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Understanding Op Amp Dynamic Response In A Type-2 Compensator (Part 1): The Open-Loop Gain

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A compensator is an electronic filter tuned to make a control system fast and stable during dynamic operation. In the vast majority of studies, the compensator is an active circuit built around an operational amplifier (op amp) whose characteristics are considered perfect. If this approach suffices in low-bandwidth systems, nowadays power converters cross over at or beyond 100 kHz to ensure a transient response that is fast enough to limit the output voltage drop in spite of a small output capacitive bank. In these applications, calculations considering a perfect op amp no longer work and induce severe gain and phase distortions in the end.

However, by accounting for the effects of the finite open-loop gain and the two low- and high-frequency poles of the selected op amp on the compensator's overall response, you can select the right op amp model without altering the gain and phase characteristics you need at crossover. Here in part 1 of this two-part article, we'll analyze the impact of the open-loop gain on the compensator's response, purposely ignoring the low- and highfrequency poles. Part 2 will explore the effect of the extra poles and show how they can potentially degrade the final result.

This part begins with a review of the three types of compensation circuits used in power supplies followed by an explanation of a circuit analysis method known as Fast Analytical Circuits Techniques (FACTs), which we'll use to determine the response of compensators. Here in part 1, we'll use FACTs to derive a transfer function for a type 2 compensator—one that accounts for the open-loop gain of the op amp. Finally, we'll compare results obtained with our derived type 2 compensator transfer function to that of the conventional transfer function, which assumes an ideal op amp.

Different Types Of Compensators

The role of a compensator is to shape the frequency response of a given circuit—a buck converter for example so that once the loop is closed, the control system exhibits the wanted crossover frequency f_c and adequate phase/gain margins. The compensator forces the 0-dB crossover point by providing some *mid-band* gain or attenuation at f_c . Phase margin φ_m is adjusted by the amount of *phase boost* the compensator exhibits also at f_c . Finally, the gain margin depends on the compensator's capability to roll-off the gain after crossover.

There are different types of compensators and those found in switching converters are commonly named type 1, type 2 and type 3. All three versions feature a pole at the origin to offer the maximum available quasi-static gain (s = 0) for a precise output variable.

A type-1 compensator is a simple integrator not providing phase boost at all. A type 2 builds on a type 1 and adds a pole/zero pair to offer a maximum phase boost of 90°. Finally, a type-3 circuit provides another pole-zero pair and can boost the phase up to 180°. Fig. 1 shows the frequency response (magnitude and phase) of the three compensators and their respective transfer function expressions. More information on these circuits can be found in reference 1.

The type-2 compensator is a popular implementation found in current-mode power supplies where the maximum phase boost of 90° offers plenty of compensation possibilities. Its implementation around an op amp appears in Fig. 2.

You can observe a resistive divider sensing the monitored variable (V_{out} , the output voltage in this example) and a few passive components forming the filter. To determine the transfer function of this converter, we will first consider the op amp open-loop gain A_{OL} and see how it affects the final expression. The transfer function G of this circuit is the mathematical relationship linking the excitation signal V_{out} to the output response V_{FB} .





Fig. 1. When designing a power supply compensator, you select the type of compensator according to the amount of phase boost you want.



Fig. 2. In this compensator, we consider the op amp with a finite open-loop gain but we do not account for its internal poles yet.

A Quick Introduction To Fast Analytical Techniques

Numerous methods exist to determine the dynamic response of this filter. In this article, we will use the Fast Analytical Circuits Techniques (FACTs) described in references 2 and 3. The basic principle behind these FACTs is to determine the circuit time constants under two different conditions: when the excitation signal disappears (V_{out} is reduced to 0 V) and when the response is nulled ($V_{FB} = 0$). Using this method, you will appreciate how quick and intuitive it is to determine a particular transfer function.

