

## ***Estimating Output Current Tolerance Of A Primary-Side-Regulated Constant-Current Flyback Converter (Part 2): Monte Carlo Analysis***

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Part 1 of this article presented an analytical model of the primary-side-regulated flyback converter operating under constant-current control. We also verified the accuracy of this model by comparing it against experimental results.<sup>[1]</sup> This model will allow us to estimate the accuracy of the flyback output current in LED driver applications. Now in part 2, we'll begin to use the model to determine the worst-case output current accuracy of the example flyback converter design by performing a Monte Carlo analysis.

### ***Worst-Case Circuit Analysis (WCCA)***

WCCA allows assessing the risk and margin of a given circuit design. Indeed, electronic circuits are designed using specific component and parameter values. But components are not perfect; their real values and parameters vary due to manufacturing tolerances and the effect of time and environment. Typically, when a WCCA is performed, the effects of initial tolerance, operating temperature, and aging are considered for the parts. In addition, space applications consider the effect of radiation on the part value.

For this analysis, we will only consider the effect of initial tolerance on the values for the resistors, capacitors and inductor. In addition, we will consider the effect of temperature and aging for the controller only, since all its parameters are specified for a junction temperature ranging from -40°C to 125°C and the datasheet limits includes aging tests.

WCCA encompasses several methods used to analyze the margins of a system. Among them, we have:<sup>[2,3]</sup>

- Extreme value analysis (EVA)
- Sensitivity analysis
- Monte Carlo analysis (MCA)

An EVA consists of finding the parameters that maximize and minimize the system/circuit output (the output current in our case) and stacking their effects to find the worst value of the circuit output. It requires having an explicit function to describe the system output variation with its parameters, so that we can deduce which direction (max/min) of parameter change results in the worst value of the circuit output. This can be done by inspection if we have a simple rational expression for the system. But for more complicated expressions involving perhaps non-linear functions, determining the worst set of parameters may be very difficult. For these difficult cases, we may use sensitivity analysis.

The worst case (most extreme) value of a monotonic system will occur when all of its parameters peak at their maximum or minimum value. Thus, the results of an EVA for such systems represent its worst possible performance characteristics and can consequently be very pessimistic. For a non-monotonic system, the min/max solutions of the function may not necessarily be found when all parameters are pushed to one extreme or the other; they might be found when some parameters are around the middle of their range.

A sensitivity analysis will allow studying the contribution of each parameter (for example  $V_{REF}$ ,  $R_{sense}$ ,  $L_p$ , ...) to the system output variation (ex:  $I_{out}$ ). Circuit sensitivity is a measure of how a circuit will react to an incremental change in a single component parameter when all other parameters are held constant. Practically, it involves differentiation of the system output with respect to all the considered variables. Sensitivity analysis is typically used when it is difficult or impossible to determine by inspection the worst combination of circuit parameters for the extreme value analysis.

Monte Carlo analysis is a statistical methodology generally based on circuit simulation to evaluate the performance of a system. It is an assessment of confidence intervals of a sampled data system. Circuit

simulation programs such as SPICE generally provide scripts for Monte Carlo simulation. This simulation consists of generating random values following a defined probability distribution (normal or uniform) for each parameter and running several simulations to calculate the output value of the system.

A Monte Carlo simulation usually produces a histogram showing the spread of output values over a number of runs. This histogram often looks like a Gaussian curve and the simulator also gives the output mean and standard deviations. The results of the Monte Carlo simulation are analyzed using one of these two methods:<sup>[4]</sup>

- Tolerance intervals for normal distribution (see the Appendix 1 for terminology)
- Distribution-free tolerance intervals.

For both methods, the analysis result will be an interval for the output value representing a population coverage associated with a confidence level.

The distribution-free tolerance interval method is used if the SPICE Monte Carlo output is not Gaussian. In this method, the tolerance interval (see Appendix 1 again) is given by the smallest and the largest data value from a sample of N measurements or simulations in our case. The number of measurements N will set the confidence and coverage levels. Reference [6] gives a way to calculate the number of simulations needed in order to use the minimum and the maximum result points as the tolerance interval for a given population coverage/confidence level. Based on this, it is determined that:

- 2,456 simulations must be carried out to get a 99.73% population coverage and 99% confidence level
- 1,439 simulations must be carried out to get a 99.73% population coverage and 90% confidence level
- 388 simulations must be carried out to get a 99% population coverage and 90% confidence level.

