Eddy-Current Effects In Magnetic Design (Part 5): Winding Design Optimization

by Dennis Feucht, Innovatia Laboratories, Cayo, Belize

In Part 4, Dowell's equation was presented. It provides a graphic way of determining winding loss for a given wire size and number of layers. Having that capability, we now progress to the problem of how to optimize wire size for minimum winding resistance.

Given the two winding design parameters, ξ and M (with frequency, f given), the winding design goal is not to minimize F_R in itself but to minimize winding loss,

$$\overline{P}_w(\xi) = R_w(\xi, M) \cdot \tilde{i}^2 = F_R(\xi, M) \cdot R_{w0}(\xi) \cdot \tilde{i}^2$$

To achieve this goal, we will transition from F_R to F_r, which is proportional to $\overline{P}_w(\xi)$. Whereas F_R is the resistance ratio with constant wire size and varying frequency, F_r instead has constant frequency with varying wire size. We can thereby find the optimal wire size using F_r.

Ultimately, we will find there are two possible solutions for minimizing eddy-current effects —a low-ξ *solution* involving a smaller wire size and a high-ξ solution involving a larger wire size. Each solution serves to minimize the contribution of eddy-current effects to winding losses, but each will be applicable under different winding requirements and operating conditions. However, we'll begin by analyzing the other resistive component of winding loss, R_{w0}.

The Path To Lowest Winding Loss

P_w in the equation above is minimized by minimizing both $F_R(\xi, M)$ and static (0 Hz) resistance, R_{w0}, though these reductions conflict. As square or round wire size increases, ξ_h or ξ_r increases, and F_R increases, whereas R_{w0} decreases with larger wire. R_{w0} is expressed for foil in ξ as

$$R_{w0}(\xi) = \frac{\rho \cdot l_w}{A_c} = \frac{\rho \cdot l_w}{w \cdot h} = \frac{\rho \cdot l_w}{w \cdot \delta \cdot (h/\delta)} = \frac{R_{\delta f}}{\xi} \ , \ R_{\delta f} = \frac{\rho \cdot l_w}{w \cdot \delta}$$

R_{w0} is expressed for square wire in ξ_h as

$$R_{w0}(\xi_h) = \frac{\rho \cdot l_w}{A_c} = \frac{\rho \cdot l_w}{h^2} = \frac{\rho \cdot l_w}{\delta^2 \cdot \left(\frac{h}{\delta}\right)^2} = \frac{R_{\delta h}}{\xi_h^2} \ , \ R_{\delta h} = \frac{\rho \cdot l_w}{\delta^2}$$

R_{w0} is expressed for round wire in ξ_r as

$$R_{w0}(\xi_r) = \frac{\rho \cdot l_w}{A_c} = \frac{\rho \cdot l_w}{\pi \cdot r_c^2} = \frac{\rho \cdot l_w}{\pi \cdot \delta^2 \cdot \left(\frac{r_c}{\delta}\right)^2} = \frac{R_{\delta r}}{\xi_r^2} \ , \ R_{\delta r} = \frac{\rho \cdot l_w}{\pi \cdot \delta^2}$$

$R_{\delta x}$ is the resistance of a conductor of cross-sectional area set by the skin depth. $R_{\delta x}$ and ξ_x vary the same with f (in δ), making R_{w0} independent of f and static (hence the 0 subscript.) At a fixed f, $R_{\delta x}$ is constant and ξ_x is only a function of the conductor sizing dimension (h or r_c). $R_{\delta x}$ is thus independent of ξ_x at a fixed f.

For foil or wide circuit-board power traces ($w >> h$), w is independent of δ, and only the h dimension affects δ. Foil R_{w0} varies only with $1/\xi$. For square and round wire, both of the wire cross-sectional dimensions vary with the skin-depth dimension, resulting in R_{w0} that varies by $1/\xi_x^2$.

