

## Selecting An Inductor Value For A DC-DC Boost Converter

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The boost topology is fundamental in the field of power electronics, but selection of the inductor value is not always as straightforward as often assumed. In the dc-dc boost converter, the chosen inductor value ( $L_1$  in Fig. 1) has an impact on input current ripple, output capacitor size and transient response. Choosing the value correctly can help in optimizing the size and cost of the converter and ensure operation in the desired conduction mode.

In this article, methods for calculating the inductor value to maintain the required ripple current and chosen conduction mode over a range of input voltage are described. A mathematical approach for computing the upper and lower mode boundaries of input voltage is presented, and the use of the WebDesigner online design tool from ON Semiconductor to accelerate these design steps is discussed.

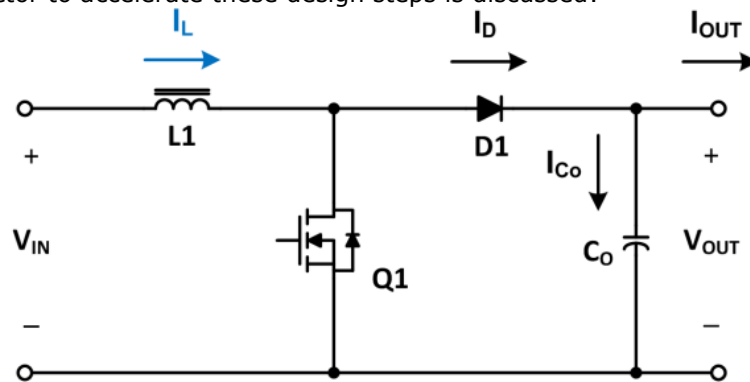


Fig. 1. Simplified schematic of boost converter.

### Conduction Mode

The conduction mode of the boost converter is determined by the magnitude of peak-to-peak inductor ripple current ( $\Delta I_L$ ) relative to the dc input current ( $I_{IN}$ ). This ratio can be defined as inductor ripple factor ( $K_{RF}$ ). Greater inductance results in less ripple current and lower  $K_{RF}$ .

$$K_{RF} = \frac{\Delta I_L}{I_{IN}} \quad (1)$$

$$\text{where } I_{IN} = \frac{P_{IN}}{V_{IN}} \quad (2)$$

In continuous conduction mode (CCM), instantaneous inductor current never reaches zero during a normal switching cycle (Fig. 2a). CCM is therefore maintained when  $\Delta I_L$  is less than twice  $I_{IN}$ , or  $K_{RF} < 2$ . Either the MOSFET or diode must be conducting in CCM. This mode is usually preferred for medium- and high-power converters to minimize the peak and RMS currents in the components.

Discontinuous conduction mode (DCM) occurs when  $K_{RF} > 2$  and the inductor current is allowed to decay to zero during each switching cycle (Fig. 2b). Inductor current remains at zero until the start of the next switching cycle, with neither the diode nor MOSFET in conduction. This nonconduction time will be referred to as  $t_{idle}$  here. DCM offers a smaller inductor value and eliminates output diode reverse-recovery losses.

