized with an integrated circuit current-to-voltage converter or another operational amplifier circuit.

The low-frequency and high-frequency inductor current analogs are combined in the operational amplifier to form a complete and precise replica of the inductor current. The inductor current analog at the operational amplifier output is then combined with the slope compensation to form the inverting input to the modulator comparator.

A simpler and cheaper alternative to the Fig. 6 circuit with similar characteristics and performance is illustrated in Fig. 7. The Fig. 7 circuit also forms a temporally more precise analog of the inductor current. A slow, bandwidth-limited signal is formed by sensing the inductor current using a current-to-voltage converter. A high-bandwidth ac ramp signal is generated by forming a triangle wave by applying the voltage at the modulator output (node A) to an integrating capacitor, C_2 , at node B through resistor R_1 .

The signal from the current-to-voltage converter is passed through the low-pass filter formed by R_2 and C_2 , and then combined with the ac ramp at node B. The low-frequency and dc components of the modulator signal at node A are attenuated or eliminated, respectively, by C_1 . The waveforms associated with the Fig. 7 circuit are illustrated in Fig. 8.

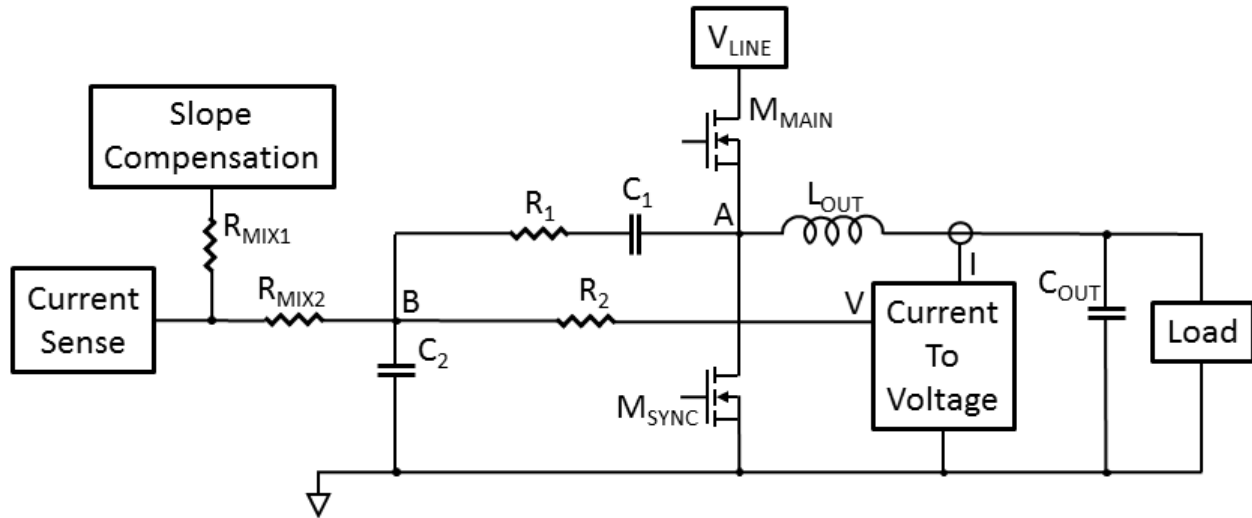


Fig. 7. A circuit employing a combination of slow inductor current feedback and fast modulator feedback.