To find foil thickness for minimum R_w, it is expressed for foil (or single, wide rectangular conductors such as circuit-board traces) by substituting the above expression for R_{w0} into

$$R_w(\xi) = F_R(\xi)\cdot\frac{R_{\delta f}}{\xi} = \frac{F_R(\xi)}{\xi}\cdot R_{\delta f} = F_r(\xi)\cdot R_{\delta f} \Rightarrow F_r(\xi) = \frac{F_R(\xi)}{\xi}\text{, foil}$$

For square and round wire,

$$R_w(\xi) = F_R(\xi)\cdot R_{w0} = F_R(\xi)\cdot\frac{R_{\delta x}}{\xi_x^2} = \frac{F_R(\xi)}{\xi_x^2}\cdot R_{\delta x} = F_r(\xi_x)\cdot R_{\delta x}$$

where ξ_x is either ξ_h or ξ_r, $R_{\delta x}$ is either $R_{\delta h}$ or $R_{\delta r}$ and where F_r is R_w normalized to $R_{\delta x}$, a constant that varies with wire size;

$$F_r(\xi_x) = \frac{F_R(\xi)}{\xi_x^2}\text{, x = r for round wire, x = h for square wire}$$

We can find the conductor size corresponding to a minimum R_w because ξ_x varies only with conductor size at a fixed f. Where $F_r(\xi_x)$ is minimum, so is $R_w(\xi)$. Layer plots of F_r (ξ_r, M) are shown below in Fig. 1 for round wire.

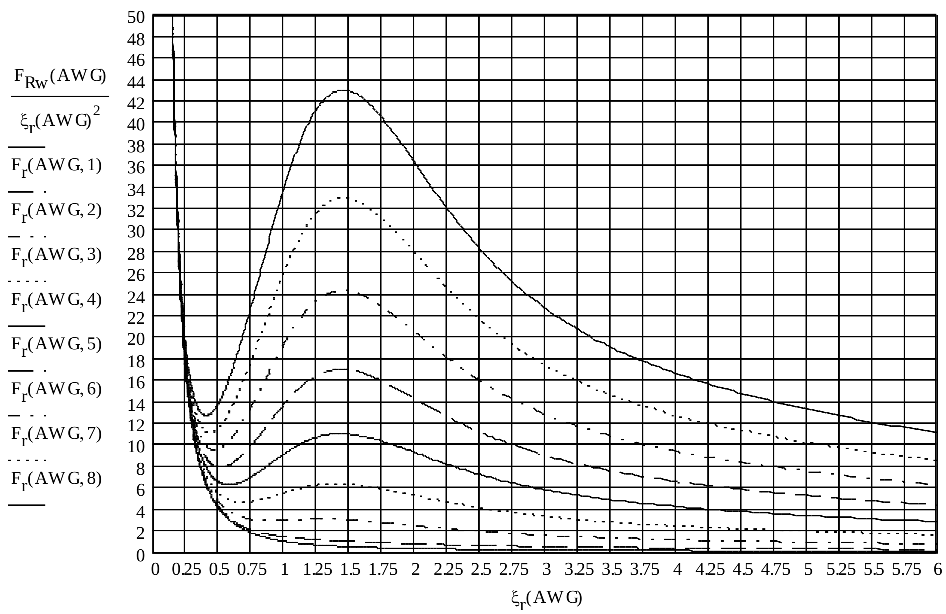

Fig.1. Constant-frequency resistance factor, F_r , plotted against round-wire ξ_r with layers, $M = 1$ to 8 (top plot) and isolated-wire F_{rw} (bottom plot) for reference. In the low-ξ_r region, F_r asymptotically decreases by $1/\xi_r^2$, reaches a minimum at $F_{rv}(\xi_{rv})$, increases in the medium-ξ_r region, and decreases again in the high-ξ_r region.

The same graph as Fig. 1 with logarithmic scales is shown in Fig. 2.