Having a single desktop on hand for the simulations, 2,456 and 1,439 simulations are not practical as it would take some days to complete this many simulations of the PSR constant-current flyback with only one desktop. On the other hand, 388 simulations are achievable.

Having an analytical model for our system will allow carrying out an MCA much faster than with a SPICE simulator. Indeed, the PSR flyback is a switched-mode power supply as opposed to a linear converter. A transient simulation must be performed first until steady-state is reached to measure the output current value. Only a few seconds will be needed for Mathcad to calculate the output-current distribution considering 2,456 or more different values for the parameters while several hours would be needed with SPICE to run 2,456 simulations in a transient analysis.

In order to perform the WCCA, we need to identify the parameters which will be considered for the study. The analytical model presented in the previous part contains the variables influencing the output current. These parameters are summarized in Fig. 1 which shows a schematic of the flyback constant-current converter with the components or parameters considered for the study highlighted in blue. In addition, there is also the propagation delay  $t_{prop}$  to consider.

The goal of the study will be to calculate the output current dispersion at a given input voltage and output load, so  $V_{in}$  and  $V_{out}$  will be considered constant. The turns ratio  $N_{sp}$  will also be considered constant. For the controller, we will treat the reference voltage  $V_{REF}$  and the line feedforward voltage to current converter  $K_{LFF}$  as varying parameters. For the components around the controller,  $R_{BOU}$  and  $R_{BOL}$  are the line voltage sensing resistors used to build the line feedforward offset current affecting the current sense voltage through  $R_{LFF}$ .

The flyback transformer primary and leakage inductances ( $L_p$ ,  $L_{leak}$ ) and the sense resistor directly influence the current setpoint so they are also within the scope of this study. The RCD clamp resistor fixes the clamping voltage, which influences the output current as shown in equation (20) in part 1.<sup>[1]</sup> So  $R_{clamp}$  also needs to be studied.

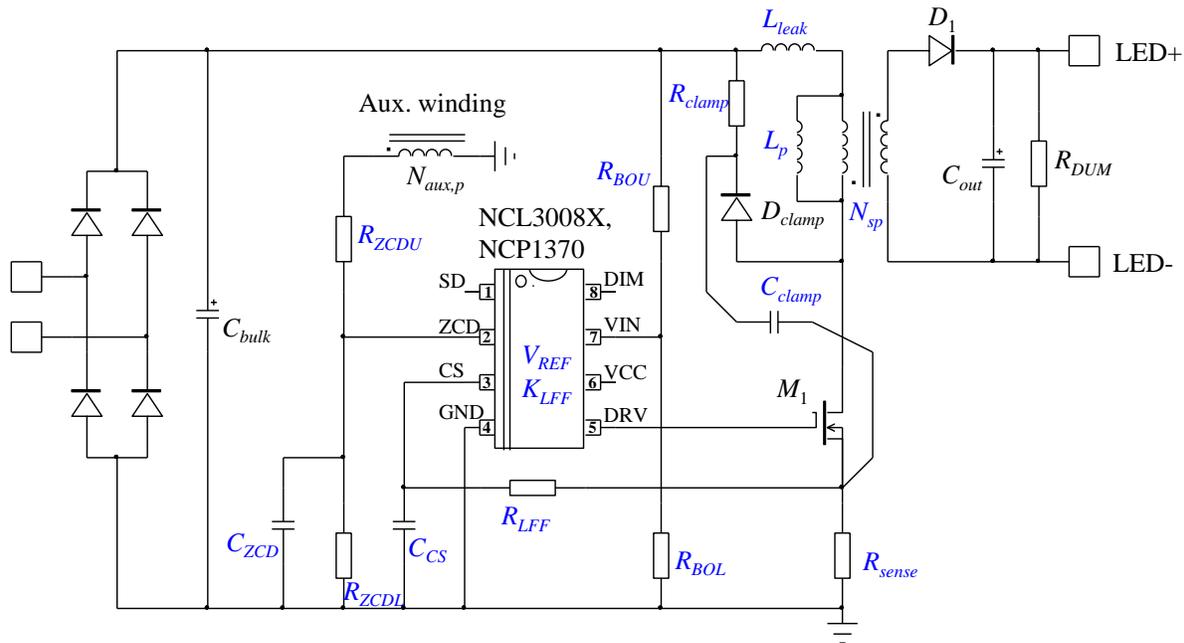


Fig. 1. A primary-side constant-current flyback converter with its components for the worst-case-analysis study.