As shown in the references, the transfer function of a first-order system featuring a non-zero quasi-static gain can be expressed in the following form.



$$G(s) = G_0 \frac{N(s)}{D(s)}$$

(1)

The leading term G_0 is the gain exhibited by the system for s = 0. That term carries the unit if any. Here, because we talk about a gain in volts/volt (V/V), there is no unit and G is dimensionless. The numerator N(s) hosts the zeros of the transfer function. Mathematically, a zero is a particular point s_z for which the response is null.

Theoretically and considering an excitation signal covering the entire *s*-plane (and not the vertical axis only as in a harmonic mode), a zero manifests itself by the nulling of the output response when the input signal is tuned to the zero angular frequency s_z . Some particular impedance combination in the circuit blocks the signal propagation and the response is 0 V despite the presence of an excitation source. The zeros are numerator roots. Please note that this is a convenient mathematical abstraction that offers tremendous help in finding the zeros by inspection, without writing a line of algebra. More details on this approach can be found in reference 4.

The denominator D(s) is formed by the circuit's natural time constants. These time constants, $\tau = RC$ or $\tau = L/R$, are obtained by setting the excitation signal to zero and determining the resistance "seen" from the considered capacitor or inductor in this configuration. By "seeing," I mean you imagine placing an ohm-meter across the temporarily removed capacitor or inductor and read the resistance it displays.

This is a quite simple exercise actually. Look at the Fig. 3 passive circuit where you see an injection source—the stimulus—biasing the left-side of the network. The input signal propagates through meshes and nodes to form the response observed across resistor R_3 .



Fig. 3. Determining the time constant of a circuit requires setting the excitation to zero and looking at the resistance seen by the energy storage elements, which requires that they be temporarily removed from the circuit.

To determine the time constant of this example circuit, we will set the excitation to zero (a 0-V voltage source is replaced by a short circuit and a 0-A current source is open circuited) and remove the capacitor. Then, we connect (in our head) an ohm-meter to determine the resistance offered by the capacitor terminals. Fig. 4 guides you in these steps.





Fig. 4. After replacing the 0-V source by a short circuit, you determine the resistance seen from the capacitor terminals.

If you run the exercise on Fig. 4, you "see" r_c in series with R_4 in series with the parallel connection of R_1 , R_2 all paralleled with R_3 . The time constant of this circuit is simply the product of R and C_1 as shown in equation 2.

$$\tau_1 = \left[r_C + \left(R_4 + R_1 \| R_2 \right) \| R_3 \right] C_1$$
(2)

We can show that the pole of a first-order system is the inverse of its time constant.

$$\omega_{p} = \frac{1}{\tau_{1}} = \frac{1}{\left[r_{c} + \left(R_{4} + R_{1} \parallel R_{2}\right) \parallel R_{3}\right]C_{1}}$$
(3)

Now, what is the quasi-static gain of this circuit for s = 0? In dc conditions, a capacitor becomes an open circuit while an inductor becomes a short circuit. Apply this concept to the Fig. 3 circuit and redraw it as shown in Fig. 5. In your head, you cut the connection before R_4 and you see a resistive divider involving R_1 and R_2 . Thévenin voltage across R_2 is therefore as shown in equation 4.

$$V_{th} = V_{in} \frac{R_2}{R_1 + R_2}$$
(4)

The output resistance R_{th} is R_1 paralleled with R_2 . The complete transfer function thus involves the resistive divider made of R_4 in series with R_{th} and loaded by R_3 . Resistance r_c is out of the picture since capacitor C_1 is removed in this dc analysis. You can thus write the following expression.

$$G_0 = \frac{V_{out}}{V_{in}} = \frac{R_2}{R_2 + R_1} \frac{R_3}{R_4 + R_3 + R_1 \parallel R_2}$$
(5)





Fig. 5. To determine the static gain of the circuit in *Fig. 3,* open the capacitor in dc and calculate the transfer function of this simple resistive arrangement.