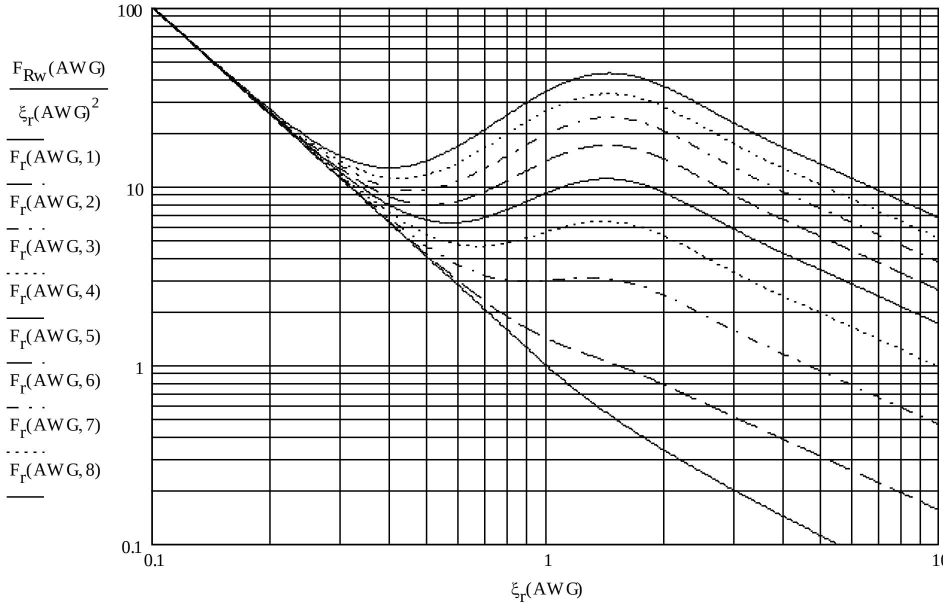

Fig. 2. Here the graph from Fig. 1 is plotted with log-log scales to show asymptotic slopes of plots: –2 for low-ξ_r *; –1 for high-*ξ_r *; and in the medium-*ξ_r *region, the slope increases by about* $(M/3)^2$*. The plot of* $M = 8$ *has twice the slope of* $F_r(\xi_r, 4)$*.* ξ_{rv} *sets the lower end of the medium-*ξ_r *region and decreases with* M *while the upper end is at* $\xi_r \approx 1.5$ *for all layers.*

F_r is asymptotic with a line of –2 slope for low ξ_r and at high ξ_r is asymptotic with a –1 slope. The minima of F_r, or F_{ropt}, also lie along a line with a –2 slope. The round-wire plot of Fig. 3 is expanded along the ξ_r axis for the low- and medium-frequency regions.