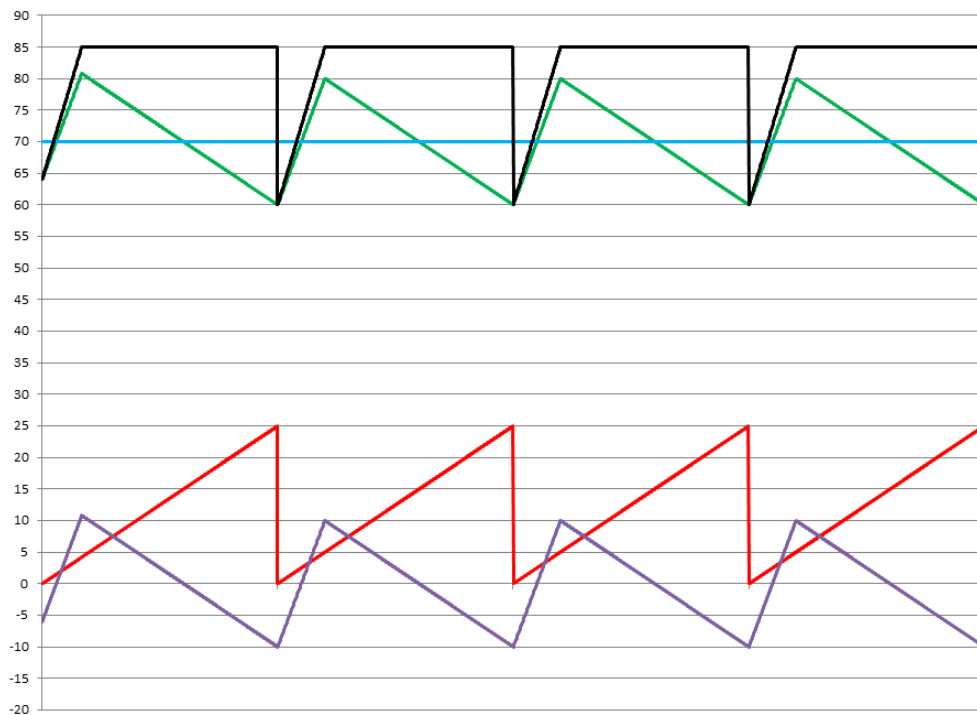


Fig. 8. Waveforms associated with the Fig. 7 circuit. The blue line represents the low-frequency portion of the inductor current analog (output of current-to-voltage converter) and the violet line represents the high-frequency ac portion of the inductor current analog ($R_1 C_1$ node, which is node B). The green line (node B) represents the combined low frequency (blue line) and high frequency (violet line) components of the inductor current analog. The black line is the sum of the slope compensation (red line) and low-frequency and high-frequency portions of the inductor current analogs (green line).

By generating an analog of the high-frequency portion of the inductor current, time delay and bandwidth limitations for the inductor current analog are eliminated. The Fig. 7 circuit behavior is similar to the ideal low duty cycle behavior illustrated in Fig. 3.

Conclusions

Subharmonic behavior in peak and average current-mode control is a common, but well known, problem for current-mode converters that operate at high duty cycles. The solution to this particular subharmonic behavior is also well known as this problem is usually eliminated by applying slope compensation. Slope compensation with a slope equal to the negative of the inductor current down slope provides optimal transient recovery for perturbations and noise.

Optimal slope compensation provides the same transient response benefits to low duty cycle systems. A less well known subharmonic problem relates to bandwidth limitations and delays in the inductor current analog that forms the inverting input to the modulator comparator. This subharmonic problem may be more difficult to deal with because the typical slope compensation remedy does not always eliminate the problem. More extreme measures are often needed to solve subharmonic issues at low duty cycle that result from delays and bandwidth limitations in the inductor current analog.

By providing a separate high-bandwidth ac inductor current analog that can be combined with a bandwidth-limited inductor current analog, the subharmonic behavior problems related to delays and bandwidth limitations can be eliminated. Though not yet commonly applied in current-mode-control solutions that are available off the shelf, the inclusion of a high-bandwidth current analog sensing mechanism is a feature that can be very easily implemented in controller ICs and given its effectiveness should be considered best practice.

About The Author



Ernie Wittenbreder is widely recognized in the industry from his seminars, publications, and many patents. His areas of expertise include topology selection, design tradeoff decision making, high-efficiency design, soft switching, gate drivers, and design for EMC. His applications experience spans a broad range including high reliability and extremely low-noise converters for military and aerospace, telecommunications, solid-state lighting, very high density power supply in package (PSIP) converters, and a wide variety of commercial and industrial applications.

Wittenbreder has developed a power converter computer simulation product and SiC gate driver IC. He also has a great deal of litigation experience in patent infringement lawsuits. Wittenbreder began his career as a physics professor and holds MS Physics and MSEE degrees and BS, math and physics. He can be reached via this [email](#).

For further reading on current-mode control, see the How2Power [Design Guide](#), locate the Design Area category and click on "Control Methods".