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Eddy-Current Effects In Magnetic Design (Part 4): Dowell's Formula

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This article series began in parts 1 and 2 with a discussion of the skin and proximity effects as they relate to transductors (transformers and coupled inductors). It continued in part 3, by converting ξ, the penetration or skin-depth ratio, to forms suitable for different wire shapes, so that real wire types can be analyzed using Dowell's equation. This allows us to graphically visualize the impact of eddy currents on wire resistance.

Here in part 4, we examine Dowell's equation to gain an understanding of how it illustrates the impact of skin effect and proximity effect at different frequencies by plotting wire resistance for different numbers of wire layers over frequency. This analysis will lead to some conclusions about where (in the frequency range) skin effect or proximity effect dominates, and how the physical winding of the transductor (spacing of turns and layers) affects the accuracy of Dowell's formula.

Dowell's Equation

In the reference work of Dowell, $\xi = h/\delta$ applies to a pair of parallel bars. A close geometric approximation is to a single layer of foil with thickness, h, for which $h <<$ the foil layer winding radius. Dowell's equation in its full form is an impedance equation, with resistive and inductive components in the complex domain at a given sinusoidal frequency, f. We are interested here in the resistive part.

The resistive component has two terms, expressed in R_w normalized to its static value, R_{w0}, as $F_R = R_w(f)/R_{w0}$. At $f = 0$ Hz, $\delta \rightarrow \infty$, $\xi = 0$, and $F_R = 1$. The first term is that of the skin effect, $F_{RS}(\xi)$.

$$F_{RS}(\xi) = \xi \cdot \frac{\sinh(2\cdot\xi) + \sin(2\cdot\xi)}{\cosh(2\cdot\xi) - \cos(2\cdot\xi)}$$

For M winding layers, the field-referred current increases geometrically with M because of the proximity effect, resulting in the second term, $F_{RP}(\xi,M)$.

$$F_{RP}(\xi,M) = \frac{M^2-1}{3}\cdot\left((2\cdot\xi)\cdot\frac{\sinh(\xi)-\sin(\xi)}{\cosh(\xi)+\cos(\xi)}\right)$$

The combined effect is additive.

$$F_R(\xi,M) = F_{RS}(\xi) + F_{RP}(\xi,M)$$

Plots of $F_R(\xi, M)$ are shown in Fig. 1 for layers, $M = 1, \ldots, 8$. The conversion factors relating square-wire ξ_h and round-wire ξ_r to ξ as given in the previous part 3 are used to calculate ξ which when substituted into F_R gives foil, square- or round-wire F_R values.

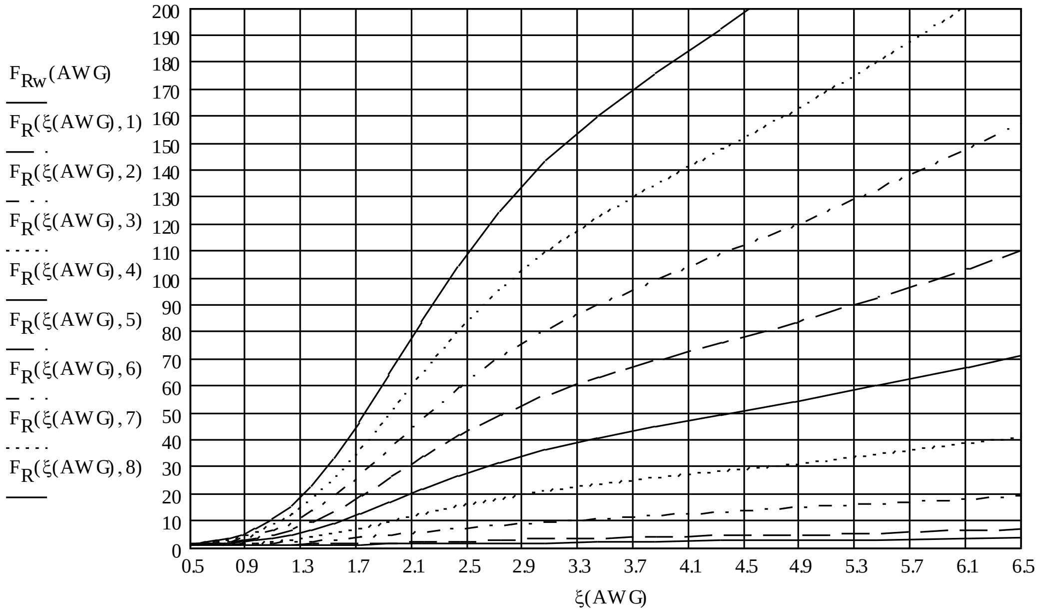

Fig. 1. Plots of Dowell's equation for constant wire size, r_c, and ξ variable with f, for 1 to 8 layers. The bottom plot is for reference—the isolated-wire skin-effect F_{rw}. The eight-layer plot is at the top. Below $\xi = 1$, F_R approaches 1.

