Magnetic-Electric Analogs Relate Magnetic Fields To Familiar Circuit Quantities

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Circuit designers are usually able to think more easily about the circuit behavior of capacitors than inductors. Inductance is the *dual* of capacitance; exchange *v* and *i* in capacitor equations and they apply to inductors. This dualism can be extended to circuit laws too, and the three most basic laws of circuits (Ohm's Law, KVL and KCL) have magnetic counterparts.[1] We will derive these equivalents and examine them. Finally, we'll explain how the magnetic counterparts to circuit laws can be applied in transformer design.

Electric And Magnetic Dualism

We learn early in our engineering education that the voltage across a capacitor cannot change instantaneously because it takes time for current to change the charge and hence the voltage. The dual for inductors is that the current in an inductor cannot change instantaneously because it takes time for the voltage across an inductor to change the flux. The *v-i* relationship for a capacitor is

$$i = C \cdot \frac{dv}{dt}$$

whereas for an inductor, *v* and *i* are interchanged, resulting in

$$v = L \cdot \frac{di}{dt}$$

The basic equations relating *C* and *L* to geometric properties also have an analog for resistance, or rather, its inverse, conductance, *G*. The three equations are

$$C = \frac{\varepsilon \cdot A}{l}$$

$$\mathcal{L} = \frac{\mu \cdot A}{l}$$

$$G = \frac{\sigma \cdot A}{l}$$

where ε is permitivity (or dielectric constant), μ is permeability, σ is conductivity, *A* is area and *l* is length or thickness.

For the three basic circuit elements, the form of their equations is identical, differing only in the material parameters: ε, μ, or σ. The $\mathcal{L}$ quantity is field-referred inductance (*permeance* or per-turn-square inductance), the inductance of a single turn. It is the circuit inductance referred to the magnetic field. There is no capacitance analog for this aspect of inductors because electric fields do not couple as do magnetic fields. Except for fringing, electric field lines are constrained to stay largely between capacitor plates. But magnetic fields form closed magnetic loops instead and any closed conductive loop within the magnetic-field loop is coupled to it.

Some analogous electric-magnetic circuit quantities are given in the table.

Table. Electric circuit quantities and their magnetic counterparts.

Electric circuit quantity	Magnetic field quantity
Current, i	Magnetic field flux, ϕ
Voltage, v	Field current, $Ni = N \cdot i$ ("MMF")
Conductance, G	Field inductance, $\mathcal{L}$ (Permeance)

Magnetic flux is like current in that it flows in closed loops through materials of high permeability (such as iron or ferrites) just as current flows through materials of high conductivity (conductors). MMF, which historically stood for "magnetomotive force," is not a force but a magnetic quantity analogous to voltage. It is more accurately named the *field-referred current*, or simply *field current*. A current flowing through N loops or *turns* of a coil produces a "scalar magnetic potential" (the correct name for this quantity) that is related to the flux by the analog of Ohm's Law—a magnetic Ohm's Law (MΩL):

$$\phi = \mathcal{L} \cdot (N \cdot i) \qquad \text{M}\Omega\text{L}$$

MΩL is analogous to Ohm's Law when expressed using conductance:

$$i = G \cdot v \qquad \Omega\text{L}$$

$Ni = N \cdot i$ can be viewed as the electric-circuit inductor terminal current, i, referred to the magnetic field circuit, where it is N times larger. Ni causes flux to flow in a closed magnetic loop, and the amount depends on the field inductance, $\mathcal{L}$, around the loop.

MΩL can also be expressed referred to the electric circuit as terminal quantities, where flux ϕ is referred to the circuit by turns, N as the *circuit-referred flux*, or simply *circuit flux*, λ:

$$\lambda = N \cdot \phi = N \cdot (\mathcal{L} \cdot N \cdot i) = (N^2 \cdot \mathcal{L}) \cdot i = L \cdot i$$

Inductance, L, is the field inductance times the turns squared. λ and charge, q, are both electric circuit quantities, and each relates *electrically* to L or C through the relations:

$$\lambda = L \cdot i$$

$$q = C \cdot v$$

Both λ and q are electric-circuit quantities as used in electric circuit analysis. λ is referred to the magnetic circuit or field (by referral factor N) as ϕ, just as current is also referred to the field as field current by N. When both quantities are combined in inductance,

$$L = \frac{\lambda}{i} = \frac{N \cdot \phi}{(N \cdot i)/N} = N^2 \cdot \frac{\phi}{Ni} = N^2 \cdot \mathcal{L}$$

Magnetic Kirchhoff's Laws

Kirchhoff's current and voltage laws also have magnetic analogs. The outline of their derivations will be given here. Start with the analog of Kirchhoff's current law (KCL). For electric circuits,

$$\sum_{node} i = 0 \qquad \text{KCL}$$

The sum of the currents entering and leaving a node is zero. In other words, nodes do not accumulate charge; what comes in must go out. To derive the magnetic analog (MKCL), we begin with one of the four basic Maxwell fields equations, Gauss's Law:

$$\oint_S \mathbf{B} \cdot \hat{\mathbf{n}} \cdot da = 0$$

where **B** is the magnetic-field density vector (in units of V·s/m²) followed by a unit normal vector pointing outward at right angles from the surface through which the flux goes, and integrated over the area, *S*, of the closed surface.

For those not familiar with vector calculus, this says that a closed surface (imagine a cylinder) with a magnetic field of flux density *B* going in one end of it and coming out the other end (perpendicular to the surface) must have a total flux of zero. In other words, the amount of B-field flux coming out must equal the amount going in to the closed surface somewhere else. Like nodes of a circuit, the closed surface "node" must have as much flux coming out of it downstream from the direction of flux flow as goes in upstream.

