Eddy-Current Effects In Magnetic Design (Part 6): Winding Bundles

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This article series closes with an additional design consideration when multistrand windings or *bundles* are used to reduce winding resistance because of eddy-current effects. Strands have an equivalent number of layers within a bundle and the layer value for use in the Dowell formula (and graphs) of the previous part 5 can be approximated from the number of strands with the same current polarity.

This part derives the formula for that approximation, which allows calculation of the layer value, *M*, from parameters such as number of strands, strands per layer, and strand packing factor. Then, we delve into eddy current effects encountered in the special case of unibundle construction where primary and secondary strands are all twisted in the same bundle.

In this unibundle situation, we have to consider two possibilities—one is the simple case where strand placement within the bundle is controlled such that strands with opposing currents are distributed uniformly within the bundle. The other is the more complex case where these strands are randomly distributed. This forces us into an analysis of probabilities that leads to derivation of a special approximation of *M* based on the number of strands in the bundle.

Defining The Parameters

Winding parameters for strands, bundles, layers, and windings are shown in Fig. 1.

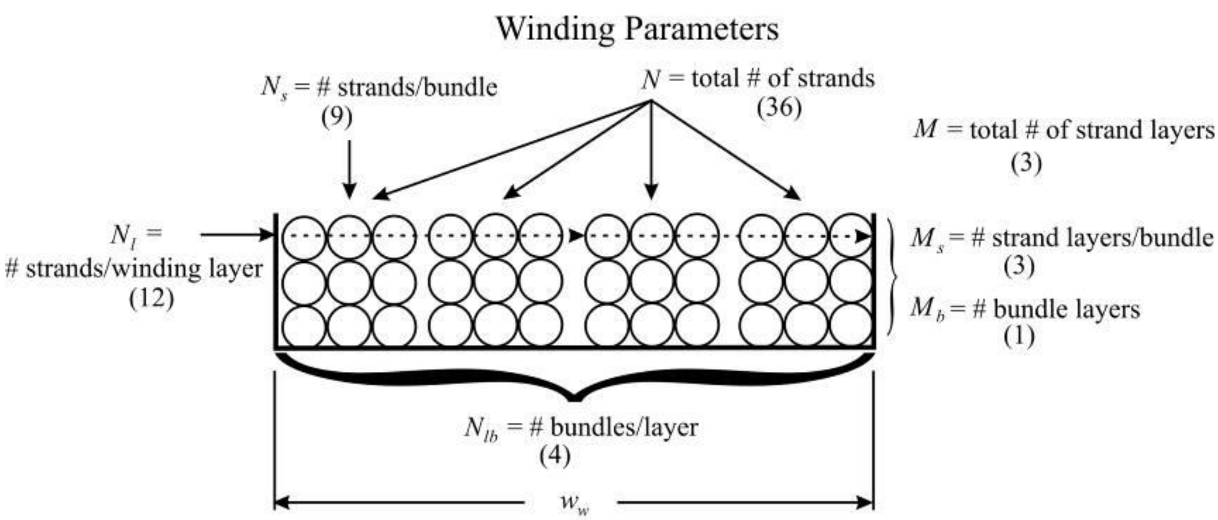

Fig. 1. Definitions of various quantities related to windings when winding turns are bundles of wire strands.

A winding bundle has N_s strands and M_s layers of strands within it while a winding layer has N_{lb} bundles and N_l strands across the window (bobbin) width, w_w. *N* is the total number of winding strands and *M* is the total number of layers of winding strands.

In a transductor, the primary and secondary windings have currents of opposite polarity, and each of these windings can be subdivided into multiple primary or secondary sub-windings. A *winding* is thus the set of sub-windings, bundled or single-strand, with the same current polarity. All windings, taken together, are *the winding*.

