# Ferrite Core Magnetics (Part 2): Gapped Ferrite Cores

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Gapped ferrite cores are used to store transfer energy, as are iron powder cores. Yet they differ in important ways. This article describes how the differences affect the design of inductors, transformers and coupled inductors, and how transfer power is maximized for high-ripple cores. Transformer optimization results from part 1[1] are also derived from the standpoint of large-ripple design. We begin by explaining how calculation of the operating point *H* differs for ferrite gapped cores versus powder cores, and how to find the operating point $N\bar{i}$ and the saturation-limited turns, $N_i$ for gapped ferrite cores.

## Impact Of Gap Length And Saturation Characteristics

Ferrites differ from iron powder cores in two important ways. First, changing the gap length of a ferrite core leaves the material properties of the ferrite unchanged, but for powder cores, $\mu(f_s)$ and $\bar{p}_c(\hat{B}_\sim, f_s)$ can change with a change in $\mu$. Each value of $\mu$ *(permeability)* or $\mathcal{L}$ (field inductance) is treated as a distinct material.

Second, ferrite $k_{sat} \approx 1$ until *B* increases to some value where $k_{sat}$ plunges abruptly to a low value. When used as inductors, this corner value must not be exceeded by the peak *B*,

$$\hat{B} = \bar{B} \cdot (1+\gamma) \le B_{sat} \;\Rightarrow\; \bar{B} \le \frac{\hat{B}}{1+\gamma}$$

The typical ferrite *B*(*H*) curve from part 1 is repeated below in Fig. 1.

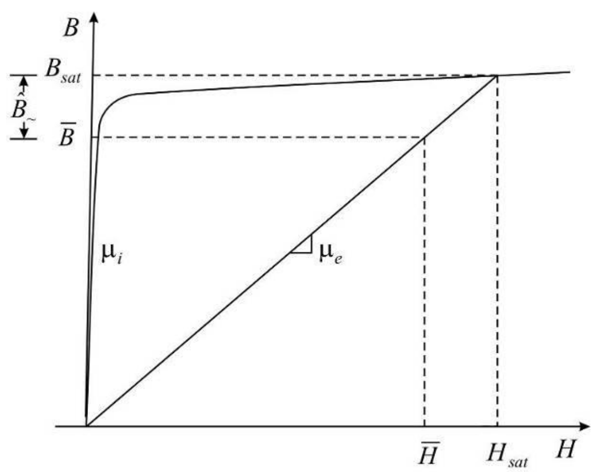


*Fig .1 A B(H) curve for a typical ferrite core.*

The core operating-point is at the *B* average of

$$\bar{B} = \frac{\hat{B}}{1+\gamma}$$

Then by definition of ripple factor, $\gamma$,

$$\gamma = \frac{\hat{B}_\sim}{\bar{B}} = \frac{\hat{B}_\sim}{\hat{B}} \cdot (1+\gamma) \;\Rightarrow\; \gamma = \frac{\hat{B}_\sim}{\hat{B} - \hat{B}_\sim}$$

where $\hat{B}_{\sim}$ is the *B* ripple amplitude, or Δ*B*/2. For ferrites,

$$N\bar{i} = \frac{\overline{B} \cdot A}{\mathcal{L}} = \frac{\hat{B} \cdot A}{(1+\gamma) \cdot \mathcal{L}} = \frac{\hat{\phi}}{(1+\gamma) \cdot \mathcal{L}}$$

where the $\mathcal{L}$ (or $\mu$) is determined by choice of a particular core. The peak *H* for ferrites can be calculated from $B_{sat}$ as

$$\hat{H} \leq H_{sat} = \frac{B_{sat}}{\mu} = \frac{B_{sat} \cdot A}{\mathcal{L} \cdot l}$$

The operating-point on the gapped curve is at $\overline{H}$, placed sufficiently below $H_{sat}$ so that $\hat{H}$ never exceeds it. For powder cores, the operating-point $H(k_{sat}) = \overline{H}$ is read from the saturation curve of $\mu(k_{sat})$; for ferrite cores, it is instead calculated from the above equation for $H_{sat}$ and adjusted downward by a chosen design safety margin to $\hat{H}$.

Air gaps in the magnetic path provide a place to store magnetic energy, and they keep $\mu(H)$ (or $\mathcal{L}(Ni)$ or $L(i)$) flat out to a larger value of *H* before falling off, though with the disadvantage of a lower $\mu$. Gapped cores are specified in $\mathcal{L}$ or $\mu$, and have a linear $B(H)$ curve below saturation. Because of linearity, both the $\mu$ and $\mathcal{L}$ total (or "dc") and incremental (or "ac") values are equal. Consequently, the incremental values for gapped cores in ferrite core catalogs can be used to find the operating-point $N\bar{i}$ using the equation above. For ferrites, $N_i$ (the saturation limited turns) is calculated from this $N\bar{i}$.