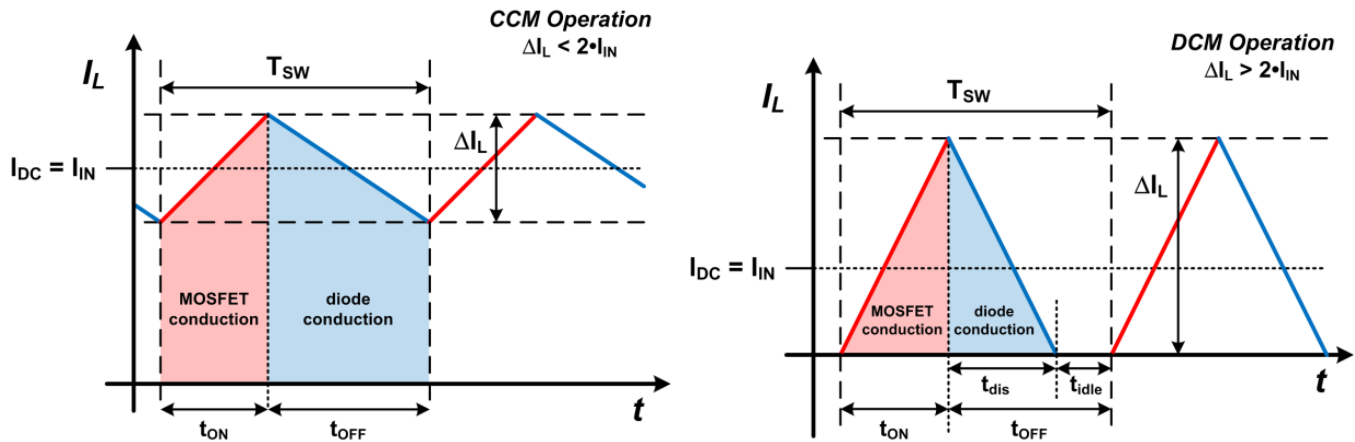


Fig. 2. Comparing boost converter operation under continuous conduction mode (CCM) (a) and discontinuous conduction mode (DCM) (b). In CCM, the inductor current never goes to zero during a switching cycle, but in DCM, it does.

When  $K_{RF} = 2$ , the converter is said to be in critical conduction mode (CrCM) or boundary conduction mode (BCM). In this mode, inductor current reaches zero at the end of the cycle, just as the MOSFET is turned on at the beginning of the next cycle. For applications requiring a range of input voltage ( $V_{IN}$ ), fixed-frequency converters are typically designed to operate in a single desired conduction mode (CCM or DCM) over the specified range of  $V_{IN}$  with max load.

As load is reduced, CCM converters will eventually enter DCM operation. The load at which the conduction mode changes for a given  $V_{IN}$  is the *critical load* ( $I_{CRIT}$ ). The inductor value resulting in CrCM/BCM at a given  $V_{IN}$  is known as *critical inductance* ( $L_{CRIT}$ ), typically specified at max load.

### Ripple Current Vs $V_{IN}$

For the boost converter with a fixed output voltage operating in continuous-conduction mode, it is well known that maximum inductor ripple current occurs when input voltage is half of output voltage ( $V_{OUT}$ ), i.e. when duty cycle ( $D$ ) is 50% (Fig. 3). This can be shown mathematically by setting the derivative (slope of the tangent) of the ripple current with respect to  $D$  equal to zero and solving for  $D$ . For the sake of simplicity, it is assumed that converter efficiency is 100%.

With

$$D = t_{ON} \cdot F_{SW} \quad (3)$$

$$F_{SW} = \frac{1}{T_{SW}} \quad (4)$$

$$\Delta I_L = \frac{V_{IN} \cdot D}{L \cdot F_{SW}} \quad (5)$$

and by inductor volt-second balancing for CCM or CrCM :

$$V_{IN} = V_{OUT} \cdot (1 - D) \quad (6)$$

then

$$\Delta I_L = \frac{V_{OUT} \cdot (D - D^2)}{L \cdot F_{SW}} \quad (7)$$

Setting the derivative to zero,

$$\frac{d}{dD} \Delta I_L = \frac{V_{OUT} \cdot (1 - 2D)}{L \cdot F_{SW}} = 0 \quad (8)$$

gives us

$$D = \frac{1}{2} \quad (9)$$

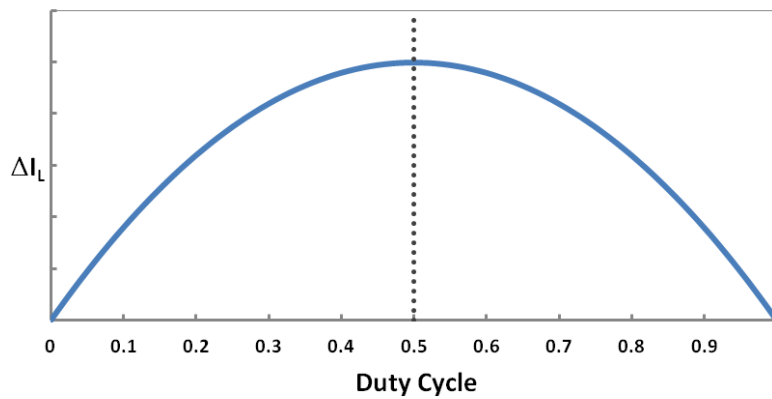


Fig. 3. Inductor ripple current in CCM.

