Replacing Power Supply Output Capacitors Requires Equivalent Impedances

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Very often designers have to replace a power supply's output filter capacitors with equivalent counterparts due to the obsolescence of the original parts or because the manufacturer has gone out of business. To maintain power supply performance, the replacement capacitors should provide the same voltage ripple across the load as the original parts did.

Since the voltage ripple across a capacitor is a function of the capacitor's impedance, we have to create two equivalent series schematics of the original capacitors and replacement capacitors and equate their impedances. To find the equivalent impedance of a number of parallel-connected capacitors of different chemistries, we have to find the resulting impedance of parallel-connected capacitors. In this article, we will solve this problem in symbolic form, enabling us to plug-in parametric values when necessary. After deriving the necessary formulas, we will demonstrate their use with a numerical example.

In this discussion, equivalency means that the total impedance of the replacement capacitors is equal to or less than the total impedance of the original capacitors. Capacitor manufacturers seldom provide the equivalent series inductance (ESL) value. More often the capacitor's resonant diagram is available. However, we can use this diagram as the source for the inductance calculation.

Accounting for capacitor ESL is essential to capacitor replacement in the power supply output filter. A previous treatment of this subject (see the reference) described a method of capacitor replacement with equivalent parts. However, that method was incomplete because it did not consider ESL.

Typically, with the power supply output filter, engineers try to achieve the lowest amount of voltage ripple across the filter capacitors, or a specified maximum amplitude of ripple to keep costs low. Considering the ESL of the capacitors being replaced is necessary to maintaining or even reducing the voltage ripple obtained with the replacement capacitors.

Finding The Impedance Of Parallel-Connected Capacitors

Let's start with two parallel-connected capacitors, C1 and C2. We'll designate the associated ESRs of these capacitors as R_{C1} and R_{C2} and their reactances as X_{L1} and X_{C1} for C1 and X_{L2} and X_{C2} for C2.

Let one capacitor have

$$Z_{C1}(i\omega) = R_{C1} + i\omega \cdot (X_{L1} - X_{C1})$$

(1)

Designating $X_1 = X_{L1} - X_{C1}$, we obtain

$$Z_{C1}(i\omega) = R_{C1} + i\omega \cdot X_1$$

(2)

Similarly, the other one is:

$$Z_{C2}(i\omega) = R_{C2} + i\omega \cdot X_2$$

(3)

The moduli of these two impedance are

$$Z_{C1}(\omega) = \sqrt{R_{C1}^2 + X_1^2}$$

(4)

$$Z_{C2}(\omega) = \sqrt{R_{C2}^2 + X_2^2}$$

(5)

where $X_1 = \omega \cdot X_1$ and $X_2 = \omega \cdot X_2$.

The parallel connection of these two impedances gives

$$Z_{tot}(\omega) = \frac{Z_{C1}(\omega) \cdot Z_{C2}(\omega)}{Z_{C2}(\omega) + Z_{C1}(\omega)}$$

(6)

After plugging in equations (4) and (5) we get:

$$Z_{tot}(\omega) = \frac{\sqrt{R_{C1}^2 + X_1^2} \cdot \sqrt{R_{C2}^2 + X_2^2}}{\sqrt{R_{C2}^2 + X_2^2} + \sqrt{R_{C1}^2 + X_1^2}}$$

(7)

For each capacitor we can express its ESR value as a part of the reactance through the dissipation factor DF, which is always provided by manufacturer:

$$R_{C1} = \left((DF)_1\right) \cdot X_1 \qquad R_{C2} = \left((DF)_2\right) \cdot X_2$$

(8)

The DF is tan (δ) of these capacitors at the ripple frequency f_{ripple}.

Recollect that

$$\tan\delta = \frac{R_C}{X_C}$$

where X_C is the reactance of the capacitor.