Air gaps in ferrite cores cause the constant-$\mu$ or constant-inductance part of the curve to be extended before the plunge, though at a lower value of $\mu$ (or $\mathcal{L}$ or *L*) than without the gap, as shown in the Fig. 1 graph. Then for a gapped core with an effective $\mu = \mu_e$, a maximum *H* corresponding to $\hat{B}$ is $\hat{H} = \hat{B}/\mu_e$.

### *High-Ripple Design*

High-$\mu$ ferrite cores can operate with a $\gamma$ value large enough so that the small-ripple approximation made for powder cores is not valid. For powder cores, $\overline{H}$ is used instead of $\hat{H}$ because $\hat{H}_{\sim} << \overline{H}$ so that $\hat{H} \approx \overline{H}$. The $\hat{H} - \overline{H}$ difference saturates powder cores somewhat more but only by a gradual amount. Maximum transfer power for ferrite coupled inductors is delivered at an $\overline{H}$ corresponding to $\overline{B} = \hat{B} - \hat{B}_{\sim}$. As $\hat{B}_{\sim}$ is increased to increase power, $\overline{B}$ must be reduced to keep $B(t) \leq \hat{B}$, and a reduced $\overline{B}$ (and $\overline{H}$) reduces power.

The operating-point that results in maximum power is found by setting the derivative of transfer power density to zero and solving for the maximizing condition:

$$\overline{p} = \Delta B \cdot \overline{H} = \Delta B \cdot \frac{\hat{B} - \Delta B/2}{\mu} = \frac{1}{\mu} \cdot [\hat{B} \cdot \Delta B - \tfrac{1}{2} \cdot (\Delta B)^2]$$

Then

$$\frac{d\overline{p}}{d\Delta B} = \frac{1}{\mu} \cdot [\hat{B} - \Delta B] = 0 \Rightarrow \Delta B = \hat{B}$$

The op-pt condition is thus

$$\overline{H}_{max} = \frac{1}{\mu} \cdot \frac{\hat{B}}{2} \Rightarrow \overline{B} = \frac{\hat{B}}{2}$$

Transfer power is maximized when

$$\overline{B} = \frac{\hat{B}}{2}, \max \overline{P}$$

Then

$$\gamma_{max} = \frac{\Delta B / 2}{\overline{B}} = \frac{\hat{B}/2}{\hat{B}/2} = 1$$

Thus magnetic materials with the highest $\gamma$ values are optimal for maximizing transfer power in coupled inductors as they were for transformers. The high-$\gamma$ value for ferrites used in inductors poses a design tradeoff between ripple and power. Less ripple results in less transfer power. Powder cores have an advantage over ferrites with their lower $\gamma_{opt}$ and higher power density, though at lower frequencies.

## *Maximum Power With Large Ripple*

A design goal for high-$\gamma$ waveforms is to either maximize transfer power or given a required power, to minimize core size (volume). The graph in Fig. 2 shows what happens when $N$ is varied over a range from $N_\lambda$ (the core-loss minimum turns) to $N_i$ (the saturation-limited turns). The transfer power remains constant by holding circuit input flux ripple, $\Delta\lambda$ and circuit primary current $\bar{i}_g$ constant.

