*Optimizing Transformer Winding Design For Max Efficiency Over Output Current Range**

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In a previous series of articles on interwinding power transfer in transformers and coupled inductors, design formulas were derived for maximum power transfer having peak winding transfer efficiency, η_{max}, using the transfer voltage ratio, υ_p and the ratio of winding to effective core resistance, β_p.[1,2,3,] In this article, some extended development of power-transfer theory applies the additional constraint of holding the secondary current (i.e. the load current) constant as β_p is varied.

The migration of the operating-point (op-pt) at η_{max} on one efficiency curve to another η curve at a different β_p under constant output current shows that as winding resistance is varied in design, the η op-pt can move to either side of the new η_{max}. As current is varied, the op-pt on a given η curve also moves and efficiency changes. A slight variation in current for op-pts to the right of η_{max} causes a large change in η, especially with low winding resistance (or β_p).

This article analyzes and derives the theoretical optimal peak efficiency for a power converter when its load (and that of its transformer) spans a range and uses that information to design the transformer. This is a topic that has not been explained clearly in the literature.

The method explained here involves designing the transformer to center the current range around the efficiency peak. But, as the article shows, the peak efficiency occurs at a fixed average current op-pt. At ever higher efficiencies, the peak becomes sharper and at the extremes of the load current range, the efficiency can actually be lower than for a design of lower peak efficiency.

Knowing this, designers can design the transformer for a lower peak efficiency (through adjustment of winding resistance) in cases where the goal is higher efficiency over the current range. The basic results of this analytical approach seem to fit typical efficiency curves in PWM controller data sheets.

Efficiency As A Function of Transformer Voltage And Resistance Ratios

In part 3[3] of the power-transfer series of articles, the general model for power transfer was given and is repeated here in Fig. 1.

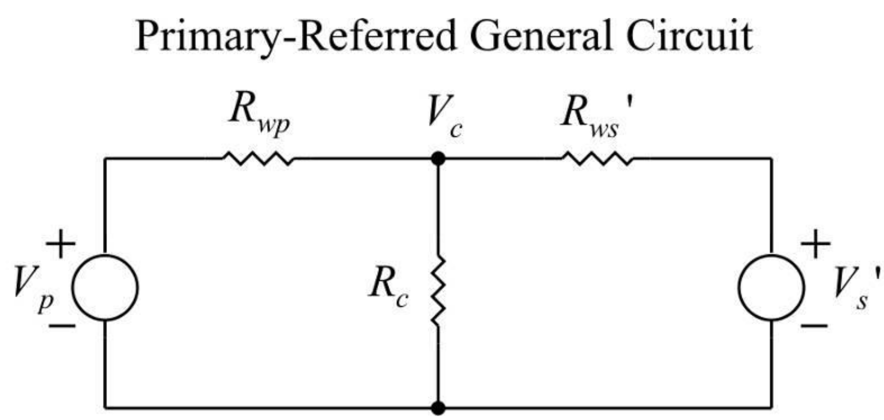

Fig. 1. General transductor circuit model for deriving design formulas, referred to primary winding (by ') and with a voltage source as output load. The voltage source, V_s, approximates a large output capacitor and voltage regulation but can be varied for other loads.

Interwinding power transfer with winding and core losses can be expressed as a function of the primary-referred *voltage ratio*, $\upsilon_p = V_s'/V_p$, (where V_s' = primary-referred secondary voltage and V_p = primary voltage) and the primary-referred *resistance ratio*, $\beta_p = R_{wp}/R_c$ (where R_{wp} is the primary winding resistance and R_c is the primary-referred equivalent core resistance). The voltage ratio, υ is simply the voltage attenuation (or gain < 1) from the primary to the primary-referred secondary winding terminals. Magnetic design determines the value of R_c leaving β_p as a measure of winding resistance, and is a design parameter for winding design.