Included for comparison in the plots is F_{Rw}, the skin effect of an isolated single wire. The isolated-wire skin effect of F_{Rw} (solid bottom curve) is weak in its effect relative to F_{RP}. F_{Rw} increases monotonically with ξ, and for $\xi >> 1$, the skin effect approaches the asymptotic slope of ½. $F_R(\xi, 1)$ (lowest dashed curve, second curve up from bottom) is the single-layer F_R. As M increases, so do the asymptotic slopes at high ξ.

F_R has three identifiable regions, described in detail by Kazimierczuk[4] Frequency affects F_R through δ, which is in ξ. ξ varies inversely with δ, and δ varies inversely with f; thus ξ varies directly with f (by $f^{1/2}$), and the horizontal axis of the $F_R(\xi)$ plot is also interpretable as a frequency axis.

The left part of the curves is the *low-ξ* or *low-frequency region*. Kazimierczuk defines this arbitrarily as the region where $F_R \leq F_{RB} = 1.05$. That is, a 5% rise in the curves indicates the onset of significant frequency effects. This region is of design interest because minimization of winding resistance, R_w, and hence winding power loss, P_w—a major winding design goal—can be achieved by operating windings in the low-frequency region.

This low-ξ region typically has $\xi < 1$ and is dominated by static resistance. An expanded view of it is shown on the graph of Fig. 2 for $\xi \leq 1$.

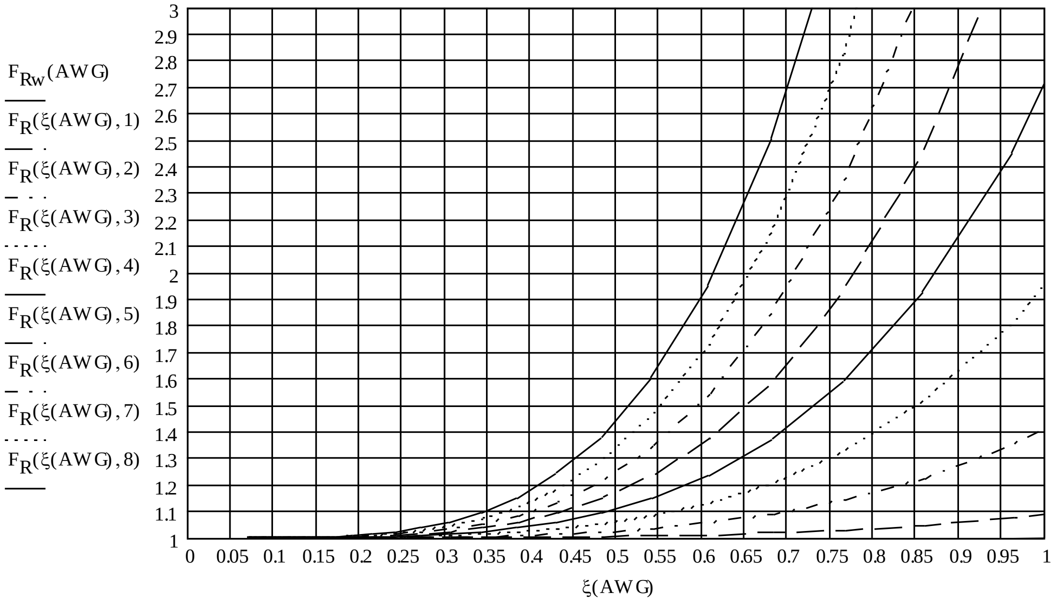

Fig. 2. A closer look at $F_R(\xi, M)$ near the origin. $F_R \approx 1$ only in the very-low-ξ region, where $\xi \leq 0.2$.

The *medium-ξ* or *medium-frequency region* is typically between $1 \leq \xi \leq 3$. In this region, the F_R curves rise quickly because of the proximity effect, which dominates this region. The low- and medium-frequency regions are shown on the Fig. 3 graph.

