Transformer Design (Part 1): Maximizing Core Utilization

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In this article, the utilization of the core in transformer design is considered by reviewing earlier work on the optimal turns that maximize use of the core in the magnetic part of a coupled-inductor design. Important differences between coupled inductor and transformer design are identified and different criteria are derived for making the same maximum use of a transformer core, and for the kind of application that can make it worthwhile.

In previous writings,[1-3] the magnetic design of coupled inductors was based on making full use of the core. This use was quantified as core utilization, the fraction of use to the maximum possible use. How much can be used of a core depends on its structural limits.

If it dissipates too much power and its temperature exceeds the Curie temperature, its permeability plummets and it is not much better than air in its magnetic properties. If a winding is driven with too much current, then when referred to the core field by the number of turns, *N*, as $N \cdot i = Ni$, it can exceed a design saturation limit, as quantified by the saturation factor, k_{sat}, the fraction of permeability or inductance at a given *Ni* over its zero-current value. Saturation, like power loss, also causes the core to lose its permeability in that increasing current decreases it until it is not much better than air or plastic.

Review of Optimal Magnetic Design

The central design parameter in coupled-inductor core utilization relates circuit to core field, *N*. With a minimum of turns, the field ripple amplitude, $\hat{B}_{\sim}$, which is found on the horizontal axis of core catalog power-loss graphs, increases until the power-loss limit is reached at a number of turns of

$$N_{\lambda} = \frac{\Delta\lambda}{\Delta\phi_{\lambda}} = \frac{\Delta\lambda}{\Delta\phi(\overline{p}_c)} = \frac{\Delta\lambda}{\Delta B \cdot A} = \frac{\Delta\lambda}{(2 \cdot \hat{B}_{\sim}) \cdot A}$$

where *A* is the core magnetic cross-sectional area and $\Delta\lambda = V_p \cdot \Delta t$ is the circuit flux, determined by winding voltage, V_p and its duration, Δt. These are circuit design parameters and are usually given for magnetics design. The field flux ripple, $\Delta\phi_{\lambda}$ is the $\Delta\phi$ corresponding to maximum allowable power loss in the core, which depends on core size.

The upper limit on *N* is saturation. For coupled inductors operating with ripple current much less than average current, the average *Ni*, or $N\bar{i}$ is the magnetic operating-point *Ni*. At the maximum allowable saturation, or minimum k_{sat}, the number of turns is

$$N_i = \frac{N\bar{i}(k_{sat})}{\bar{i}}$$

where $\bar{i}$ is the average winding current corresponding to k_{sat} and $N\bar{i}(k_{sat}) = \overline{H}(k_{sat}) \cdot l$, where *l* is the core magnetic path length and $\overline{H}(k_{sat})$ is the field intensity, as plotted in core catalogs to characterize saturation. (The plots are actually given by core manufacturers in the inverse form of $k_{sat}(\overline{H})$, with k_{sat} on the vertical axis of the graph.

Thus, the design range of *N* is between these limits;

$$N_{\lambda} \le N \le \min\{N_i, N_w\}$$

where N_w is the number of turns that fits the winding window. For full core utilization, power-loss and saturation are increased to the maximum allowable values by increasing $\Delta\lambda$ to increase N_{λ}, and average current is increased, to decrease Ni until $N = N_\lambda = Ni$ at full core utilization. Then maximum power is transferred from winding through the core to winding;

$$P = \Delta\phi_\lambda \cdot N\bar{i}(k_{sat}) \cdot f$$

where f is the frequency of the waveform driving the core. The above power equation is equivalent to setting the expressions for $N_\lambda = N_i$ and solving for power.

Given $\Delta\lambda$ and $\bar{i}$ from circuit design, and k_{sat} as input parameters to the design, they determine transfer power. Then the smallest core is a fully-utilized core, which has minimum $\Delta\phi_\lambda$ and minimum $N\bar{i}(k_{sat})$ such that $N_{opt} = N_\lambda = N_i$. This optimization scheme applies to coupled inductors, which have a winding → core → winding power flow. For transformers, the flow is more direct: winding → winding, a small amount of power transferred through the core from magnetizing current, Δi. Typically for transformers, magnetizing current ripple, $\Delta i_m << \hat{i}$, the winding peak current.