The power transfer or interwinding *efficiency* for a winding referral ratio equal to the turns ratio ($R_{ws}' = R_{wp}$), with primary power, P_p and secondary power, P_s is

$$\eta(\upsilon_p, \beta_p) = \frac{P_s}{P_p} = \frac{\upsilon_p \cdot (1 - (\beta_p + 1) \cdot \upsilon_p)}{(\beta_p + 1) - \upsilon_p}$$

η is plotted in Fig. 2 with υ_p as a variable and β_p as parameter with four values ranging from 0.01 to 0.1. β_p varies proportionally with R_{wp}, and its lowest value on the graph corresponds to the η curve (solid) with the highest peak efficiency of max-η or η_{max} at highest $\upsilon_p = \upsilon_{pmax}$. The voltage drop across the winding resistances is $V_w = (1 - \upsilon_p) \cdot V_p$. Ideally, $\upsilon_p = 1$, and that occurs only when there is no winding resistance and $\beta_p = 0$.

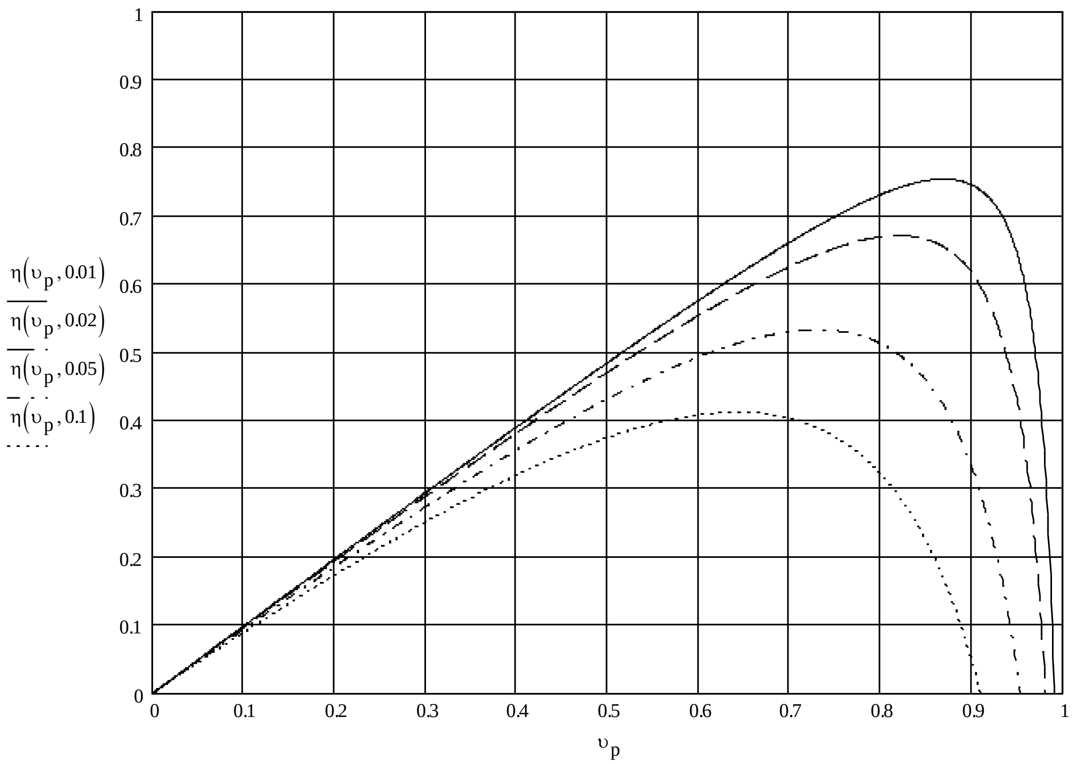

Fig. 2. Plots of primary-to-secondary power transfer (efficiency), $\eta(\upsilon_p, \beta_p)$ as a function of the primary-to-secondary voltage transfer ratio, υ_p and parameter, β_p. As winding resistance and hence β_p decrease, maximum η at η_{max} increases along with $\upsilon_p = \upsilon_{pmax}$ at η_{max}. Ideally, $\beta_p = 0$; then $\upsilon_{pmax} = 1$ and $\eta_{max} = 1$ at the upper-right corner of the graph.