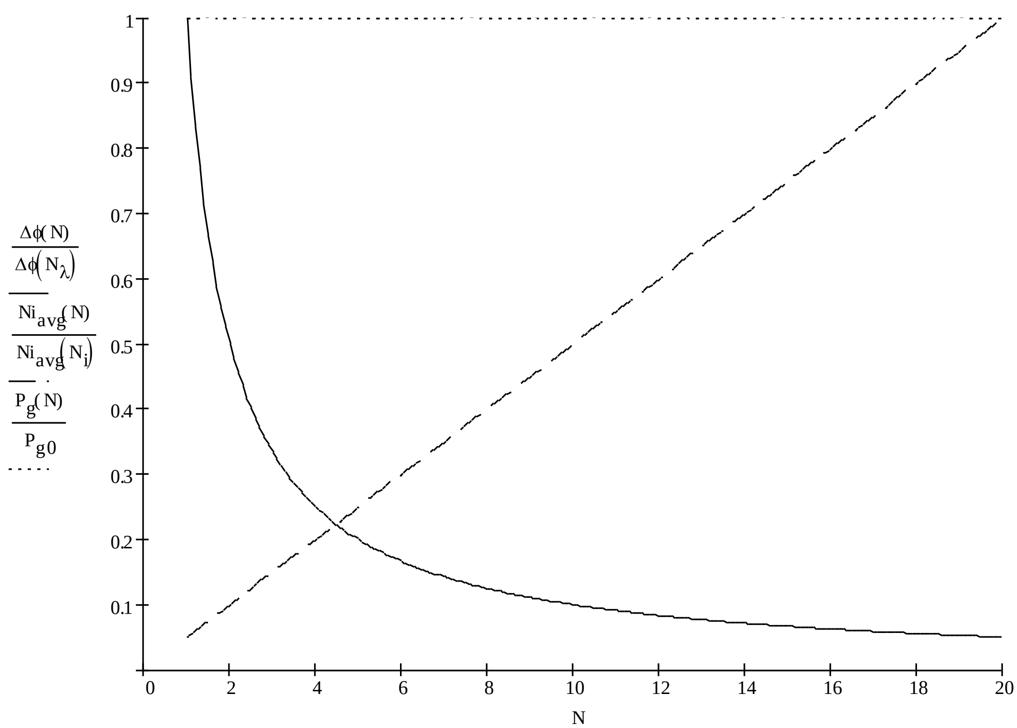


*Fig. 2. Flux ripple in the core, $\Delta\phi$, the field current in the core, $N\bar{i}$, and the input power, $P_g$, plotted as a function of N (normalized to operating-point values). The flux ripple in the core, $\Delta\phi$ decreases with increasing N while the field current of the core increases as a function of N.*

The flux ripple in the core, $\Delta\phi = \Delta\lambda/N$ decreases with increasing *N* while the field current of the core increases by $N\bar{i} = N \cdot I$. $N\bar{i}$ is always *N* times some average current. For a cycle, it is designated here as $\bar{i}$ while for an on-time average amplitude, it is $I$.

As *N* increases, the field-current operating-point, $N\bar{i}$ increases which would increase power. However, $\Delta\phi$ decreases, and power, $\Delta\phi \cdot N\bar{i}$, remains the same. As *N* increases, circuit inductance increases and current ripple decreases causing $\Delta\phi$ to decrease. Therefore, it is the combined effects of both $\Delta\phi$ and $N\bar{i}$ with *N* that affect transfer power. When power is allowed to vary, it is maximum at an *N* where the fractional change in each is equal and opposite;

$$\frac{d\Delta\phi}{dN\bar{i}} = -\frac{\Delta\phi}{N\bar{i}} \Rightarrow \frac{d\Delta\phi}{\Delta\phi} = -\frac{dN\bar{i}}{N\bar{i}} \Rightarrow \frac{\Delta\phi_2}{\Delta\phi_1} = \frac{N\bar{i}_1}{N\bar{i}_2}$$

From a waveform standpoint, saturation is a core limitation on peak field current, $N\hat{i}$, especially for ferrite cores. $N\hat{i}$ can be minimized through a choice of turns, *N*. More turns increase static *NI* and ripple is reduced along with $\hat{i}$ because of increased inductance. If *N* is reduced instead, *NI* is reduced and the ripple increases as does $\hat{i}$ from reduced inductance. What then is the value of *N* that minimizes $N\hat{i}$? (An earlier version of the following derivation is presented in a *How2Power Today* article by the author[2], embellished and refined here.)

## *Optimal Turns For Maximum Power With Large Ripple*

Given a core of field inductance $\mathcal{L}$ and circuit parameters $\bar{i} = I$ and $\Delta\lambda$, assume a triangle-wave ripple of winding current. Relating $\Delta i$ and $\Delta\lambda$,

$$\Delta\lambda = L \cdot \Delta i = N^2 \cdot \mathcal{L} \cdot \Delta i$$

The peak field current corresponding to winding $\hat{i}$ is

$$N\hat{i} = N \cdot \hat{i} = N \cdot \left( I + \frac{\Delta i}{2} \right) = N \cdot I + \frac{N}{2} \cdot \frac{\Delta\lambda}{N^2 \cdot \mathcal{L}} = N \cdot I + \frac{\Delta\lambda / 2}{N \cdot \mathcal{L}}$$

To find the minimum $N\hat{i}$, differentiate it with respect to *N* and equate to zero. The *N* of minimum $N\hat{i}$ is

$$N(N\hat{i}_{\min}) = N_{\min} = \sqrt{\frac{\Delta\lambda / 2}{\mathcal{L} \cdot I}}$$