Transformer Core Utilization

How does core utilization apply in optimizing transformer magnetic design? Transformers differ from coupled inductors in that transformer waveforms are bipolar and the small-ripple assumption does not apply. The ripple factor,

$$\gamma = \frac{\Delta i/2}{\bar{i}} = \frac{\hat{i}_\sim}{I}$$

where I is the average on-time current, the current during Δt while the winding is driven by V_p. For a CCM waveform, $\gamma \le 1$, and each half-cycle of a bipolar waveform has $\gamma = 1$. For such a large γ, we can no longer let N_i be determined by the average Ni but by the peak $Ni = N\hat{i}$ instead. Then the magnetic operating-point of the core is at

$$\bar{B} = \frac{\hat{B}}{2}, \gamma = 1, \text{ unipolar (half-cycle)}$$

The peak is twice the average as is $\Delta B = \hat{B}$. Then the turns limit set by $\hat{B}$ is

$$N_i = \frac{N\hat{i}}{\hat{i}} = \frac{\Delta Ni}{\Delta i} = \frac{2 \cdot N\bar{i}}{2 \cdot I_p} = \frac{\Delta\phi_i / \mathcal{L}}{\Delta i}$$

where $\mathcal{L}$ is the inductance referred to the field of the core, or *field inductance*: A_L in core catalogs.

Besides the abandonment of the small-ripple assumption, another important difference for transformer design is that inductors have only magnetizing current, i_m, while transformers have both i_m and i_s, the winding transfer current. With two currents, i_m is controlled only by $\Delta\lambda$ and is independent of the transfer current;

$$\Delta i_m = \frac{\Delta\lambda}{L} = \frac{\Delta\lambda}{N^2 \cdot \mathcal{L}}$$

When substituted into the N_i equation,

$$N \le N_i = \frac{\Delta Ni}{\Delta i_m} = \frac{\Delta \phi_i / \mathcal{L}}{\left(\dfrac{\Delta \lambda}{N^2 \cdot \mathcal{L}} \right)}$$

Then solving for *N*,

$$N \ge N_i = \frac{\Delta \lambda}{\Delta \phi_i} \;, \; \Delta \phi_i = \Delta B \cdot A = \hat{B} \cdot A$$

Because Δi_m is dependent on *N*, N_i (like N_λ) becomes another lower bound on *N* so that

$$\max \{N_\lambda, N_i\} \le N \le N_w$$

where now the only upper limit on *N* is N_w. For maximum core utilization, $N_\lambda = N_i$ and

$$\frac{\Delta \lambda}{\Delta \phi_\lambda} = \frac{\Delta \lambda}{\Delta \phi_i} \Rightarrow \Delta \phi_\lambda = \Delta \phi_i \Rightarrow \hat{B}_\lambda = \hat{B}_i = \hat{B} \le B_{sat} \text{, max core utilization}$$

To both drive the core to maximum loss and saturation (minus a design margin, especially for ferrites), the waveform driving the core must have $\hat{B}_\sim = \hat{B}/2$, which is a waveform with $\gamma = 1$. To drive a ferrite core with such a large *B* ripple amplitude would ordinarily overheat the core, but there is another free parameter, the frequency, *f*. Transformer power transfer, unlike coupled inductors, does not depend on *f* according to

$$P = \Delta \phi_\lambda \cdot N\bar{i}(k_{sat}) \cdot f$$

leaving *f* as a free variable that can be reduced to a low frequency where core power loss is maximum with $\hat{B}_\sim = \hat{B}/2$. These are the conditions for full core utilization.

These conditions are suboptimal for switching converters with an inductor because at low *f* the inductance is suboptimally large. Consequently, full core utilization for transformers, while obtainable in low-frequency applications such as for grid-power distribution, is not a very relevant goal for switching converter transformers. What is usually achieved is to keep $N \ge N_\lambda$ to avoid overheating the core, and if the calculation of N_i is carried out, it is found to be less than N_λ and is thus not the constraining parameter. If the core does not overheat at high switching frequencies, it also does not saturate.

References

1. "Match Circuit and Field Resistances For Optimal Magnetics Design" by Dennis Feucht, How2Power Today, March 2011 issue.

2. "Utilizing Full Saturation and Power Loss To Maximize Power Transfer In Magnetic Components", by Dennis Feucht, How2Power Today, February 2011 issue.

3. *Power Magnetics Design Optimization* by Dennis Feucht, NOV17 rev., chapter 5, "Optimization," available for purchase at www.innovatia.com

About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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