As derived from analysis of the Fig. 1 circuit in Part 3,[3]

$$\eta_{max} = \upsilon_{p\,max}^2$$

Also derived was υ_p at max-η for a given β_p ;

$$\upsilon_{p\,max}(\beta_p) = (\beta_p + 1) - \sqrt{\beta_p \cdot (2 + \beta_p)} \quad \text{, max-}\eta$$

Solving $\upsilon_{pmax}(\beta_p)$ for β_p at η_{max} ,

$$\beta_{p\,max}(\upsilon_p) = \frac{(1 - \upsilon_p)^2}{2 \cdot \upsilon_p} \quad \text{, max-}\eta$$

The η curve that is maximum at υ_{pmax} has as its parameter, $\beta_p = \beta_{pmax}(\upsilon_{pmax})$.

The Impact Of Load Current

The curved plot of $\beta_{pmax}(\upsilon_p)$ is shown in Fig. 3. Each of the η plots in Fig. 2 are for a constant β_p; traversing horizontally in Fig. 3 along a constant β_p traces a given η curve of Fig. 2. For a given value of β_p, the β_{pmax} curve is intersected at υ_{pmax} in Fig. 3. At lower β_p, β_{pmax} is intersected at higher υ_p as in Fig. 2.

At the ideal (η = 1) op-pt where $\upsilon_p = 1$, then $V_s' = V_p$ and across the winding resistance is $V_w = V_p - V_s' = 0$ V. At this end of the curve, $\beta_{pmax} = 0$ and $R_w = 0\ \Omega$. The primary-referred load current is thus $I_s' = V_w / R_w = 0$ V/0 Ω—a degenerate case, though for an infinitesimal resistance, it is 0 A. At the other end of the υ_p range, $\upsilon_p = 0$ (load shorted), $V_w = V_p$ across R_w. For the ideal case of $\beta_p = 0$, then $R_w = 0\ \Omega$ and $I_s' \to \infty$. For $\beta_p > 0$ and across the operating range, I_s' varies from a maximum value of $I_s' = V_p/R_w$ at $\upsilon_p = 0$ to 0 A at $\upsilon_p = 1$.