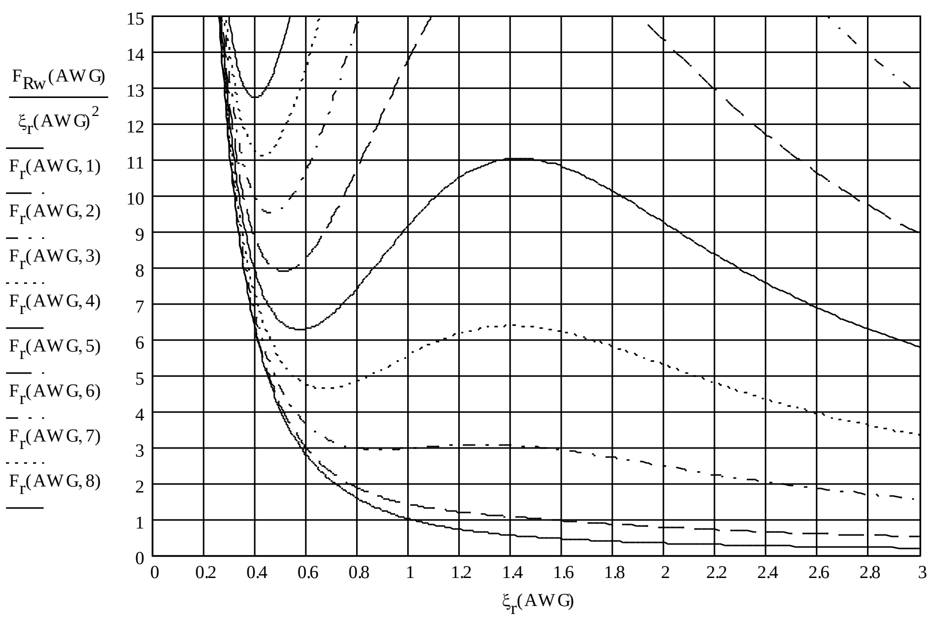

Fig. 3. Round-wire F_r *and* F_{rw} *plots, expanded along the* ξ_r *axis.*

Small wire has a lower fraction of conductive area (lower k_{pw}), and higher R_{w0}, despite low eddy-current effects. As wire size (and ξ_x) increases, R_{w0} decreases to a minimum R_w valley point at ξ_v before F_r begins to increase in the medium-ξ_r region. F_r increases to a peak value, the peak being greater with larger M. At an even larger ξ_r (and wire size), the decrease in R_{w0} dominates, and F_r decreases until at the *critical* value, $\xi = \xi_{cr}$, $F_r(\xi_{cr}) = F_r(\xi_v)$.

Above the ξ_{cr} value, R_w is reduced even further, though at the expense of a larger required winding window area, A_{ww} (and perhaps a larger core) to accommodate the larger wire size. Round-wire F_r graphs are plotted against wire gage in Figs. 4 through 8. (Additional plots at other frequencies are given in *Power Magnetics Design Optimization*, D. Feucht, www.innovatia.com.)

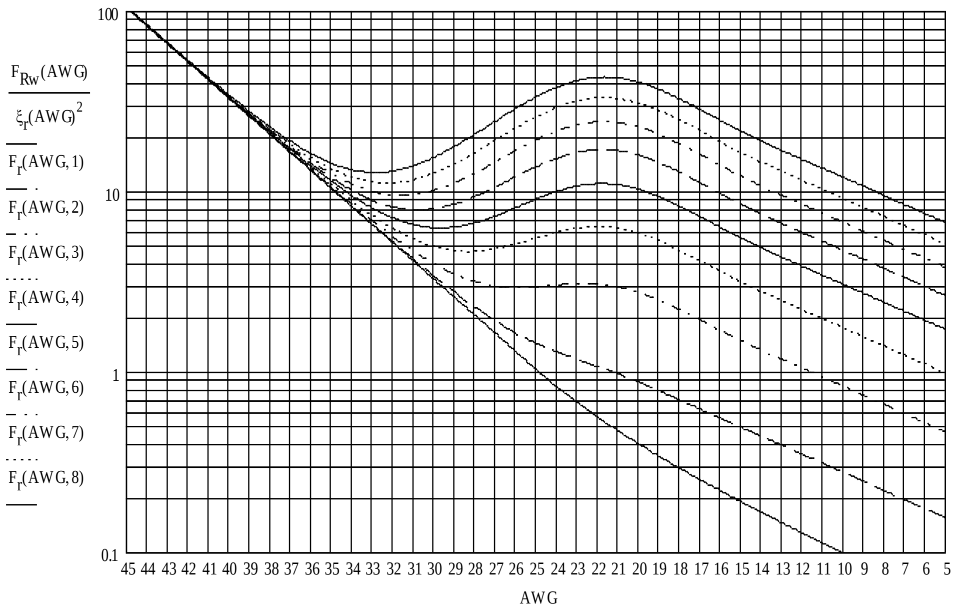

Fig. 4. Round-wire F_r and F_{rw} plots, expanded along the ξ_r axis; f = 100 kHz, $R_{\delta r}/l_w$ = 125 mΩ/m, Cu, 80°C.