Also, only the initial tolerances of the parameters are considered for the study (except for the controller parameter as explained in the previous paragraph). The goal is to provide an estimate of the expected range of the output current at the beginning of life of the converter.

### Monte Carlo Analysis

Table 1 summarizes the components that will be considered with their dispersions for the study. (See Appendix 2 for additional parameter values that are not used in the MCA, but are otherwise used in the analysis.)

The next step consists of generating vectors with random values following a certain distribution (normal or uniform) for each parameter. In order to be able to choose the correct distribution for each parameter, we need knowledge of the process used to manufacture the components or to have production data for the components. We have this knowledge for the controller, but not for the resistors or the transformer.

Reference [3] suggests that when the distribution function of a parameter is unknown, a normal distribution (also known as Gaussian) should be assumed. On the other hand, reference [7] suggests using uniform distributions for all variables as a starting point of the analysis. In the end, I chose to assign a normal distribution to  $R_{sense}$ ,  $R_{LFF}$ ,  $R_{clamp}$ ,  $R_{BOU}$ ,  $R_{BOL}$ ,  $K_{LFF}$ ,  $R_{ZCDU}$ ,  $R_{ZCDL}$ ,  $C_{ZCD}$  as well as  $t_{prop}$ .

The parameter  $V_{REF}$  is trimmed in order to provide a tight reference for the output current regulation, thus, a uniform distribution will be assigned to this parameter.

Table 1. Parameter variation

| Parameter   | Description  | Typical value      | Variation (%)  | Variation   | Distribution |
|-------------|--|--------------------|--|---|--------------|
| $V_{REF}$   | Constant current reference                                   | 500 mV             | $\pm 1.6\%$<br>( $T_j = 0^\circ\text{C}$ to $85^\circ\text{C}$ )   | $\Delta V_{REF} = \pm 8 \text{ mV}$               | Uniform      |
| $R_{sense}$ | Sense resistance   | 3 $\Omega$         | $\pm 1\%$  | $\Delta R_{sense} = \pm 30 \text{ m}\Omega$       | Gaussian     |
| $L_p$       | Primary inductance   | 1860 $\mu\text{H}$ | $\pm 7\%$  | $\Delta L_p = \pm 130.2 \text{ }\mu\text{H}$      | Gaussian     |
| $R_{LFF}$   | CS offset resistor in $V_{CS(offset)}$ parameter             | 1.6 k $\Omega$     | $\pm 1\%$  | $\Delta R_{LFF} = \pm 16 \text{ }\Omega$          | Gaussian     |
| $K_{LFF}$   | Controller parameter for generating $V_{CS(offset)}$         | 17 $\mu\text{A/V}$ | $\pm 12\%$<br>( $T_j = -40^\circ\text{C}$ to $125^\circ\text{C}$ ) | $\Delta K_{LFF} = \pm 2.04 \text{ }\mu\text{A/V}$ | Gaussian     |
| $R_{BOU}$   | Brown-out resistance (needed to calculate $V_{CS(offset)}$ ) | 9.9 M $\Omega$     | $\pm 1\%$  | $\Delta R_{BOU} = \pm 99 \text{ k}\Omega$         | Gaussian     |
| $R_{BOL}$   | Brown-out resistance (needed to calculate $V_{CS(offset)}$ ) | 100 k $\Omega$     | $\pm 1\%$  | $\Delta R_{BOL} = \pm 1 \text{ k}\Omega$          | Gaussian     |
| $R_{clamp}$ | RCD clamp resistance   | 270 k $\Omega$     | $\pm 5\%$  | $\Delta R_{clamp} = \pm 13.5 \text{ k}\Omega$     | Gaussian     |
| $t_{prop}$  | Propagation delay  | 140 ns             | $\pm 14\%$   | $\Delta t_{prop} = \pm 19.6 \text{ ns}$           | Gaussian     |
| $R_{ZCDU}$  | ZCD pin upper resistor                                       | 24 k $\Omega$      | $\pm 1\%$  | $\Delta R_{ZCDU} = \pm 240 \text{ }\Omega$        | Gaussian     |
| $R_{ZCDL}$  | ZCD pin lower resistor                                       | 8.2 k $\Omega$     | $\pm 1\%$  | $\Delta R_{ZCDL} = \pm 82 \text{ }\Omega$         | Gaussian     |
| $C_{ZCD}$   | ZCD pin capacitor  | 22 pF              | $\pm 5\%$  | $\Delta C_{ZCD} = \pm 1.1 \text{ pF}$             | Gaussian     |
| $C_{CS}$    | CS pin capacitor   | 27 pF              | $\pm 5\%$  | $\Delta C_{CS} = \pm 1.35 \text{ pF}$             | Gaussian     |