### CCM Operation

To choose the inductor value ( $L$ ) for a CCM boost converter, a maximum  $K_{RF}$  value is selected to ensure CCM operation across the input voltage range and to limit the peak current handled by the MOSFET, diode, and output capacitor. A *minimum* inductor value is then calculated. Maximum  $K_{RF}$  is commonly chosen between 0.3 and 0.6, but can be up to 2.0 for CCM. We have shown that the max value of ripple current  $\Delta I_L$  occurs when  $D = 0.5$ , but what duty cycle results in the maximum  $K_{RF}$ ? We can use the derivative method to find out.

Assuming  $\eta = 100\%$ , then

$$P_{IN} = P_{OUT} = V_{OUT} \cdot I_{OUT} \quad (10)$$

and substituting equations (2), (6), (7) and (10) into (1) yields:

$$K_{RF} = \frac{\Delta I_L}{I_{IN}} = \frac{V_{out} \cdot (D - 2D^2 + D^3)}{L \cdot F_{SW} \cdot I_{OUT}} \quad (11)$$

$$\frac{d}{dD} K_{RF} = \frac{V_{out} \cdot (1 - 4D + 3D^2)}{L \cdot F_{SW} \cdot I_{OUT}} = 0 \quad (12)$$

Solving for  $D$ , we get

$$D = \left[ \frac{1}{3}, 1 \right] \quad (13)$$

The extraneous solution  $D = 1$  is ignored since it is physically impossible in steady state (duty cycle must be less than 1.0 for the boost converter). Therefore ripple factor  $K_{RF}$  is maximum when  $D = \frac{1}{3}$ , or  $V_{IN} = \frac{2}{3}V_{OUT}$ , as shown in Fig. 4. Maximum  $L_{MIN}$ ,  $L_{CRIT}$  and  $I_{CRIT}$  occur at the same point, as can be shown using the same technique.

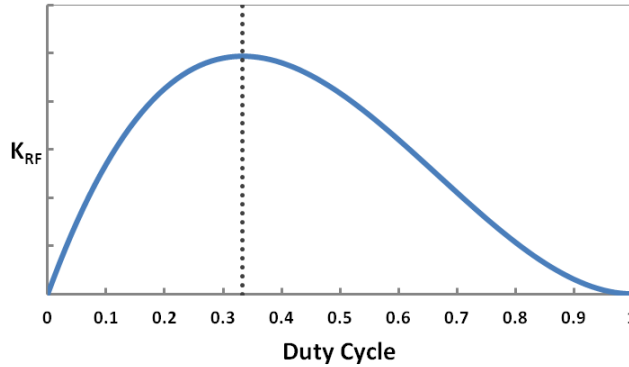


Fig. 4. Max CCM ripple factor  $K_{RF}$  at  $D = \frac{1}{3}$ .

For CCM operation, the minimum inductor value ( $L_{MIN}$ ) should be calculated at an actual operating input voltage ( $V_{IN(CCM)}$ ) nearest to  $\frac{2}{3}V_{OUT}$ . Depending on the specific input voltage range of the application,  $V_{IN(CCM)}$  could occur at minimum  $V_{IN}$ , maximum  $V_{IN}$ , or somewhere in between. Solving (5) for  $L$  and rewriting in terms of  $K_{RF}$  at  $V_{IN(CCM)}$  gives

$$L_{MIN} = \frac{V_{IN} \cdot D}{\Delta I_L \cdot F_{SW}} = \frac{V_{IN(CCM)}^2 \cdot (V_{OUT} - V_{IN(CCM)})}{K_{RF} \cdot V_{OUT}^2 \cdot I_{OUT} \cdot F_{SW}} \quad (14)$$

where  $V_{IN(CCM)}$  is the actual operating  $V_{IN}$  closest to  $\frac{2}{3}V_{OUT}$ .