We are almost there but are missing the zero. How do we know if there is zero by the way? Well, here is a useful trick: consider the circuit of Fig. 3 and in your head, short capacitor C_1 . Now, assume you excite that circuit featuring the shorted capacitor. Would you be able to observe a response at V_{out} on an oscilloscope for instance? Certainly, r_c shorts R_3 and despite a probably low amplitude, the input signal can still propagate and there is a response.

If the answer to this exercise is "yes, there is still a response despite the short circuit of C_1 " then there is a zero associated with C_1 . If you deal with a circuit involving an inductor L_1 , then carry the same exercise but with the inductor open-circuited. If you still have response in this mode, then you have a zero involving L_1 .

We said in preamble that a zero manifests itself in a circuit by blocking the propagation of the excitation signal and it creates an output null. If we consider a transformed circuit—in which C_1 is replaced by $1/sC_1$ —as shown in Fig. 6, what particular condition would imply a nulled response when a stimulus biases the network? Having a nulled response simply mean that the current circulating in R_3 is 0. If there is no current in a resistance, there is no voltage across its terminals and V_{out} is 0 V. It is not a short circuit but rather a virtual ground.



Fig. 6. In this transformed circuit, when the series connection of r_c and C_1 becomes a transformed short circuit, the response disappears.

If we have no current in R_3 , then the series connection of r_c and $1/sC_1$ creates a transformed short circuit as defined in equation 6.



$$Z_1(s_z) = r_C + \frac{1}{s_z C_1} = 0$$
(6)

The root s_z is the zero location we want:

$$s_z = -\frac{1}{r_C C_1} \tag{7}$$

leading to the following zero.

$$\omega_z = \frac{1}{r_C C_1} \tag{8}$$

We can now assemble all these results to form the final transfer function characterizing the Fig. 3 circuit.

$$G(s) = \frac{R_2}{R_2 + R_1} \frac{R_3}{R_4 + R_3 + R_1 \parallel R_2} \frac{1 + sr_c C_1}{s \left[r_c + \left(R_4 + R_1 \parallel R_2 \right) \parallel R_3 \right] C_1} = G_0 \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}$$
(9)

This is what is called a *low-entropy* expression in which you can immediately distinguish a gain, a pole and a zero. A *high-entropy* expression would be that obtained by applying the brute-force approach to the original circuit when considering an impedance divider for instance, the expression in equation 10 is obtained.

$$G(s) = \frac{R_2}{R_2 + R_1} \frac{R_3 \| \left(r_C + \frac{1}{sC_1} \right)}{R_3 \| \left(r_C + \frac{1}{sC_1} \right) + R_4 + R_1 \| R_2}$$
(10)

Not only could you make mistakes in deriving the expression—I certainly would!—but formatting the result to something like equation 9 would require more energy. Also, please note that we did not write a single line of algebra when writing (9). If we later identify a mistake, then it is easy to come back to one of the individual drawings and fix it separately. The correction in equation 9 would then be easy. Now try to run the same correction in equation 10 and you will probably have to start again from scratch.

You check that expressions 9 and 10 are identical by plotting their frequency response in a Mathcad sheet as shown in Fig. 7.





Fig. 7. A quick check in Mathcad tells you if the expression you have derived using FACTs matches the response returned by the raw expression.

This quick introduction to FACTs is intended to show how pleasant and efficient it is to use them over simple and more complex circuits. By splitting a complicated architecture into simple separate circuits, you can quickly write a transfer function sometimes just by inspection, as we did. Now that we have introduced the tool, let's apply it to our type-2 compensator.

FACTs Applied To The Type-2 Compensator

To efficiently apply FACTs to the Fig. 2 circuit (which for convenience is shown again in Fig. 8) we start by counting the energy storage elements: C_1 and C_2 .





Fig. 8. The type-2 compensator has two storage elements, C_1 and C_2 .