We consider a dimension of 2,456 elements for the vectors meaning that the software will generate a vector containing 2,456 random values for each parameter. As an example, using Mathcad, the command line **runif(size, lower\_limit, higher\_limit)** returns a vector with *size* elements falling between *lower\_limit* and *higher\_limit* following a uniform distribution.

To define a normal distribution with Mathcad, the standard deviation of the considered parameter is needed. Since we will assign a normal distribution to the resistors and primary inductance, we must extract the standard deviation from the initial tolerance of the parameter.

Fig. 2 plots the probability density function (PDF) of a random variable having a standard normal distribution. The standard normal distribution is a normal distribution with a mean value equal to zero and a unity standard deviation. It can be observed that about 99.73% of the random values drawn from a normal distribution are within  $\pm 3$  standard deviations ( $\sigma$ ) away from the mean (the three-sigma rule). Seen another way, the probability of a random variable value being outside  $\pm 3\sigma$  from the mean is 0.0027. We can use this as a starting point and place the limits at  $\pm 3\sigma$ . Based on this, we can define the standard deviation of  $R_{sense}$  as:

$$\sigma_{R_{sense}} = \frac{\Delta R_{sense}}{3} \quad (1)$$

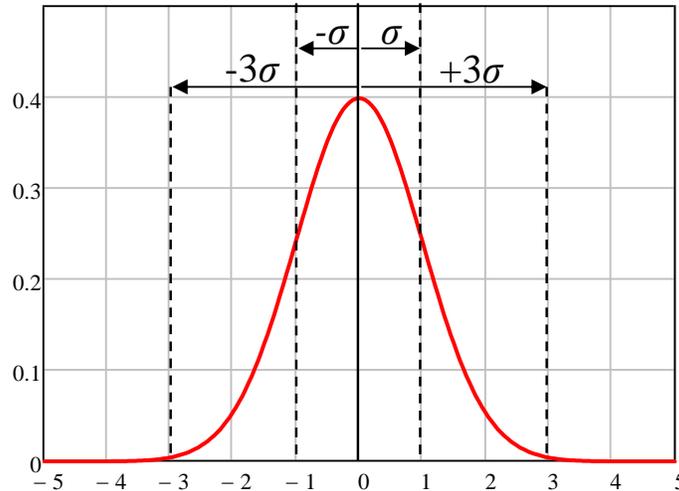


Fig. 2. A standard normal distribution.

Using Mathcad, the normal distribution law is assigned to the parameter using the keyword **norm**.

If the random values of  $R_{sense}$  follow a Gaussian distribution, 99.73% of the values are within  $\pm 3$  standard deviations: For  $R_{sense}$ , we obtained the histogram shown in Fig. 3a.

To plot the histogram, the values in the  $R_{sense}$  array have been sorted into 50 bins (see the Appendix 1 for the terminology). The span of one bin is roughly 1.4 mΩ. The vertical axis represents the number of values falling in a particular bin.

For example, about 275 samples are in the range of 2.9993 Ω to 3.0007 Ω. Looking at this graph, it can be observed that some values are outside the  $\pm 1\%$  limits of  $R_{sense}$  represented by  $R_{sense,LL}$  and  $R_{sense,UL}$  on the histogram. Since the manufacturer's specification is also statistical, we cannot be entirely sure that we will not find resistor values outside this  $\pm 1\%$  initial tolerance range, unless 100% of the resistors are tested in production. If 100% of the resistors are tested, it may be more precise to assume a truncated normal PDF (Fig. 3b).