Estimating The Number Layers In A Bundle

The number of layers, M_s, of strands of wire in a bundle can be approximated as M_{sh} by configuring the N_s strands of the bundle as square with $\sqrt{N_s}$ strands per side. Then the strand layers per bundle,

$$M_s \approx M_{sh} = \sqrt{N_s}$$

Another approximation of bundle strand layers, which we'll call M_{sb}, is the number of wires of packed radius, r_{cwp} (wire radius including k_p, the wire packaging factor) required to span the twist circle (or circumference) of a bundle with radius r_{bw}:

$$M_s \approx M_{sb} = \frac{r_{bw}}{r_{cwp}}$$

Strand packing is the same within a bundle as for unbundled layers: $k_{pb} = k_{pf} \approx \pi/4$. The round bundle area is

$$\pi \cdot r_{bw}^2 = \frac{N_s \cdot A_c}{k_p} = \frac{N_s \cdot A_{cw}}{k_{pb}} = \frac{N_s \cdot (\pi \cdot r_{cw}^2)}{k_{pb}} = N_s \cdot \pi \cdot \left(\frac{r_{cw}}{\sqrt{k_{pb}}} \right)^2 \Rightarrow r_{bw} = \sqrt{N_s} \cdot \left(\frac{r_{cw}}{\sqrt{k_{pb}}} \right)$$

where for a bundle, $k_p = k_{pb} \cdot k_{pw}$ and porosity, k_{pw} relates $A_c = k_{pw} \cdot A_{cw}$, where A_{cw} is the insulated-wire area. Solving for r_{bw} and substituting into bundle layers,

$$M_{sb} = \frac{r_{bw}}{r_{cwp}} = \frac{r_{bw}}{r_{cw} / \sqrt{k_{pb}}} = \sqrt{N_s}$$

Then $M_{sb} = M_{sh}$ and $M_s \approx \sqrt{N_s}$.

M_{sb} can be expressed alternatively as

$$M_{sb} = \frac{r_{bw}}{r_{cwp}} = \frac{r_{bw}}{r_{cw} / \sqrt{k_{pb}}} = \sqrt{\frac{\pi}{4}} \cdot \frac{r_{bw}}{r_{cw}} \approx (0.886) \cdot \frac{r_{bw}}{r_{cw}}$$

If square and round bundle areas are equated,

$$A_{sq} = \left[\sqrt{N_s} \cdot (2 \cdot r_{cw}) \right]^2 = N_s \cdot 4 \cdot r_{cw}^2 = A_r = \pi \cdot r_{bw}^2$$

Solving for r_{bw}/r_{cw},

$$\frac{r_{bw}}{r_{cw}} = \sqrt{\frac{N_s}{k_{pb}}} = \sqrt{\frac{4}{\pi}} \cdot \sqrt{N_s}$$

Substituting into M_{sb},

$$M_{sb} = \frac{r_{bw}}{r_{cw} / \sqrt{\pi / 4}} = \sqrt{\frac{\pi}{4}} \cdot \frac{r_{bw}}{r_{cw}} = \sqrt{\frac{\pi}{4}} \cdot \left(\sqrt{\frac{4}{\pi}} \cdot \sqrt{N_s} \right) = \sqrt{N_s}$$

The total number of layers of a winding with M_b bundle layers is

$$M = M_s \cdot M_b$$

and applies to strands with current of the same polarity (no interleaving). Thus

$$M \approx \sqrt{N_s} \cdot M_b$$

The number of winding bundle turns is N_b. The total number of winding strand turns is

$$N = N_s \cdot N_b$$

For N_{lb} bundle turns per layer and window winding layer width w_w,

$$N_{lb} = \frac{N_b}{M_b} = \frac{w_w}{2 \cdot r_{bw}} = \frac{w_w}{2 \cdot (\sqrt{N_s / k_{pb}} \cdot r_{cw})} = \sqrt{k_{pb}} \cdot \frac{N_l}{\sqrt{N_s}} \;\; , \;\; N_l = \frac{w_w}{2 \cdot r_{cw}}$$

where N_l = the total number of strands across the winding at a given layer, in all the bundles spanning the window. Then, M, the total number of strand layers is

$$M \approx \sqrt{N_s} \cdot M_b = \sqrt{N_s} \cdot \frac{N_b}{N_{lb}} = \frac{\sqrt{N_s} \cdot N_b}{N_l / \sqrt{N_s / k_{pb}}} = \sqrt{k_{pb}} \cdot \frac{N}{N_l}$$

Therefore, for fixed k_{pb} (the packing factor for strands within a bundle), the total number of layers does not depend on any bundle-related quantities, but rather on the total number of strands, N, and the number of strands per winding layer, N_l. So bundling does not affect the strand layer count.