For critical inductance vs.  $V_{IN}$  and  $I_{OUT}$ ,  $K_{RF} = 2$  giving

$$L_{CRIT} = \frac{V_{IN}^2 \cdot (V_{OUT} - V_{IN})}{2 \cdot V_{OUT}^2 \cdot I_{OUT} \cdot F_{SW}} \quad (15)$$

Critical load ( $I_{CRIT}$ ) occurs when  $K_{RF} = 2$  for a given  $V_{IN}$  and  $L$  value:

$$I_{CRIT} = \frac{V_{IN}^2 \cdot (V_{OUT} - V_{IN})}{2 \cdot V_{OUT}^2 \cdot L \cdot F_{SW}} \quad (16)$$

### DCM Operation

As shown in Fig. 5, DCM operation is maintained when the inductor value is less than  $L_{CRIT}$  at the operating  $V_{IN}$  and output current ( $I_{OUT}$ ). For a DCM converter, a minimum idle time can be chosen to ensure DCM operation across the input voltage range. Minimum  $t_{idle}$  is typically 3% to 5% of the switching period, but may be longer at the expense of increased peak device currents. A *maximum* inductor value ( $L_{MAX}$ ) is then calculated using the

minimum  $t_{idle}$ .  $L_{MAX}$  must be below the lowest  $L_{CRIT}$  in the  $V_{IN}$  range. CrCM results when the inductor value equals  $L_{CRIT}$  ( $t_{idle} = 0$ ) for a given  $V_{IN}$ .

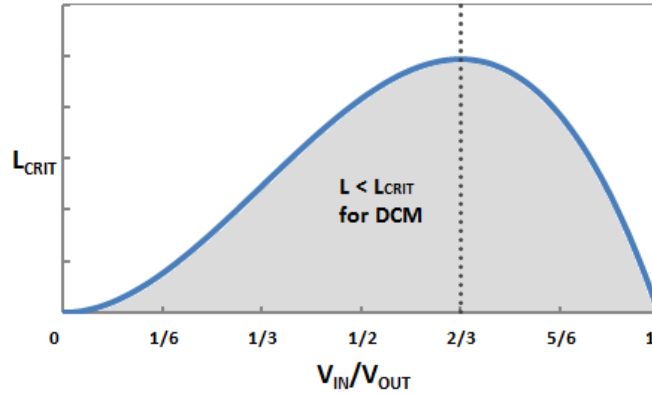


Fig. 5.  $L_{CRIT}$  vs. normalized  $V_{IN}$ .

To calculate  $L_{MAX}$  for a chosen minimum idle time ( $t_{idle(min)}$ ), first use the DCM volt-second balance equation to find  $t_{ON(max)}$  (maximum permissible MOSFET on-time) vs.  $V_{IN}$ , where  $t_{dis}$  is the inductor discharge time.

$$V_{IN} \cdot t_{ON} = (V_{OUT} - V_{IN}) \cdot t_{dis} \quad (17)$$

with

$$t_{dis} = \frac{1}{F_{SW}} - t_{ON} - t_{idle} \quad (18)$$

gives

$$t_{ON(max)} = \left(1 - \frac{V_{IN}}{V_{OUT}}\right) \cdot \left(\frac{1}{F_{SW}} - t_{idle(min)}\right) \quad (19)$$

The average (dc) inductor current is equal to the converter dc input current, and  $t_{dis}$  can be expressed in terms of  $t_{ON}$  by rearranging (17). We will again assume  $P_{IN} = P_{OUT}$  for the sake of simplicity.

$$\frac{\Delta I_L}{2} \cdot (t_{ON} + t_{dis}) \cdot F_{SW} = \frac{P_{IN}}{V_{IN}} \quad (20)$$

where

$$t_{dis} = \frac{V_{IN} \cdot t_{ON}}{V_{OUT} - V_{IN}} \quad (21)$$

Substituting equations (3), (5), (10), (19) and (21) into (20) and solving for  $L$  at  $V_{IN(DCM)}$  yields

$$L_{MAX} = \frac{V_{IN(DCM)}^2 \cdot (V_{OUT} - V_{IN(DCM)}) \cdot (1 - t_{idle(min)} \cdot F_{SW})^2}{2 \cdot I_{OUT} \cdot V_{OUT}^2 \cdot F_{SW}} \quad (22)$$