Considering their independent state variables— C_1 and C_2 are not in series or in parallel for instance—this is a second-order system. Such a system can be expressed in the following form considering a non-zero quasi-static gain.

$$G(s) = G_0 \frac{1 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2}$$
(11)

For a second-order system, we can show that the denominator obeys the following formula.

$$D(s) = 1 + b_1 s + b_2 s^2 = 1 + s(\tau_1 + \tau_2) + s^2(\tau_2 \tau_1^2)$$
(12)

The coefficient for *s* is simply the sum of the time constants determined for a zeroed excitation. The coefficient for s^2 is slightly more complex as it introduced a new notation: τ_1^2 . This notation means that you look at the resistance "seen" from C_1 's terminals while C_2 is replaced by a short circuit. A bit mysterious at first sight but nothing insurmountable as we will see in a few lines.

Following the path adopted to solve the Fig. 3 circuit, we can study the system for s = 0. This is what is shown in Fig. 9. During the analysis, V_{ref} is a perfect source and its dynamic response is zero (its voltage is fixed regardless of the modulation we apply). As such, it naturally disappears from the small-signal circuit and takes the form of a short circuit in ac analysis.





Fig. 9. In dc, open all the capacitors: the op amp runs in an open-loop configuration.

The op amp delivers a voltage equal to ε times the open-loop gain A_{OL} . The voltage at the inverting pin involves the low-side resistance R_{lower} and ε is a non-zero value in this case.

$$G_0 = -\frac{R_{lower}}{R_{lower} + R_1} A_{OL}$$
(13)

We have two capacitors in this circuit hence two individual time constants. To determine the first time constant involving C_2 , we will set the excitation signal to zero and we determine the resistance seen from the C_2 connecting terminals while C_1 is removed from the circuit. The sketch appears in Fig. 10.



Fig. 10. The first time constant involves capacitor C₂: what resistance do you see between its terminals?

If inspection worked well in the previous example, the presence of a voltage-controlled source—the op amp forbids that simple approach in this case. To determine the resistance offered by C_2 's terminals, we can connect a test generator I_T and determine the voltage V_T at its terminals. V_T/I_T will then give us the resistance we want. The sketch involving the current source appears in Fig. 11.





Fig. 11. To determine the resistance seen between C₂'s *terminals, you install a test generator and measure the voltage across the terminals.*

The first simple equation you can write involves ε . The voltage between the op amp input pins is minus the voltage across the paralleled combination of R_1 and R_{lower} .

$$\varepsilon = -I_T \left(R_1 \parallel R_{lower} \right) \tag{14}$$

The op amp output scales ε by the open-loop gain A_{OL} as in (15).

$$V_{FB} = \varepsilon A_{OL} \tag{15}$$

Substituting equation 14 in (15) yields the following.

$$V_{FB} = -I_T \left(R_1 \parallel R_{lower} \right) A_{OL}$$
(16)

 V_T is the voltage across the current source. In its left-side terminal you have minus ε while the right-side is biased to V_{FB} as expressed in equation 17.

$$V_T = -\varepsilon - V_{FB} = I_T \left(R_1 \parallel R_{lower} \right) - V_{FB}$$
(17)

If we extract V_{FB} from (17), equate the result with (16), we have the expression shown below.

$$V_{T} = I_{T} \left(R_{1} || R_{lower} \right) \left(1 + A_{OL} \right)$$
(18)

Therefore, our resistance is simply:



$$R = \frac{V_T}{I_T} = \left(R_1 \parallel R_{lower}\right) \left(1 + A_{OL}\right)$$

which leads to an expression for the first time constant, τ_2 .

$$\tau_2 = C_2 \left[\left(R_1 \, \| \, R_{lower} \right) \left(1 + A_{OL} \right) \right] \tag{20}$$

The second time constant involving C_1 requires an updated schematic shown in Fig. 12. We did not install a current generator because the result is obvious: the resistance seen between C_1 's terminals is simply that already determined for C_2 with R_2 in series.