Reducing ξ, the penetration or skin-depth ratio, to reduce F_r, the resistance ratio, has diminishing benefit when a decrease in ξ causes an increase in M that results in an increase in F_r. When the additional strands needed to reduce ξ form additional layers, they can limit the advantage of adding more strands. A graphical optimization using the F_r graphs (or their equations) from part 5 can lead to an optimum M and ξ.

Unibundle Eddy-Current Effects

In single-bundle (*unibundle*) winding construction, primary and secondary strands are twisted together so that their fields mostly cancel, having opposing currents of the same field-referred current, Ni. The strands are interchangeable, have the same voltage induced across them, and conduct the same amount of current. For uniform power distribution in the winding, half are primary and half are secondary strands, configured in either series or parallel to effect a desired turns ratio.

The eddy-current effects of bundled strands with unipolar currents have been described by Charles R. Sullivan.[1] Four categories result from combinations of skin or proximity effect on strand or bundle:

- Strand skin effect—exists in the strands of bundled wire.
- Bundle skin effect—current crowds to the outer strands of a bundle. For twisted wire, it is minimized and will be considered negligible.
- Strand proximity effect—the "internal" effect within the bundle affects wires in proximity. The effective number of layers depends on the wire configuration. "External" effects are from B-fields from adjacent bundles. Because field current, Ni = 0, for a unibundle, external fields are negligible. Adjacent strands of adjacent bundles have some effect, but if the bundle is configured so that opposing fields are close by, then close bundles have the same effect on each other as close strands within a bundle, and the proximity effect is negligible.
- Bundle proximity effect—adjacent bundle turns have field current, Ni = 0, for a unibundle, and external fields from adjacent bundles are negligible.

Consequently, only the strand skin effect and internal strand proximity effect are significant in unibundle windings. The skin effect of a wire has been characterized in part 1. The strand proximity effect depends on opposing current distribution within the bundle. If strands with opposing currents are distributed uniformly throughout the bundle, the proximity effect is minimized and $M_b \approx 1$.

The bundle can be considered to be composed of two windings of opposite-polarity current. Each winding has an equal number of strands, each having the same magnitude of current. Then winding loss is equally distributed between windings and field-referred current, $Ni = 0$ A for the bundle. Multiple primary or secondary windings are strand subdivisions of the two windings.

If winding strand placement is random, a layer approximation can be made for Dowell's formula by assuming that all strands have the same probability of being anywhere in the bundle cross-section. Then strands of the same current polarity that are "clumped" together (not interleaved) will have a strand proximity effect.

For a random placement of $N_s/2$ of the strands for each winding, it is possible that two or more of the strands will be together in adjacent "clumps" that increase the proximity effect in a winding. *Our goal is to derive a formula for the average strands per clump and use it with the* M_s *approximation.*

From combinatoric math, the number of different subsets of k strands out of a set of n strands is the binomial coefficient,

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!} = \frac{n \cdot (n-1) \cdot \cdots \cdot (n-(k-1))}{k!}, \; k = 1, 2, \ldots, n$$

The strands are the same size but distinguishable from each other, as though numbered, so that distinct subsets of them can be made. The k strands within a subset can be in any order or sequence. Subsets of k are unordered *combinations*, though within them are $k!$ possible orderings. The number of different sequences or orderings of k strands out of n is $n!/(n - k)!$. The $k!$ in the denominator of the binomial coefficient divides out the orderings, leaving the number of unordered combinations.