$L_{MAX}$  follows a curve similar to  $L_{CRIT}$  and is also at peak with  $V_{IN} = \frac{2}{3}V_{OUT}$ . To guarantee the minimum  $t_{idle}$ , calculate the *lowest*  $L_{MAX}$  value at an actual operating input voltage ( $V_{IN(DCM)}$ ) *opposite* this operating point.

$V_{IN(DCM)}$  will be equal to either minimum or maximum operating  $V_{IN}$  depending on the actual input voltage range of the application. If the entire input voltage range falls either above or below  $\frac{2}{3}V_{OUT}$  (inclusive),  $V_{IN(DCM)}$  is the input voltage furthest from  $\frac{2}{3}V_{OUT}$ . If the input voltage range spans  $\frac{2}{3}V_{OUT}$  calculate the inductance at both min and max  $V_{IN}$  and choose the lower (worst case) inductance value. Alternatively, evaluate  $L_{MAX}$  graphically with respect to  $V_{IN}$  to determine worst case.

### Input Voltage-Mode Boundaries

When the output current of a boost converter is less than the maximum value of  $I_{CRIT}$  vs.  $V_{IN}$ , CCM operation will result if the input voltage is increased above an upper-mode boundary or decreased below a lower-mode boundary where  $I_{OUT}$  is greater than  $I_{CRIT}$ . DCM operation occurs between the two  $V_{IN}$  mode boundaries where  $I_{OUT}$  is less than  $I_{CRIT}$ . To find these conduction-mode boundaries of  $V_{IN}$  graphically, plot the critical load (with the selected inductor) vs. input voltage and the output current of interest on the same graph. Then find the two  $V_{IN}$  values on the X-axis corresponding to the intersections of the two curves (Fig. 6).

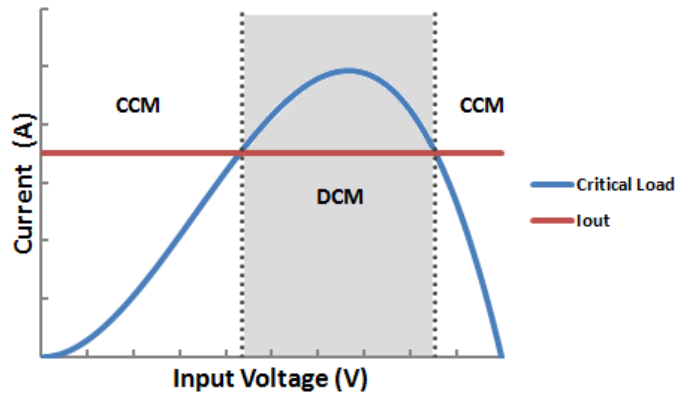


Fig. 6. Input voltage-mode boundaries.

To find the mode boundaries of  $V_{IN}$  algebraically, first set the expression for critical load equal to the output current of interest to find the intersections:

$$I_{CRIT} = \frac{V_{IN}^2 \cdot (V_{OUT} - V_{IN})}{2 \cdot V_{OUT}^2 \cdot L \cdot F_{SW}} = I_{OUT} \quad (23)$$

This can be rewritten as a cubic equation with the constants lumped into the term  $K_{CM}$ :

$$V_{IN}^3 - V_{OUT} \cdot V_{IN}^2 + K_{CM} = 0 \quad (24)$$

with

$$K_{CM} = 2 \cdot L \cdot F_{SW} \cdot V_{OUT}^2 \cdot I_{OUT} \quad (25)$$

Here, the three solutions for the cubic equation of general form  $x^3 + ax^2 + bx + c = 0$  can be found using the trigonometric version of the cubic formula.<sup>[1,2]</sup> In this case, the "b" coefficient of the  $x^1$  term is zero. We define the solutions as a vector  $V_{MB}$ .