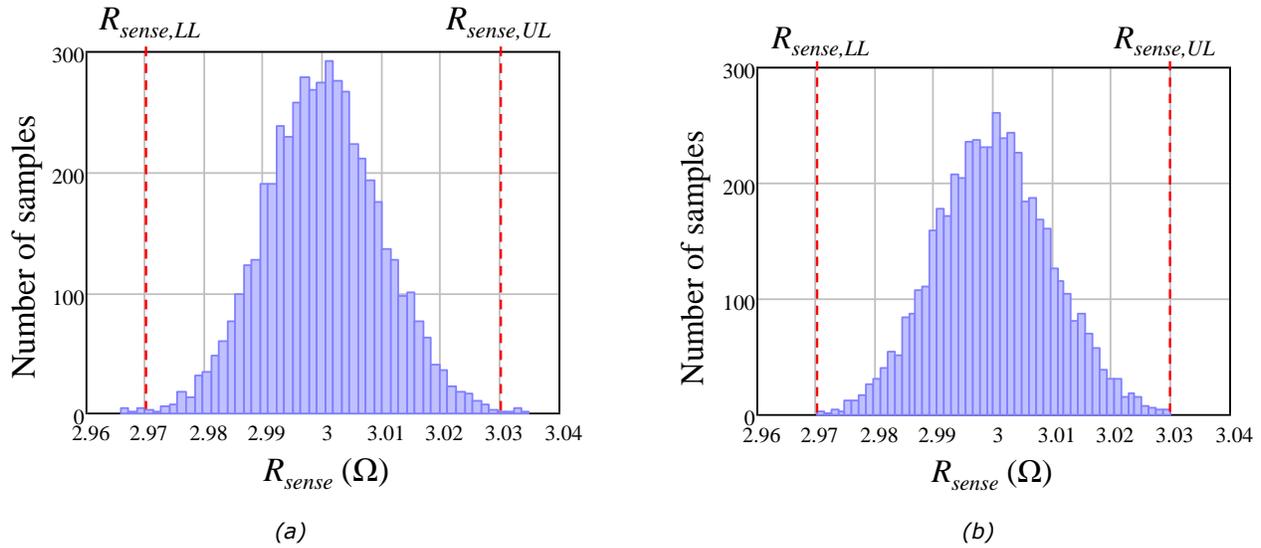


Fig. 3. Sense resistor normal distribution (a) and sense resistor normal distribution clamped at  $\pm 1\%$  (b).

After defining the vectors for each of the parameters mentioned in Table 1, we can calculate the vectors for  $I_{L,pk}$  and  $V_{clamp}$  and finally obtain the histogram for the possible values of  $I_{out}$  shown in Fig. 4. Using Mathcad, we can extract the average output current, its maximum and minimum values at  $V_{in} = 162$  V and  $V_{out} = 20$  V:

- $I_{out,mean} = 479$  mA
- $\sigma_{Iout} = 4.99$  mA
- $I_{out,min} = 465.3$  mA
- $I_{out,max} = 492.2$  mA.

The resulting output current distribution is not Gaussian. Thus, we will use the distribution-free interval method to interpret the results. For 2,456 samples, the interval for the output current is given by  $[I_{out,min}; I_{out,max}]$  with a 99.73%/99% population coverage/confidence level.

Initially, the targeted output current for this design is 480 mA:  $I_{out,nom} = 480$  mA.

Finally, if we define the accuracy of  $I_{out}$  as a deviation from the target value  $I_{out,nom}$ , we have:

$$\delta I_{out-} = 100 \frac{I_{out,min} - I_{out,nom}}{I_{out,nom}} = 100 \frac{465.3\text{m} - 480\text{m}}{480\text{m}} = -3.1\% \quad (2)$$

$$\delta I_{out+} = 100 \frac{I_{out,max} - I_{out,nom}}{I_{out,nom}} = 100 \frac{492.2\text{m} - 480\text{m}}{480\text{m}} = 2.5\% \quad (3)$$

The output current tolerance is around  $\pm 3\%$  considering a temperature range of  $0^\circ\text{C}$  to  $85^\circ\text{C}$  for the controller parameters. In order to do a comparison against SPICE simulation results, we also used the analytical model to get the output current spread considering 388 random values for all the parameters of interest, which results in a 99%/90% population coverage/confidence level. The results are summarized in Table 2.