A unibundle has $N_s/2$ strands of each polarity. The number of combinations of N_s different strands in subsets of $N_s/2$ strands is

$$\binom{N_s}{N_s/2} = \frac{N_s!}{(N_s/2)! \cdot (N_s - N_s/2)!} = \frac{N_s!}{[(N_s/2)!]^2}$$

The binomial coefficient of $N_s/2$ strands out of N_s can be constructed intuitively. The probability of placing a strand for the first placement is $N_s/2$ out of N_s, or ½. For the next placement, $(N_s/2) - 1$ strands of the same winding remain out of $N_s - 1$ total strands. The probability of choosing a second strand is

$$\frac{(N_s/2)-1}{N_s-1}$$

Then for $k = N_s/2$,

$$\frac{N_s/2}{N_s} \cdot \frac{(N_s/2)-1}{N_s-1} \cdot \cdots \cdot \frac{(N_s/2)-((N_s/2)-1)}{N_s-((N_s/2)-1)} = \frac{[(N_s/2)!]^2}{N_s!} = 1/\binom{N_s}{N_s/2}$$

For $N_s = 10$ strands, the probability that a particular five-strand sequence will be chosen at random is

$$\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6} = \frac{1}{2} \cdot \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{1}{6} = \frac{1}{252}$$

Adjacency is defined with the proximity effect in mind as a set of strands that form a *simply-connected convex region* or *clump* of strands wherein no line drawn within the area occupied by the set intersects a strand of the opposite-polarity winding. Strands on the perimeter of the bundle have fewer adjacent strands, by about ½ from inner strands, though for increasing N_s, the perimeter strands increase by only $N_s^{1/2}$. A clump of k strands that includes perimeter strands is treated the same as a clump of interior strands.

The bundle cross-section presents randomly-placed strands of the two windings with their opposing current polarities. Each winding has $N_s/2$ strands. Suppose we randomly choose a distinct $N_s/2$ strands in the bundle and determine how many combinations we could choose that would have k of the $N_s/2$ chosen strands be of the same winding. The number of combinations is the product of the combinations of k strands from a winding of $N_s/2$ times the number of combinations of the remaining $N_s - N_s/2 = N_s/2$ strands of the other winding from which the remaining $N_s/2 - k$ strands are chosen;

$$\binom{N_s/2}{k}\cdot\binom{N_s - N_s/2}{N_s/2-k} = \binom{N_s/2}{k}\cdot\binom{N_s/2}{N_s/2-k}$$

The need for the product is that after k winding strands are chosen of the given winding, the remaining $N_s/2 - k$ strands must be chosen from the other winding. This second factor is the number of combinations of strands of the other winding from which the remaining $N_s/2 - k$ strands are chosen to complete the $N_s/2$ selected strands. Both conditions must be true and the combinations multiply.

Binomial coefficients are symmetric over the range of k in that

$$\binom{n}{n-k} = \frac{n!}{(n-k)!\cdot(n-(n-k))!} = \frac{n!}{(n-k)!\cdot k!} = \frac{n!}{k!\cdot(n-k)!} = \binom{n}{k}$$

Consequently, the number of k-strand combinations from the same winding in $N_s/2$ winding strands chosen out of the N_s of the bundle is

$$\binom{N_s/2}{k}\cdot\binom{N_s - N_s/2}{N_s/2-k} = \binom{N_s/2}{k}^2$$

The probability that k out of $N_s/2$ winding strands is in a selection of $N_s/2$ strands chosen randomly from a bundle of N_s strands is

$$h(k) = \frac{\binom{N_s/2}{k}^2}{\binom{N_s}{N_s/2}}$$

This is a *hypergeometric probability* function of k.