Substituting $N_{\min}$ for *N* in the equation for $N\hat{i}$, then $N\hat{i}_{\min}$ is

$$N\hat{i}_{\min} = 2 \cdot N_{\min} \cdot I$$

The $N\hat{i}$ equation can be put into polynomial form in variable *N*:

$$N^2 - \left( \frac{N\hat{i}}{I} \right) \cdot N + \left( \frac{\Delta\lambda / 2}{\mathcal{L} \cdot I} \right) = 0$$

When solved for $N$ with $N_{\min}$ substituted in the constant term,

$$N = \frac{N\hat{i}}{2 \cdot I} \pm \sqrt{\left(\frac{N\hat{i}}{2 \cdot I}\right)^2 - N_{\min}^2} = \frac{N\hat{i}}{2 \cdot I} \cdot \left(1 \pm \sqrt{1 - \left(\frac{2 \cdot I \cdot N_{\min}}{N\hat{i}}\right)^2}\right)$$

This is the more general turns function for a given $N\hat{i}$ and $N_{\min}$. It can be expressed more simply in ripple factor,

$$\gamma = \frac{\Delta i / 2}{I} = \frac{\Delta\lambda / 2}{L \cdot I} = \frac{\Delta\lambda / 2}{N^2 \cdot \mathcal{L} \cdot I} = \left(\frac{N_{\min}}{N}\right)^2$$

Then for minimum field current, $N = N_{\min}$, $\gamma_{\min} = 1$. This is $\gamma$ of a CCM waveform at the boundary between DCM and CCM; it has a minimum $N\hat{i}$.

To express $N$ in $\gamma$, the expression under the radical converts to

$$\left(\frac{2 \cdot I \cdot N_{\min}}{N\hat{i}}\right)^2 = \left(\frac{N_{\min}}{N}\right)^2 \cdot \left(\frac{2 \cdot I}{\hat{i}}\right)^2 = \gamma \cdot \left(\frac{2}{\gamma + 1}\right)^2$$

by applying $N\hat{i} = N \cdot \hat{i}$ and $\hat{i} = (\gamma + 1) \cdot I$. When substituted into $N$ above, $N$ simplifies to

$$N = \frac{N\hat{i}}{2 \cdot I} \cdot \left(1 \pm \frac{\gamma - 1}{\gamma + 1}\right)$$

Now define "peak $N_i$" based on peak current:

$$\hat{N}_i \equiv \frac{N\hat{i}}{\hat{i}}$$

When substituted, $N$ reduces to

$$N = \gamma \cdot \hat{N}_i, \hat{N}_i$$

Two values of $N$ satisfy a given $N\hat{i}$: $\hat{N}_i$ with a lower-ripple, higher-$I$ waveform, and $\gamma \cdot \hat{N}_i$ with higher ripple and lower $I$. Then for $N = N_{\min}$, $N\hat{i}$ is minimized. Consequently, maximum power is achieved whenever $N\hat{i}$ is minimized.

As an example using a powder core, a T50B-26 Fe-pwd core has $\mathcal{L}_0$ = 43.5 nH. Let $I$ = 0.3 A and $\Delta\lambda$ = (5 V)·(5 μs) = 25 μV·s. Then $N_{\min}$ = 30.95 and the following plot results. At $\gamma$ = 1, $N\hat{i}$ = $N_{\min}^2 \cdot I$, with one $N$ value.