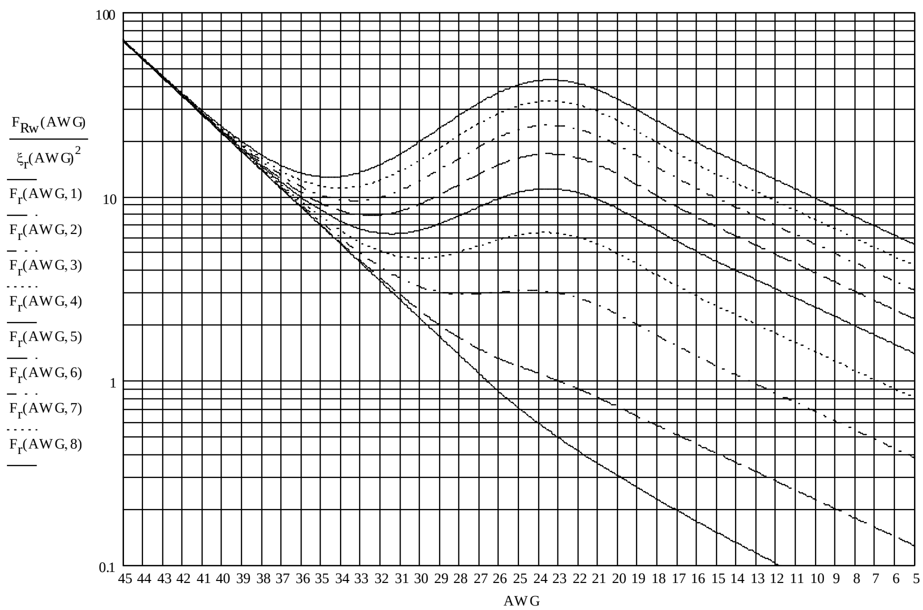

Fig. 5. Round-wire F_r and F_{rw} plots, expanded along the ξ_r axis; f = 150 kHz, $R_{\delta r}/l_w$ = 188 mΩ/m, Cu, 80°C.

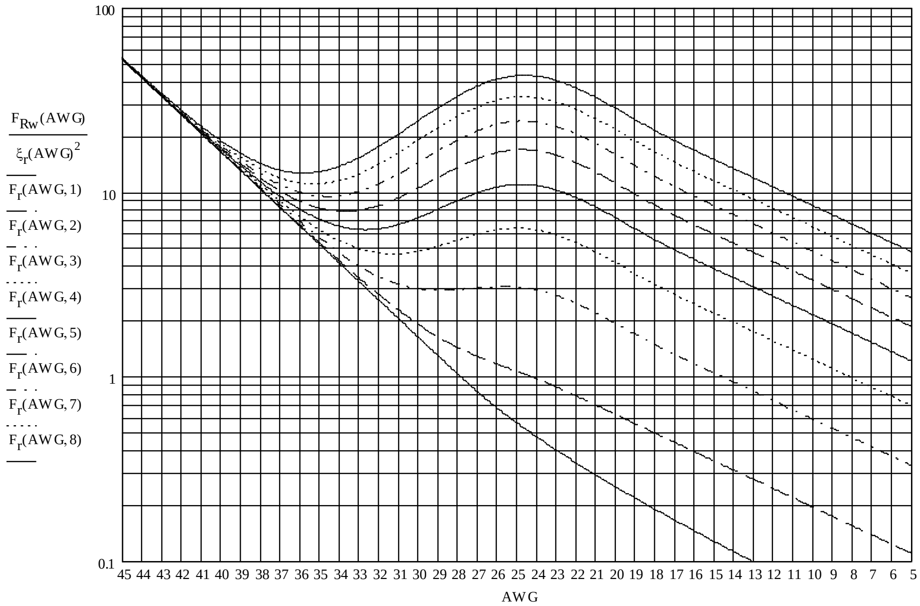

Fig. 6. Round-wire F_r and F_{rw} plots, expanded along the ξ_r axis; f_s = 200 kHz, $R_{\delta r}/l_w$ = 251 mΩ/m, Cu, 80°C.