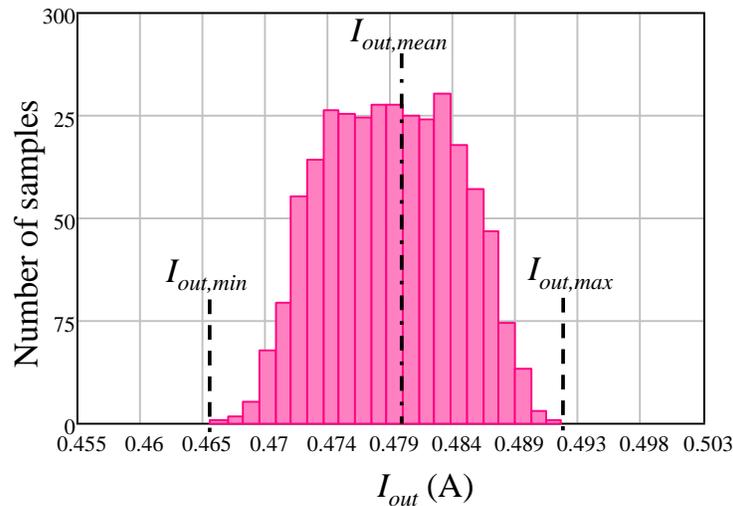


Fig 4. Output current distribution.

For comparison, the same exercise was carried out using SIMPLIS, which is a simulation program specifically designed to handle the simulation challenges of switched-mode power supplies. Like SPICE, SIMPLIS works at the component level but is able to perform a transient analysis of a switching circuit 10 to 50 times faster owing to its piece-wise-linear (PWL) modeling approach.

A behavioral simulation model of the PSR controller (NCL30082) was developed to perform the transient simulations needed to find the steady-state value of the output current. A Monte Carlo run was made of 388 transient simulations. For each simulation, a random value chosen from either a normal distribution or a uniform distribution was assigned to the parameters of interest for the study. The averaged output current was computed for each transient simulation over 50 switching cycles and finally, we obtained the results summarized in Table 2.

The simulator directly computes the average and the standard deviation values of the output current from the 388 simulations. A Monte Carlo simulation lasted approximately 2 hours, while less than 1 minute was needed to get results from the analytical model over 388 or 2,456 random values.

Table 2. MCA results of analytical model versus SPICE simulator (SIMPLIS).

|   | Spice simulator | Analytical Model |               |
|---|-----------------|------------------|---------------|
|   | 388 runs        | 388 samples      | 2,456 samples |
| Population coverage / Confidence level (%/%)                    | 99/90           | 99/90            | 99.73/99      |
| $I_{out,mean}$ (mA)   | 479.2           | 479.1            | 479           |
| $\sigma_{I_{out}}$ (mA)   | 4.94            | 5.00             | 4.93          |
| $I_{out,min}$ (mA)  | 466.5           | 467.5            | 465.3         |
| $I_{out,max}$ (mA)  | 492.8           | 493.8            | 492.2         |
| $I_{out}$ tolerance $\delta I_{out-}$ ; $\delta I_{out+}$ (%;%) | -2.8 and +2.7   | -2.6 and +2.9    | -3.1 and +2.5 |

We see that the SIMPLIS results are very close to the results given by the analytical model.

## Conclusion

When building a particular converter, it is always important to check for the performance variability knowing the natural spread of components used in the circuit. While powering an LED string from the mains may look like a trivial exercise, ensuring that these LEDs receive a well-regulated and constant current when powered by the end-user actually involves a great deal of statistical analysis. Running this analysis is of utmost importance if you want to ensure the highest product quality.

This part 2 has explored the Monte Carlo analysis, which is one of the worst-case circuit analysis techniques, to estimate the output current tolerance of a PSR CC flyback converter. This Monte Carlo analysis has shown that the output current variation is roughly within  $\pm 3\%$  of the mean output current for a population coverage/confidence level of 99.73%/90%.