When the geometry of the magnetic circuit is simple—and for many practical magnetics problems, it is—the above equation reduces to

$$\sum_{\text{"node"}} B \cdot A = \sum \phi = \sum \boldsymbol{\mathcal{L}} \cdot (Ni) = 0 \qquad \text{MKCL}$$

The magnetic flux,

$$\phi = B \cdot A$$

The flux density, *B*, over cross-sectional area *A* of the flux flow path is the total flux through that area. These expressions for magnetic flux must sum to zero. In other words, flux, like current, can only exist along closed paths. That is why most magnetic cores have closed paths for the flux, such as E-cores, pot cores, or toroids. Open cores, such as drum cores, contain the flux over only part of the magnetic circuit, which is then completed through air or whatever ambient medium the core is immersed in.

Kirchhoff's voltage law states that the sum of voltages around a closed loop is zero:

$$\sum_{loop} v = 0 \qquad \text{KVL}$$

By convention, voltage drops (+ to −) are positive and sources or rises (− to +) are negative. The magnetic analog is derived from the relevant part of another of Maxwell's equations, Ampere's Law (in part):

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot \hat{\mathbf{n}} da = Ni$$

To envision this equation, imagine a closed loop enclosing a planar surface, like a hoop, through which magnetic flux flows. Around the periphery of the hoop itself flows field current, *Ni*. The equation can be simplified for simpler geometry to

$$\sum_{loop} H \cdot l = N \cdot i$$

where *N* accounts for multiple loops in series with circuit current *i* flowing around them. *H* is the magnetic field intensity or strength, in units of A/m, and *l* is the circumference of the flux loop which is the magnetic path.

The magnetic field quantities *B* and *H* are related by the permeability;

$$B = \mu \cdot H$$

Applying this materials relation to the above equation;

$$\sum_{loop}\left(\frac{B}{\mu}\right)\cdot l = \sum\frac{B\cdot A}{\left(\frac{\mu\cdot A}{l}\right)} = \sum\frac{\phi}{\mathcal{L}} = N\cdot i = Ni$$

And by MΩL,

$$\frac{\phi}{\mathcal{L}} = Ni$$

The sum of the magnetic *Ni* drops around the loop must equal the source on the right side of the equation;

$$\sum_{loop} Ni = 0 \qquad \text{MKVL}$$

where *Ni* on the right is included as a source. Together, the wire loops through which the magnetic flux loop travels have a net *Ni* of zero. For example, if a transformer has a 10-turn primary and 5-turn secondary, and if 1 A is flowing in the primary, then 2 A must flow in the secondary in a direction that cancels the $N \cdot i$ of the primary. If the secondary winding is left open, the *Ni* drop in the core around the magnetic path equals the *Ni* of the winding source and the magnetic component becomes an inductor.

Application

The magnetic form of the basic circuit laws are useful when designing magnetic components. When transformers and inductors are used as circuit components, their terminal equations can be used and the field equations ignored. However, a good designer has some insight into what is going on within.

In transformer design, the primary (the source winding) of N_p turns and current i_p creates a *B*-field in the magnetic core. Ferrite and powdered-iron core databooks give, for each core, the effective values of flux-path cross-sectional area, *A*, and path length, *l*. They also give relative permeability, μ_r ($\mu = \mu_r \cdot \mu_0$, where $\mu_0 \cong 0.4 \cdot \pi$ μH/m ≅ 1.26 μH/m) or alternatively, the more useful quantity, field inductance, $\mathcal{L}$ (as A_L), for each air-gap width. Additionally, saturation values of *B* and magnetic power-loss curves are given in core catalogs. From this data and MΩL, the number of turns *N* can be calculated for a given core loss from

$$\hat{B}_{\sim} = \frac{\mathcal{L}\cdot(N\cdot\hat{i}_{\sim})}{A}$$

where $\hat{B}_{\sim}$ is the *B* ripple amplitude of the core loss curves on the horizontal axis and $\hat{i}_{\sim}$ is the current ripple amplitude. Core size affects *A* and $\mathcal{L}$ values while $\hat{B}_{\sim}$ for a given power loss and frequency depends on the core material. *N* is also a design parameter, which can be optimized as part of the magnetic design of the component.

For an air gap in a magnetic path, the gap has a different μ (for air, $\mu \cong \mu_0$) than the core material. MKVL is applied by summing the source *Ni* from the driven winding, and *Ni* drops around the flux loop. These drops are in the core and the air gap. Each "conducts" the same flux (the analog of current) but with different values of $\mathcal{L}$, the analog of conductance. ($1/\mathcal{L}$ is *reluctance*, the analog of resistance.) Then applying MKVL,

$$Ni = \left(\frac{\phi}{\mathcal{L}_{\text{core}}} + \frac{\phi}{\mathcal{L}_{\text{gap}}}\right)$$

E-cores have three legs. *Ni* is applied to the center leg, and two flux loops, one per outer leg, result in two magnetic circuits. MKVL can be applied to each circuit, with the common source.

The three basic magnetic circuits laws are the foundation for analysis and design of magnetic circuits. Keep in mind that the magnetic path is well contained within the core material only if the material permeability, $\mu >> \mu_0$ of the surrounding air. Then the core geometric parameters of path cross-sectional area, *A*, and path length, *l*, apply. A more complete treatment of the subject is found in the beginning chapters of most magnetics or motor textbooks, or in the author's book.[2]

References

1. "Magnetic Component Theory Is Simpler If You Grasp Reference Frames" by Dennis Feucht, How2Power Today, June 2017 issue.

2. *Power Magnetics Design Optimization* by Dennis Feucht, available for purchase at www.innovatia.com.

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About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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