With

$$q = \frac{a^3}{27} - \frac{a \cdot b}{6} + \frac{c}{2} = \frac{K_{CM}}{2} - \frac{V_{OUT}^3}{27} \quad (26)$$

$$p = \frac{b}{3} - \frac{a^2}{9} = -\frac{V_{OUT}^2}{9} \quad (27)$$

and

$$\theta = \arccos\left(\frac{-q}{\sqrt{-p^3}}\right) = \arccos\left(1 - \frac{27 \cdot K_{CM}}{2 \cdot V_{OUT}^3}\right) \quad (28)$$

$$V_{MB} = \begin{bmatrix} 2\sqrt{-p} \cdot \cos\left(\frac{\theta}{3}\right) - \frac{a}{3} \\ 2\sqrt{-p} \cdot \cos\left(\frac{\theta+2\pi}{3}\right) - \frac{a}{3} \\ 2\sqrt{-p} \cdot \cos\left(\frac{\theta+4\pi}{3}\right) - \frac{a}{3} \end{bmatrix} V = \begin{bmatrix} \frac{V_{OUT}}{3} \cdot \left(2 \cdot \cos\left(\frac{\theta}{3}\right) + 1\right) \\ \frac{V_{OUT}}{3} \cdot \left(2 \cdot \cos\left(\frac{\theta+2\pi}{3}\right) + 1\right) \\ \frac{V_{OUT}}{3} \cdot \left(2 \cdot \cos\left(\frac{\theta+4\pi}{3}\right) + 1\right) \end{bmatrix} V \quad (29)$$

Any solutions of  $V_{MB} \leq 0$  or  $V_{MB} > V_{OUT}$  can be ignored due to the physical constraints of the boost converter. Two real solutions are left as valid values of  $V_{IN}$  at the mode boundaries.

### Mode Boundaries—Design Example

Let's assume a DCM boost converter with the following specs:

$$V_{OUT} = 12 \text{ V}$$

$$I_{OUT} = 1 \text{ A}$$

$$L = 6 \text{ } \mu\text{H}$$

$$F_{SW} = 100 \text{ kHz}$$

First, compute  $K_{CM}$  and  $\theta$  using (25) and (28):

$$K_{CM} = 2 \cdot (6 \cdot 10^{-6}) \cdot (100 \cdot 10^3) \cdot 12^2 \cdot 1 = 1728$$

$$\theta = \arccos\left(1 - \frac{27 \cdot 1728}{2 \cdot 12^3}\right) = 1.928 \text{ rad} = .614\pi \text{ rad}$$

Enter  $V_{OUT}$  and the calculated value of  $\theta$  into (29) to find the  $V_{IN}$  values at the mode boundaries:

$$V_{MB} = \begin{bmatrix} \frac{12}{3} \cdot \left(2 \cdot \cos\left(\frac{.614\pi}{3}\right) + 1\right) \\ \frac{12}{3} \cdot \left(2 \cdot \cos\left(\frac{.614\pi + 2\pi}{3}\right) + 1\right) \\ \frac{12}{3} \cdot \left(2 \cdot \cos\left(\frac{.614\pi + 4\pi}{3}\right) + 1\right) \end{bmatrix} V$$

$$= \begin{bmatrix} 10.40 \\ -3.36 \\ 4.95 \end{bmatrix} \text{ V}$$

Ignoring the extraneous solution (-3.36 V), we have the two input voltage-mode boundaries at 4.95 V and 10.40 V. These calculated values match the intersections depicted graphically as shown in Fig. 7.

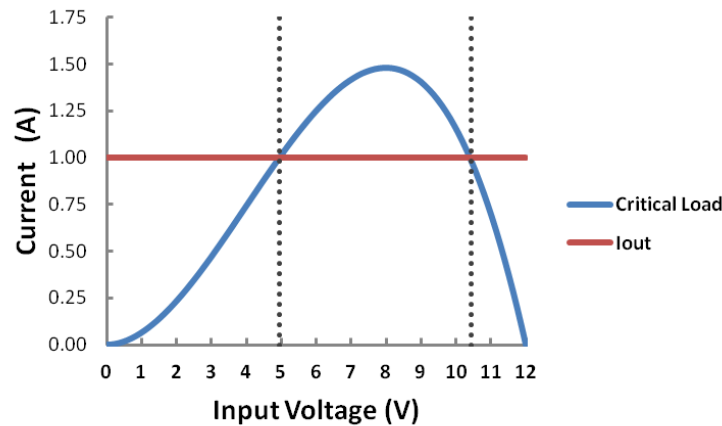


Fig. 7. Calculated mode boundaries.