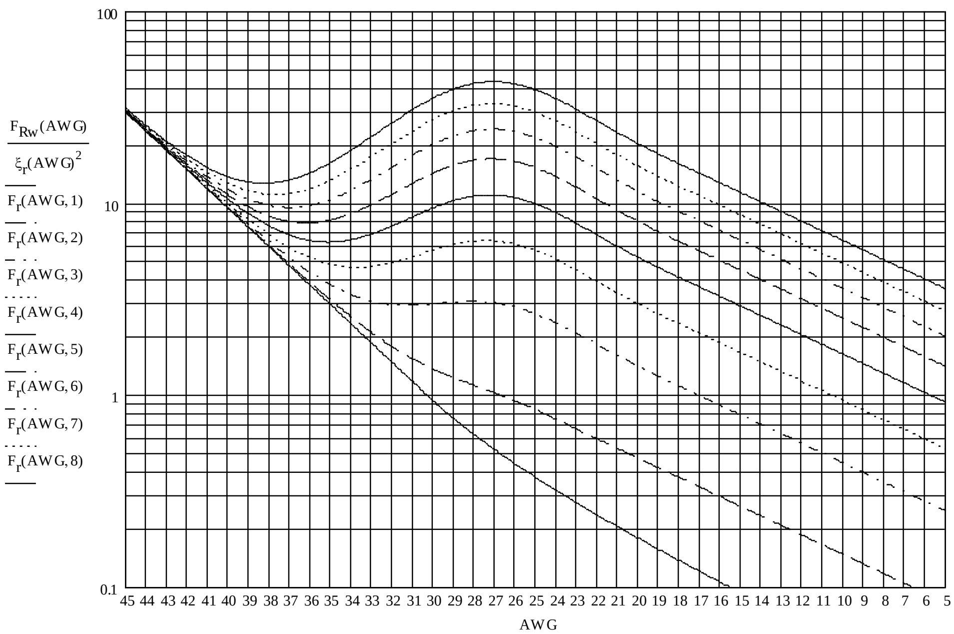

Fig. 7. Round-wire F_r and F_{rw} plots, expanded along the ξ_r axis; f_s = 350 kHz, $R_{\delta r}/l_w$ = 440 mΩ/m, Cu, 80°C.

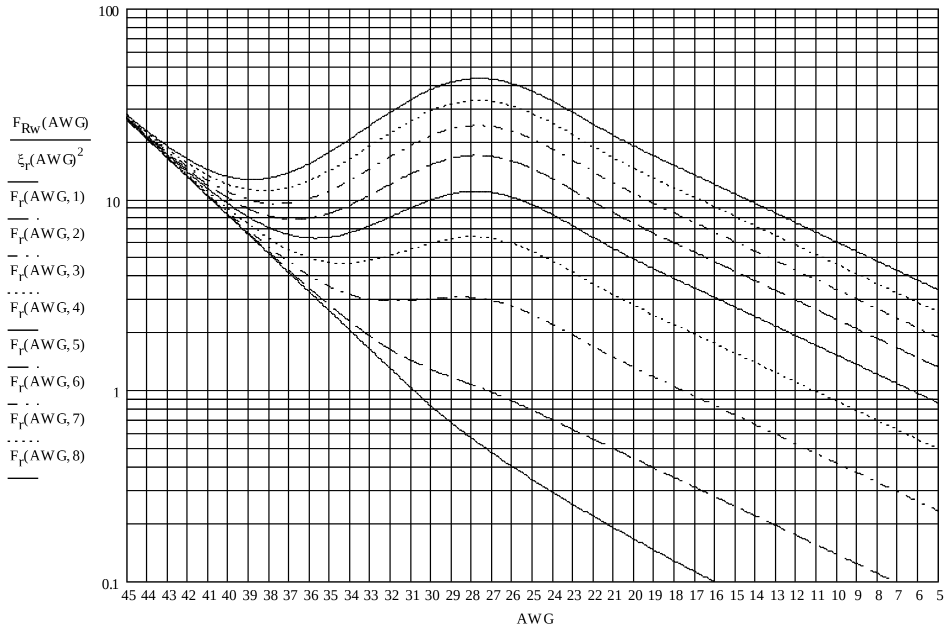

Fig. 8. Round-wire F_r and F_{rw} plots, expanded along the ξ_r axis; f_s = 400 kHz, $R_{\delta r}/l_w$ = 502 mΩ/m, Cu, 80°C.

What F_r shows is that either sufficiently large or optimally small wire both minimize eddy-current effects. The low-ξ solution is usually preferred because of the smaller winding area, A_{ww}, and a lower winding material cost.

As layers increase, ξ_v decreases somewhat. At 8 layers, $\xi_{rv}(8) \approx 0.41$—about a #38 wire at 350 kHz. As wire size decreases, for the same A_{ww}, M must increase. Eventually, ξ_{rv} is of wire above #40 AWG, which (like wire below #16 AWG) becomes hard to work with. Foil for large wire and commercial Litz wire for small wire become alternatives.