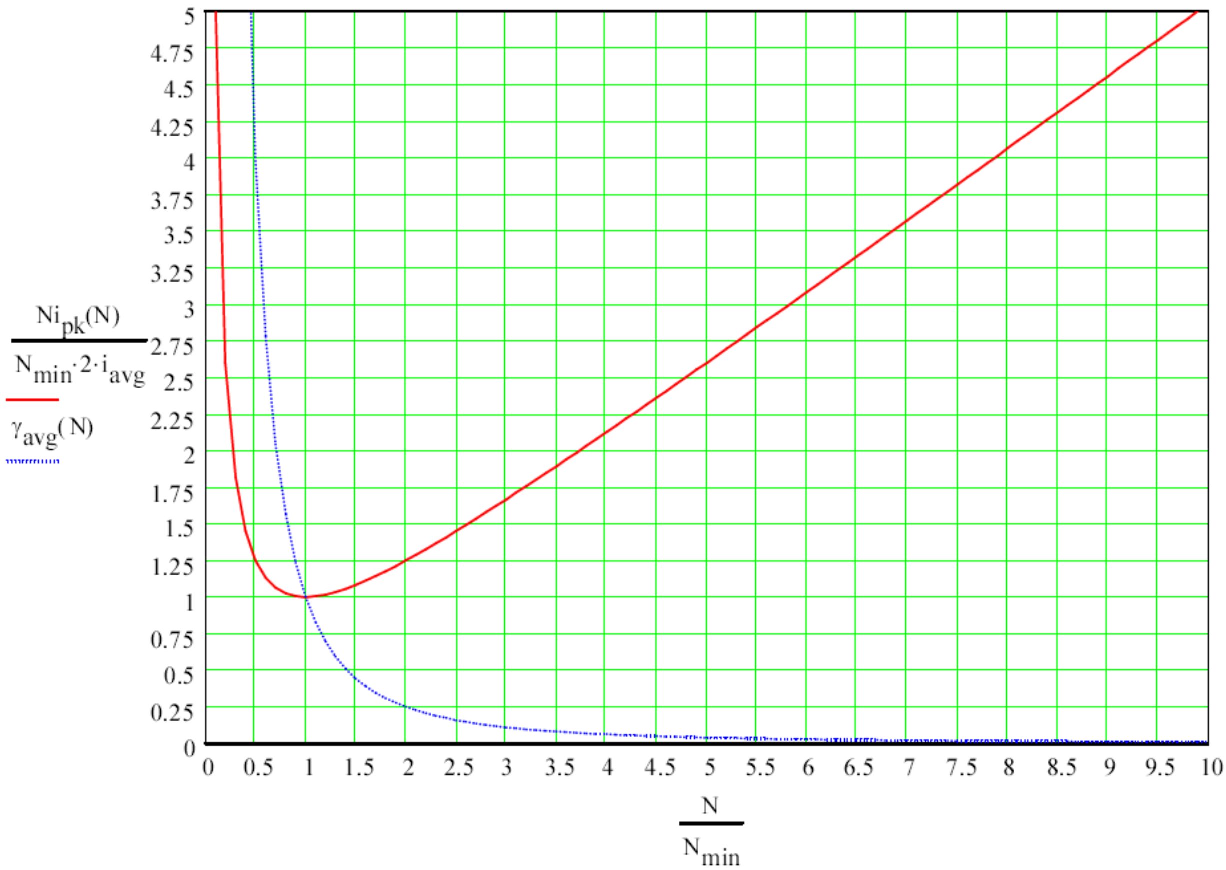


*Fig. 3. Red plot: normalized field current versus normalized turns. When N = $N_{min}$, as derived, then field current is minimum. Blue plot: ripple factor decreases with turns because ΔB and hence ripple decrease. The ripple factor at the minimum field current is $\gamma$ = 1, at the boundary between DCM and CCM.*

This result is also the same as for the full-utilization transformer design from part 1, where

$$\Delta\phi_{sat} = \Delta\phi_{\lambda} = \Delta\phi(\overline{p}_c)$$

This can be derived starting from a previous equations

$$N_{\min} = \sqrt{\frac{\Delta\lambda/2}{\mathcal{L}\cdot I}}$$

and $N\hat{i}_{\min} = 2\cdot N_{\min}\cdot I$ ;

$$N_{\min}^2\cdot\mathcal{L}\cdot I = N_{\min}\cdot\mathcal{L}\cdot\frac{N\hat{i}_{\min}}{2} = \frac{\Delta\lambda}{2} \Rightarrow \mathcal{L}\cdot N\hat{i}_{\min} = \frac{\Delta\lambda}{N_{\min}} = \Delta\phi_{\lambda}$$

From $\Delta\phi_{sat}$ in part 1 and setting $N\hat{i} = N\hat{i}_{\min}$ ,

$$\Delta\phi_{sat} = 2\cdot\mathcal{L}\cdot N\hat{i} = \mathcal{L}\cdot[N_{\min}\cdot 2\cdot I] = \mathcal{L}\cdot N\hat{i}_{\min} = \frac{\Delta\lambda}{N_{\min}} = \Delta\phi_{\lambda}$$

Therefore,

$$\Delta\phi_{sat} = \Delta\phi_{\lambda}$$

Thus a transformer design optimized to have a fully-utilized core has a magnetizing current waveform with unipolar (half-cycle) $\gamma_m = \gamma_{opt} = 1$.

**References:**

1. "Ferrite Core Magnetics (Part 1): Ungapped Ferrite Cores" by Dennis Feucht, How2Power Today, August 2017 issue.

2. "How to Minimize Core Saturation" by Dennis Feucht, How2Power Today, November 2012 issue.

## About The Author

*Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.*

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