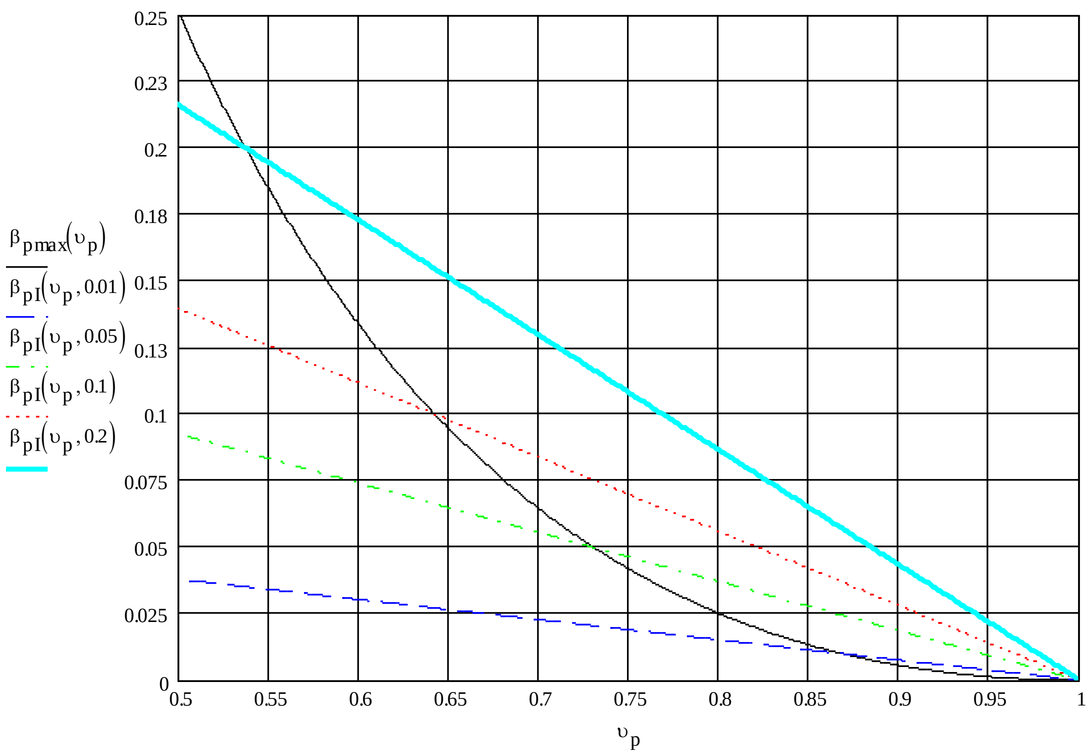

Fig. 3. The concave curve is $\beta_{pmax}(\upsilon_p)$ of the η_{max} loci. $\beta_{pmax}(\upsilon_p)$ determines the $\eta(\upsilon_p)$ curve at which $\upsilon_p = \upsilon_{pmax}$. Constant-current winding-resistance parameter, $\beta_{pI}(\upsilon_p, \beta_p)$ is plotted for four values of I_s', where $\beta_p/(1 - \upsilon_{pmax}(\beta_p))$ is the scaling factor of β_{pI}, constraining β_{pI} to include the max-η point for the curve for which $\beta_p = \beta_{pmax}$.

In voltage-output converters, the secondary voltage is held constant by regulation and a large capacitor. The load typically sinks current, I_s'. At a constant load current, it is of interest to know how the operating-point (op-pt) on the efficiency curves migrates as υ_p and β_p change. For instance, suppose operation is established at a peak efficiency, η_{max} at a given υ_{pmax} and β_p. Then if the winding resistance is reduced, β_p decreases, υ_{pmax} increases, and the op-pt migrates to a higher-η curve. Where is it on the new curve if the load current is held constant? To which side of η_{max} on the new, higher η curve has it moved?

The answer to this question begins with the primary-referred secondary current equation, from the transductor model in Fig. 1 and Ohm's Law;

$$I_s' = \frac{V_p - V_s'}{R_w} = \frac{(1-\upsilon_p)\cdot V_p}{2\cdot R_{wp}} = \frac{1-\upsilon_p}{2\cdot \beta_p}\cdot\frac{V_p}{R_c}$$

V_p/R_c is constant for constant V_p and R_c, and if I_s' is also held constant, then

$$\frac{1-\upsilon_p}{2\cdot\beta_p}=\frac{I_s'\cdot R_c}{V_p}=\text{constant} \;\Rightarrow\; \frac{1-\upsilon_{p0}}{2\cdot\beta_{p0}}=\frac{1-\upsilon_p}{2\cdot\beta_p}\;,\, I_s'\text{ constant} \;\Rightarrow\; \beta_{pI}(\upsilon_p)=\left(\frac{\beta_{p0}}{1-\upsilon_{p0}}\right)\cdot(1-\upsilon_p)\,,\, I_s'\text{ constant}$$

where the constant-current $\beta_p = \beta_{pI}$. υ_{p0} and β_{p0} scale the $\beta_{pI}(\upsilon_p)$ function for the value of β_{p0} at υ_{p0} and relate it to I_s';

$$\beta_{pI}(0)=\frac{\beta_{p0}}{1-\upsilon_{p0}}=\frac{V_p}{2\cdot I_s'\cdot R_c}\quad,\, I_s'\text{ constant}$$

As I_s' increases, $\beta_{pI}(0)$ decreases. For constant I_s', $\beta_{pI}(\upsilon_p)$ is a linear, decreasing function. The scaling for $\beta_{pI}(\upsilon_p)$ is set to include a max-η point by making

$$\beta_{pI}(\upsilon_p,\beta_p)=\left(\frac{\beta_p}{1-\upsilon_{p\max}(\beta_p)}\right)\cdot(1-\upsilon_p)\quad,\, I_s'\text{ constant, max-}\eta$$

where $\beta_p = \beta_{p0}$ corresponds to the max-η point and υ_p and β_{pI} correspond to the new point on a different η curve. The β_{pI} curves are plotted in Fig. 3 for four values of β_p. They intersect the $\beta_{p\max}$ curve where the max-η peak occurs at the given I_s'. The lower the β_{pI} line on the graph, the lower is the winding resistance and the higher is I_s'. On the β_{pI} lines to the left of the $\beta_{p\max}$ intersection, the op-pt is to the left side of the max-η points of the plots, as shown in Fig. 4.