R_w is calculated from the above F_r plots for AWG values of round Cu wire using

$$\frac{R_{\delta r}}{l_w} \approx \left(1.256 \frac{\mu\Omega}{\mathrm{m \cdot Hz}}\right) \cdot f \text{ , round-wire, Cu, 80 °C}$$

The winding resistance is

$$R_w(\mathrm{AWG}, M, f) = F_r \cdot (R_{\delta r} / l_w) \cdot l_w$$

As a design example using the Fig. 5 graph for f = 150 kHz, for M = 6 layers (higher dash-dot curve), F_r is minimum at a value of 9.5 at $\xi_{rv} \approx 0.45$. this corresponds to a skin depth of

$$\delta \approx \frac{73.5\,\mathrm{mm}}{\sqrt{f/\mathrm{Hz}}} = \frac{73.5\,\mathrm{mm}}{\sqrt{150 \cdot 10^3}} = 0.19\,\mathrm{mm, Cu, 80\,°C}$$

and an optimal round wire size of

$$r_c = \xi_r \cdot \delta \approx \frac{\xi}{1.55} \cdot \delta = \frac{(0.7) \cdot (0.19\,\mathrm{mm})}{1.55} = 0.086\,\mathrm{mm}$$

For this r_c, a wire table gives a wire size between #34 and #33 AWG. This size agrees with the 150-kHz plot minimum of F_r plotted against wire gage. For the high-frequency solution, follow the $F_r(\xi_v)$ = 9.5 value across the graph to where the M = 6 curve is intersected at $\xi_{rcr} \approx 3.9$. ξ_{rcr} corresponds to a wire $r_c = (3.9)\cdot(0.19\text{ mm}) = 0.741$ mm or about #15 AWG, with 70 times larger wire area! It is likely that this solution will require a larger core to accommodate the larger wire. Hence, the low-ξ_r solution is usually preferred. For low-turns windings and high frequency, such as in planar transductors, the high-ξ_r solution becomes feasible.

On the F_r graphs, as M increases, F_r and P_w minimums increase at ξ_v values. As M increases, the variation in F_r in the medium-frequency region also increases with ξ, with higher F_r peaks and higher values of ξ_{cr}. The medium-frequency region also widens as M increases. Consequently, reduction of layers is an important design consideration because even at ξ_v, the minimum R_w increases with M. With smaller wire, additional strands reduce the increasing R_w per strand, yet eventually an increasing number of strands of decreasing size is suboptimal because strand packing factor decreases, and more of the winding area becomes winding insulation. For the same winding area, using the minimum number of layers is generally optimum.

Litz wire is produced by specialized manufacturers as a multi-stranded bundle of tiny wires with strands bundled together (by weaving instead of twisting) to minimize the layering effect. The field cancellation geometry of the weave is somewhat related to the interleaving of winding layers that sequentially alternates primary and secondary windings to reduce accumulated field-referred current, Ni. Litz wire has no opposite-polarity currents in strands, but configures bundles of twisted strands for minimization of eddy-current effects over the pitch length.

In summary, there are two solutions to the R_w minimization problem, a low-ξ and a high-ξ solution. The low-ξ solution results from the minimization of frequency effects by operation in the low-ξ region, using multiple parallel strands of wire to compensate for the low R_{w0} of their smaller wire size. The high-ξ solution results from the dominance of decreasing R_{w0} in the high-ξ region over the proximity effect. Between ξ_v and ξ_{cr} is the medium-frequency region dominated by the proximity effect. Only for $M \leq 2$ is it feasible to operate in this region.

References

1. "Eddy-Current Effects In Magnetic Design (Part 1): The Skin Effect" by Dennis Feucht, How2Power Today, August 2016 issue.

2. "Eddy-Current Effects In Magnetic Design (Part 2): The Proximity Effect" by Dennis Feucht, How2Power Today, September 2016 issue.

3. "Eddy-Current Effects In Magnetic Design (Part 3): Conductor Cross-Sectional Geometry" by Dennis Feucht, How2Power Today, October 2016 issue.

4. "Eddy-Current Effects In Magnetic Design (Part 4): Dowell's Formula" by Dennis Feucht, How2Power Today, October 2016 issue.

About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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