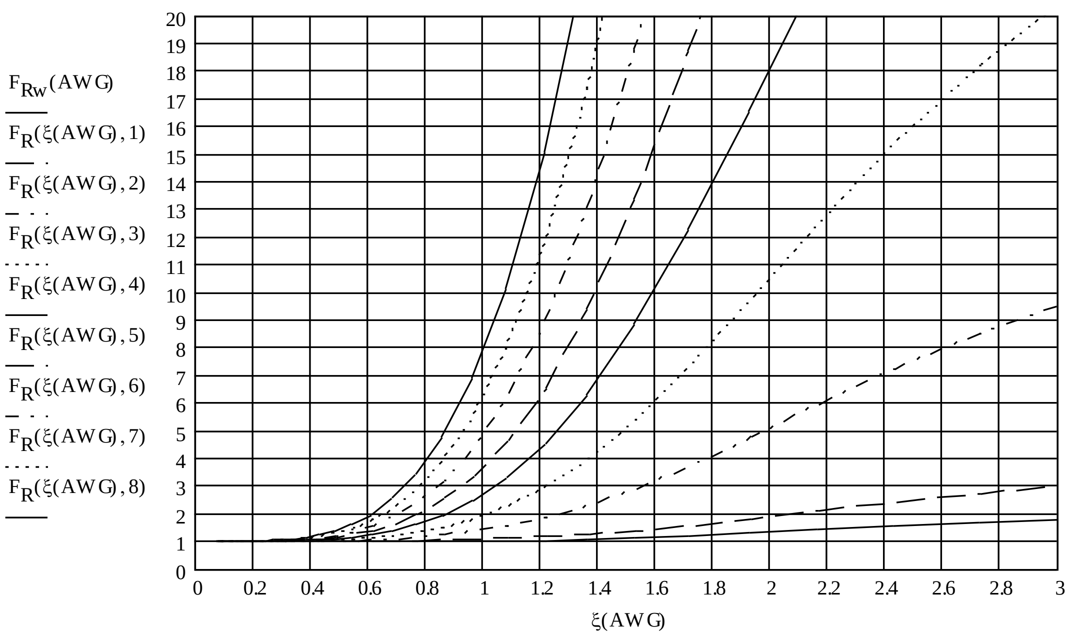

Fig. 3. Medium-ξ region shows rise in F_R. In this region the proximity effect dominates and the plots increase quickly with ξ until around $\xi = 3$ where their slopes reduce and the plots become asymptotic to lines of fixed slope.

At the onset of the *high-ξ* or *high-frequency region* ($\xi > 3$), the curves bend downward somewhat (slopes decrease) and become asymptotic, approaching a constant slope that increases parametrically in value with *M*. In this region, the increasing wire size reduces R_{w0}, which begins to dominate F_R and overcome the proximity effect by the brute dominance of a large conduction area at the large wire perimeter.

To summarize, the low-frequency and high-frequency regions are dominated by the skin effect and the medium-frequency region is dominated by the proximity effect.

Xi Nan and Charles R. Sullivan have refined F_R modeling.[5] They compare their formulation of F_R to the Dowell and modified Bessel function solutions, and compare all three to FEA simulations. (The magnetic-field solution for round wire results in modified Bessel functions.) For closely-spaced turns and layers and for $\xi < 5$, the Dowell formula error for F_R is about 5%. For $\xi > 5$, it can be in error by as much as 60% to 120%.

Good design minimizes wire spacings (porosity), which decreases the F_R error of the Dowell equation, and also keeps $\xi < 5$. Thus, for most power-magnetics design, the Dowell formula is the standard, though approximations that remove hyperbolic functions can be used to optimize winding design.

The topic of wire size optimization using the Dowell formula plots will be discussed next, in part 5.

References

1. "Eddy-Current Effects In Magnetic Design (Part 1): The Skin Effect," by Dennis Feucht, How2Power Today, August 2016 issue.

2. "Eddy-Current Effects In Magnetic Design (Part 2): The Proximity Effect," by Dennis Feucht, How2Power Today, September 2016 issue.

3. "Eddy-Current Effects In Magnetic Design (Part 3): Conductor Cross-Sectional Geometry," by Dennis Feucht, How2Power Today, October 2016 issue.

4. *High-Frequency Magnetic Components, Second Edition*, Marian K. Kazimierczuk, Wiley, 2014, 729 pages, hardback, $93.95 U.S. retail price; ISBN 978-1-118-71779-0.

5. "Simplified High-Accuracy Calculation of Eddy-Current Losses in Round-Wire Windings" by Xi Nan and Charles R. Sullivan, *IEEE PESC*, June 2004, pages 873-897.

About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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