Fig. 12. The second time constant is immediately determined since it is the resistance driving C_2 with R_2 in series.

$$\tau_{1} = C_{1} \Big[\Big(R_{1} \parallel R_{lower} \Big) \Big(1 + A_{OL} \Big) + R_{2} \Big]$$
(21)

Now we have our two time constants and we can carry on with the second-order term. We say that we need to evaluate τ_1^2 in which C_2 is replaced by a short circuit while we look at the resistance seen from C_1 's terminals. Fig. 13 shows the new sketch. Since we have a short circuit in the mesh involving R_2 , then the resistance R is simply R_2 .

$$\tau_1^2 = R_2 C_1 \tag{22}$$

This is it, we have our denominator D(s) in equation 23 if we assemble our time constants according to equation 12.

$$D(s) = 1 + s \left(C_1 \left[\left(R_1 \parallel R_{lower} \right) \left(1 + A_{OL} \right) + R_2 \right] + C_2 \left[\left(R_1 \parallel R_{lower} \right) \left(1 + A_{OL} \right) \right] \right) + s^2 C_2 \left[\left(R_1 \parallel R_{lower} \right) \left(1 + A_{OL} \right) \right] R_2 C_1$$
(23)

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Page 11 of 16

(19)





Fig. 13. The high-frequency coefficient uses a mysterious notation but nothing complicated at the end: short C_2 and determine the resistance seen from C_1 's terminals.

This second-order form can be rearranged assuming a quality factor Q much less than 1. In this case, then both poles are well separated: one dominates at low frequency while the second is located in the upper section of the spectrum. We can show that from equation 12, the two poles are defined as follows.

$$\omega_{p_1} = \frac{1}{b_1}$$
(24)
$$\omega_{p_2} = \frac{b_1}{b_2}$$
(25)

If we apply these definitions to (23), simplify and re-arrange, we obtain the next two expressions.

$$\omega_{p_1} = \frac{1}{\left(1 + A_{OL}\right) \left[\left(R_{lower} \parallel R_1\right) \left(C_1 + C_2\right) + \frac{R_2 C_1}{1 + A_{OL}} \right]}$$
(26)

$$\omega_{p_2} = \frac{\left(R_1 \parallel R_{lower}\right)\left(C_1 + C_2\right) + \frac{R_2 C_1}{1 + A_{OL}}}{C_1 C_2 R_2 \left(R_1 \parallel R_{lower}\right)}$$
(27)

Now that we have the denominator, do we have zeros in this circuit? We can apply the trick unveiled a few lines before: if we short in our head, C_1 or C_2 then C_1 and C_2 , do we have a response in these three configurations? If C_1 is shorted, we have a simple inverter involving R_2 and the other resistors: there is a zero associated with C_1 . If we short C_2 , then the op amp delivers zero: no zero with C_2 . And if both capacitors are shorted, of course, no response.

To determine the zero position, what in Fig. 14 could prevent the propagation of the stimulus and make the response a null? If the impedance provided by C_1 and R_2 becomes a transformed short circuit, then the response disappears.



$$R_2 + \frac{1}{sC_1} = 0 \tag{28}$$

Rearranging the above expression we obtain the following.

$$s_z = -\frac{1}{R_2 C_1}$$
 (29)



Fig. 14. If the impedance consisting of R_2 in series with C_1 becomes a transformed short circuit, then the response is a null: this is how the zero is created.

Equation 29 leads to the zero location below.

$$\omega_z = \frac{1}{R_2 C_1} \tag{30}$$

We now have our final transfer function:

$$G(s) = G_0 \frac{1 + \frac{s}{\omega_z}}{\left(1 + \frac{s}{\omega_{p_1}}\right) \left(1 + \frac{s}{\omega_{p_2}}\right)}$$
(31)

where G_0 , and the pole and zero locations are defined as shown in equations 32-35.

$$G_0 = -\frac{R_{lower}}{R_{lower} + R_1} A_{OL}$$
(32)