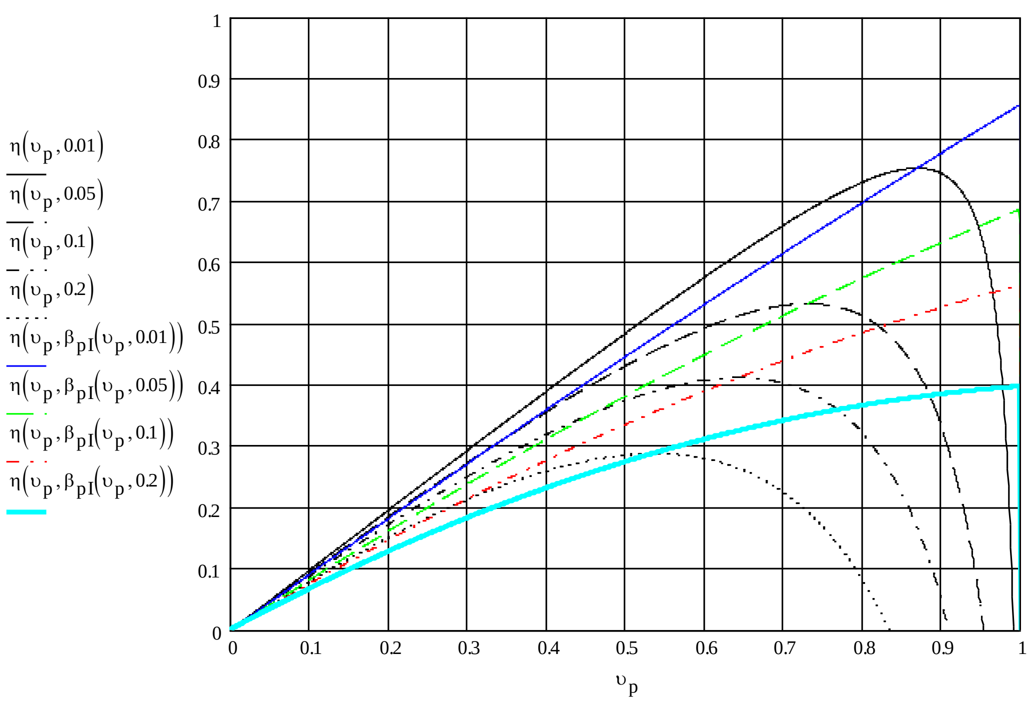

Fig. 4. Efficiency curves, $\eta(\upsilon_p, \beta_p)$, as plotted in Fig. 2, and four corresponding to the constant-current constraint, plotted for the same four values of β_p as in Fig. 3. Current is held constant by setting $\beta_p = \beta_{pI}(\upsilon_p, \beta_p)$ and letting β_p vary as β_{pI}. For constant-β_p curves, as β_p decreases, the η peaks shift to the right and upward. Each constant-current η plot intersects one constant-β_p η curve at η_{max}, and β_p is optimal at that current for max-η. For a given β_p, increasing current moves the op-pt on the η curve to the left. Where the constant-current curves intersect a given constant-β_p curve can be on either side of the peak, η_{max}.

Fig. 4 also shows that for a given magnetics design with constant β_p and η curve determined, as the η curve is traversed from right to left, I_s' increases. Consequently, the optimum current is at η_{max} for a given β_p. Maximum average efficiency over a current range can be achieved by centering the op-pts for the current extrema on each side of η_{max}, at equal values of η. Then the value of β_p—and hence winding resistance—for that η curve can be used in the winding design.

Fig. 4 also shows that maintaining high η at low current is difficult because the η curve falls off abruptly to the right of η_{max}. Operation on the left side of η_{max} ($\upsilon_p < \upsilon_{pmax}$) is preferred because the slope is less and nearly linear. Thus, changes in load current affect efficiency less.

Although higher peak efficiency can be achieved at lower R_w and β_p, the disadvantage is that a high-η curve has a narrower peak, and η becomes increasingly sensitive to changes in load current. Consequently, minimization of winding resistance, although it is generally desirable, can also result in lower efficiency of operation over a range of load current.

**Editor's Note: The technical discussion in this article was refined and republished as an appendix to the article, "Maximizing Power-Transfer Efficiency Over A Current Range," published December 2023 in How2Power Today.*

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About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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