The third part of this article will present an extreme value analysis and a sensitivity analysis of the example PSR CC flyback.

## References

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## Appendix 1: Terminology For Statistical Analysis

### Tolerance Interval

A *tolerance interval* for a measured quantity represents the interval in which you feel "some level of confidence" that a specified fraction of the population's value lies in that interval, based on a sample that you measured from this population.<sup>[5]</sup> Usually, 99.73/99 or 99/90 is used as population coverage/certainty specification. It is expressed as a percentage. The number of simulations run influences the length of the tolerance interval obtained for a given population coverage/confidence level.

### Tolerance Interval For Normal Distribution

The tolerance interval for normal distribution method is used when the population (the simulation results in our case) follows a Gaussian distribution. Reference [4] includes a nice calculator giving the tolerance interval that

results from a given number of simulations, confidence level and population coverage. The tolerance interval is expressed as a mean plus or minus a certain number of standard deviations.

### What Is A Bin? How To Read A Typical Distribution Histogram

The bar graph in Fig. 5 plots the repartition of a random discrete variable. The random variable values are split in N intervals or bins. The bar width in the graph represents the interval size. The vertical axis represents the number of elements within the different intervals. As an example, the reference voltage  $V_{REF}$  value repartition is plotted using 16 bins with 1-mV width. Reading this graph, we can see that approximately 315 samples fall in the range of [499.5 mV, 500.5 mV].

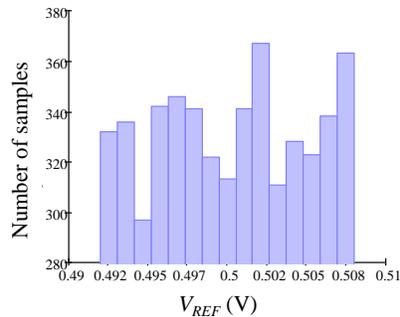


Fig. 5.  $V_{REF}$  Histogram using 16 bins.

### Appendix 2: Additional Design Information

Table summarizing the parameters values used to obtain the typical value of the output current  $I_{out}$  for the Monte Carlo Analysis.

Table. Flyback converter design values.

| Parameter                        | Value          |
|----------------------------------|----------------|
| $V_{out}$                        | 20 V           |
| $V_{in}$                         | 162 V          |
| $V_f$                            | 0.5 V          |
| $L_p$                            | 1870 $\mu$ H   |
| $k_{leak}$                       | 0.01           |
| $N_{sp}$                         | 0.17           |
| $C_{lump}$                       | 60 pF          |
| $n_v$                            | 1              |
| $R_{clamp}$                      | 270 k $\Omega$ |
| $R_{sense}$                      | 3 $\Omega$     |
| $R_{ZCDU}$                       | 24 k $\Omega$  |
| $R_{ZCDL}$                       | 8.2 k $\Omega$ |
| $C_{ZCD}$                        | 22 pF          |
| $t_{ZCD}$                        | 140 ns         |
| $R_{BOU}$                        | 9.9 M $\Omega$ |
| $R_{BOL}$                        | 100 k $\Omega$ |
| $K_{LFF}$                        | 17 $\mu$ A/V   |
| $R_{LFF}$                        | 1.6 k $\Omega$ |
| $t_{prop}$                       | 140 ns         |
| $C_{CS}$                         | 27 pF          |
| $I_{CCS}$ at $V_{in} = 162$ V dc | 6.95 $\mu$ A   |

## About The Author



*Stéphanie Cannenterre has been an application engineer at ON Semiconductor in Toulouse, France since 2006. In this role, she has developed several controllers and ASICs mainly for flyback converters. For example, she has developed the NCL3008X controller family dedicated to LED lighting. She is currently developing controllers for single-stage PSR CC/CV converters dedicated to LED lighting to further extend the NCL3008X family. Stéphanie graduated from the INSA (Institut National des Sciences Appliquées) engineering school in Toulouse with an MSEE degree. She holds five patents on power conversion.*

For more information on current-mode control, see How2Power's [Design Guide](#), locate the Design Area category and select "Control Methods".