### Accelerate Design With WebDesigner Boost Powertrain

Repeating these design calculations for different values of boost inductor can be tedious and time consuming when performed manually. Calculating the input voltage-mode boundaries is especially cumbersome and error-prone due to the complicated cubic formula. Design effort can be eased and accelerated significantly by using an online design tool like WebDesigner from ON Semiconductor.

The Boost Powertrain design module (Fig. 8) performs all of these calculations automatically (including the influence of real-world efficiency) and recommends the optimal inductor value using your application requirements. You can select a real inductor part number from the extensive built-in database or enter your own custom inductor specs. Ripple current and mode boundaries are computed instantly along with the resulting impact on the output capacitor, MOSFET and diode losses and overall efficiency.



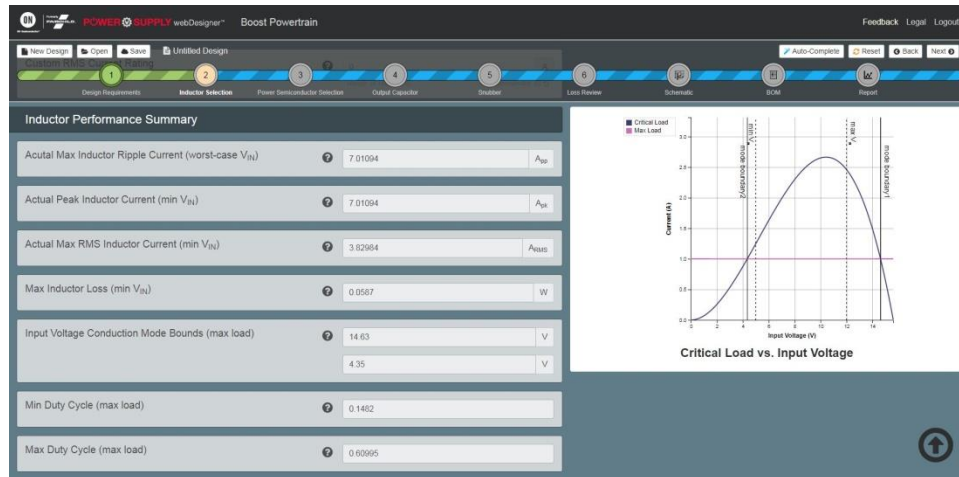


Fig.8. WebDesigner Boost Powertrain.

The WebDesigner Boost Powertrain design tool can be found on the On Semiconductor website.<sup>[3]</sup>

## Conclusion

The inductor value impacts many aspects of the boost converter, and can result in excessive cost and size or poor performance if chosen incorrectly. By understanding the relationships between inductor value, ripple current, duty cycle and conduction mode, the designer can ensure the desired performance across the input voltage range.

## References

1. *Theory of Equations*, Chapter IX by H. W. Turnbull, Edinburgh & London: Oliver and Boyd, 1952.
2. "The cubic equation—a new look at the irreducible case," *The Mathematical Gazette*, vol. 92, no. 524, pp. 264-268, July 2008.
3. [Power Supply WebDesigner Boost Powertrain](#) page.

## About The Author



*Brian Curbo is a senior principal applications engineer and a member of ON Semiconductor's online design tools group. In this role, he is responsible for creating new tools used to accelerate and enhance the design of switching power converters. Brian has over 20 years' experience in power supply for a wide range of applications including industrial and consumer electronics, telecom, and electronic lighting ballast.*

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For more information on inductor selection, see How2Power's [Design Guide](#), locate the Design Area category and select "Magnetics".