(34)



$$\omega_z = \frac{1}{R_2 C_1} \tag{33}$$

$$\omega_{p_{1}} = \frac{1}{\left(1 + A_{OL}\right) \left[\left(R_{lower} \parallel R_{1}\right) \left(C_{1} + C_{2}\right) + \frac{R_{2}C_{1}}{1 + A_{OL}} \right]}$$

$$\omega_{p_2} = \frac{\left(R_1 \parallel R_{lower}\right)\left(C_1 + C_2\right) + \frac{R_2 C_1}{1 + A_{OL}}}{C_1 C_2 R_2 \left(R_1 \parallel R_{lower}\right)}$$
(35)

Comparing The Response Between Circuits

It is now interesting to compare the dynamic response of a type-2 circuit in which we account for the open-loop gain with the response of the perfect transfer function of the type-2 compensator given below:^[1]

$$G(s) = G_0 \frac{1 + \frac{\omega_z}{s}}{1 + \frac{s}{\omega_p}}$$
(36)

where $G_{0_{r}} \omega_{Z}$ and ω_{p} are as follows.

$$G_0 = -\frac{R_2}{R_1} \frac{C_1}{C_1 + C_2}$$
(37)

$$\omega_z = \frac{1}{R_2 C_1} \tag{38}$$

$$\omega_{p} = \frac{1}{R_{2} \frac{C_{1}C_{2}}{C_{1} + C_{2}}}$$
(39)

For the sake of illustration, we compare an ideal op amp to a real op amp with 50-dB open-loop gain (a TL431 for instance) when the compensator must meet the following targets: $f_c = 10$ kHz with a 20-dB compensating gain at this frequency and the phase boost must be 65°. R_1 and R_{lower} are calculated for a 12-V output and a 2.5-V reference voltage. The two dynamic responses of equations 31 and 36 appear in Fig. 15.

The deviation of the crossover gain and phase boost are negligible. However, the gain in equation 31 is 35 dB at a 120-Hz frequency while it amounts to 45 dB with (36). Finally, the quasi-static gain is only 36.4 dB (\approx 66) for the finite- A_{OL} option while it is infinite with the perfect op amp.

What is the impact of these numbers? A lack of gain at twice the mains frequency will affect the ability of the control system to reject the rectification ripple. The output variable may be polluted by this component, especially in voltage-mode control. Also, there can be a significant static error in the controlled variable if the plant gain is low as well. But if you now select an op amp having a higher *A*_{OL}, 80 dB for instance, the discrepancies disappear and both curves are very close to each other as shown in Fig. 16.







Fig. 15. The Bode plot of the type-2 compensator in which we consider the influence of the openloop gain A_{OL} and the low-side resistor R_{lower} . The response of the compensator that accounts for finite A_{OL} does not differ too much from the response produced by the original perfect equation (the curve labeled "infinite A_{OL} ") except at low frequencies.



Fig. 16. When the open-loop gain A_{OL} increases, both curves nicely superimpose. The quasi-static gain increases to 66.3 dB versus 36 dB with the 50-dB A_{OL} gain.

Conclusion

This first part shows the effect of the open-loop gain in a compensator featuring a non-ideal operational amplifier. When the op amp is no longer considered perfect, you can see the effects of a weak open-loop gain in the low-frequency range of the dynamic response and assess performance degradation brought by this condition.

In this first part, we have only considered the open-loop gain impact. In part 2, we will complicate the analysis by adding the two low- and hi-frequency poles that integrated circuit designers naturally place in an operational amplifier to ensure its stability.

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About The Author



Christophe Basso is a technical fellow at ON Semiconductor in Toulouse, France. He has originated numerous integrated circuits among which the NCP120X series has set new standards for low standby power converters. SPICE simulation is also one of his favorite subjects and he has authored two books on the subject. Christophe's latest work is "Linear Circuit Transfer Functions: An Introduction to Fast Analytical Techniques."

Christophe received a BSEE-equivalent from the Montpellier University, France and an MSEE from the Institut National Polytechnique de Toulouse, France. He holds 18 patents on power conversion and often publishes papers in conferences and trade magazines.

For further reading on power supply compensation, see the How2Power <u>Design Guide</u>, and do a keyword search on "compensation."