The k strands of a combination are not necessarily adjacent, and often will not be. The maximum number of k-clumps is the number of combinations of the k randomly distributed strands. The sum of $k\cdot h(k)$ over k is the average of k from $k = 0$ to $N_s/2$ (in other words, the average strands per clump.) Then the layer approximation for Dowell's formula, $M_s \approx \sqrt{N_s}$, is applied to the k-strand average as

$$M_s \approx \sqrt{\bar{k}} = \sqrt{\sum_{k=0}^{N_s/2} k \cdot h(k)} = \sqrt{\frac{\sum_{k=0}^{N_s/2} k \cdot \binom{N_s/2}{k}^2}{\binom{N_s}{N_s/2}}} = \sqrt{\frac{N_s}{4}} = \frac{\sqrt{N_s}}{2}$$

which is the useful design formula we set out to derive. But how do we interpret this expression?

For $k = N_s/2$, there is only one combination in which all winding strands constitute a worst-case clump, in a bundle with two $N_s/2$-clump windings. (The same holds for $k = 0$, and its $N_s/2$-clump is of the other winding.) This is equivalent to separate primary and secondary Litz-wire bundles, twisted together as a *duobundle*. The advantage of duobundle construction is that it separates sleeved windings to avoid voltage breakdown (for reliability and safety) yet retains a reduction in eddy-current effects over that of sequential windings. Each duobundle winding has $M_s \approx \sqrt{N_s/2}$ strand magnetic layers.

For $k = 1$ of the unibundle, the resulting clumps have one strand—the best case of uniform + and – current distribution. The most frequently occurring combinations, and hence clumps, are for the average $k = (N_s/2)/2 = N_s/4$.

For example, for $N_s = 10$ strands, then the terms in the sum from the equation for M_s are as shown in the table.

Table. Calculating terms in the M_s expression above.

k	$\binom{5}{k}$	$\binom{5}{k}^2$	$k \cdot \binom{5}{k}^2$
0	1	1	0
1	5	25	25
2	10	100	200
3	10	100	300
4	5	25	100
5	1	1	5

(The row with $k = 0$ is included to show the symmetry of the distribution. It adds nothing to the average.) The sum is 630; the denominator in the M_s expression is

$$\binom{N_s}{N_s/2} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 252$$

Then the average number of k strands in a random distribution of 10 strand polarities is 630/252 = 2.50, or $N_s/4$, and $M_s \approx 1.58$. This is a worst-case value because it assumes that all combinations of k strands in a winding form clumps. The spread in what to expect of $\bar{k}$ is the hypergeometric probability variance,

$$\sigma^2 = \frac{N_s/2}{N_s - 1} \cdot \frac{N_s/2}{N_s} \cdot \frac{N_s - N_s/2}{N_s} = \frac{1}{8} \cdot \frac{N_s}{N_s - 1} \approx \frac{1}{8}, N_s >> 1$$

The deviation is the rms value of the deviation of k from the average k:

$$\sigma = \tilde{k}_{\sim} \approx \frac{1}{\sqrt{8}} = \frac{1}{2 \cdot \sqrt{2}} \approx 0.3536$$

The number of strands per clump will deviate because of the random placement of strands by about 0.354 strands from the average.

Closure

The six parts of this mini-course will hopefully enable you or others to include the effects of eddy currents in winding design. What has been covered here are the fundaments of the topic. There is more to it, including F_R approximation, approximate minimum R_w for foil and wire, winding loss from waveshape, foil, ECB traces, and winding shape, aluminum or copper wire, F_L effects, winding sequence, leakage inductance reduction, leakage inductance in winding window, and more. This is a list of section titles relating to eddy currents, following those from which this series was adapted, from the book, *Power Magnetics Design Optimization*.[2]

References

1. "Optimal Choice for Number of Strands in a Litz-Wire Transformer Winding" by Charles R. Sulivan, *IEEE Transactions on Power Electronics*, vol. 14, no. 2, pp. 283-291, MAR99.

2. *"Power Magnetics Design Optimization"* Dennis Feucht, available In PDF or print from www.innovatia.com.

3. "Eddy-Current Effects In Magnetic Design (Part 1): The Skin Effect" by Dennis Feucht, How2Power Today, August 2016 issue.

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About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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