Thus, equation (7) transforms into

$$Z_{tot}(\omega) = \frac{X_1 \cdot \sqrt{1 + ((DF)_1)^2} \cdot X_2 \cdot \sqrt{1 + ((DF)_2)^2}}{X_2 \cdot \sqrt{1 + ((DF)_2)^2} + X_1 \cdot \sqrt{1 + ((DF)_1)^2}}$$

(9)

On the other side, it should be equal to the replacement impedance:

$$Z_{rep}(\omega) = \sqrt{R_{Crepl}^2 + X_{repl}^2}$$

(10)

where

$$R_{Crepl}^2 = ((DF)_{repl})^2 \cdot X_{repl}^2$$

(11)

So equation (10) turns into:

$$Z_{repl}(\omega) = \sqrt{X_{repl}^2 + ((DF)_{repl})^2 \cdot X_{repl}^2} = X_{repl} \cdot \sqrt{1 + ((DF)_{repl})^2}$$

(12)

As $Z_{repl}(\omega)$ should be equal to $Z_{tot}(\omega)$ we can use that to obtain the value for X_{repl}.

$$X_{repl} \cdot \sqrt{1 + ((DF)_{repl})^2} = \frac{X_1 \cdot \sqrt{1 + ((DF)_1)^2} \cdot X_2 \cdot \sqrt{1 + ((DF)_2)^2}}{X_2 \cdot \sqrt{1 + ((DF)_2)^2} + X_1 \cdot \sqrt{1 + ((DF)_1)^2}}$$

and

$$X_{repl} = \frac{X_1 \cdot \sqrt{1 + ((DF)_1)^2} \cdot X_2 \cdot \sqrt{1 + ((DF)_2)^2}}{\left(X_2 \cdot \sqrt{1 + ((DF)_2)^2} + X_1 \cdot \sqrt{1 + ((DF)_1)^2}\right) \cdot \sqrt{1 + ((DF)_{repl})^2}}$$

(13)

and

$$R_{Crepl} = X_{repl} \cdot (DF)_{repl}$$

(14)

Having obtained these values, we can select a number of replacement capacitors, which when combined, would provide the same or better voltage ripple.

A Numerical Example

In this example, we will have to replace two capacitors (one electrolytic and one film) connected in parallel with ceramic capacitors. Our application specifies the following characteristics for the two capacitors in parallel:

f_{ripple} = 125 kHz

ESR = 0.001 Ω

ESL = 1 x 10^{-9} H

The first capacitor is an electrolytic with these parameters:

C1 = 1000 µF

L1 = 52 nH

DF_1 = 0.12

$$X_{L1} = 2\pi \cdot f_{ripple} \cdot L_1$$

$$X_{C1} = \frac{1}{2\pi \cdot f_{ripple} \cdot C_1}$$

$$X_1 = X_{L1} \text{-} X_{C1}$$

The second capacitor is a film cap with these parameters:

C2 = 2 µF

$$L_2 = \frac{1}{4\pi^2 \cdot C_2 \cdot f_{res}^2} = 2.585 \text{ x } 10^{-8} H$$

$$X_{L2} = 2\pi \cdot f_{ripple} \cdot L_2$$

$$X_{C2} = \frac{1}{2\pi \cdot f_{ripple} \cdot C_2}$$

$$X_1 = X_{L1} \text{-} X_{C1}$$

DF_2 = 0.0015 = 1.5 x 10^{-3}

DF_{repl} = 1.5 x 10^{-3}

Replacement capacitor parameters:

Reactance:

$$X_{repl} = \frac{X_1 \cdot \sqrt{1+((DF)_1)^2} \cdot X_2 \cdot \sqrt{1+((DF)_2)^2}}{\left(X_2 \cdot \sqrt{1+((DF)_2)^2} + X_1 \cdot \sqrt{1+((DF)_1)^2}\right) \cdot \sqrt{1+((DF)_{repl})^2}} = 0.043 \ \Omega$$

Ignoring the equivalent inductance for now, we obtain:

$$X_{repl} = \frac{1}{2\pi \cdot f_{ripple} \cdot C_{repl}}$$

$$C_{repl} := \frac{1}{2 \cdot \pi \cdot X_{repl} \cdot f_{ripple}} = 29.884 \times 10^{-6} F$$

$$L_{repl} := \frac{1}{4 \cdot \pi^2 \cdot C_{repl} \cdot f_{res}^2} = 1.73 \times 10^{-9} H$$

$$R_{Crepl} := X_{repl} \cdot (DF)_{repl} = 5.539 \times 10^{-5} \Omega$$

It is reasonable to use a few similar capacitors as replacement parts. In this case, we will use multiple VJ Hi-Rel series ceramic capacitors to obtain the capacitance equivalent to a single replacement capacitor.

C_{repl1} = 4.7 µF

Dissipation factor

DF_{repl1} = 0.0103   one capacitor value

Resonant frequency

f_{res1} = 4 MHz

Single replacement capacitor reactance

$$X_{repl1} = \frac{1}{2\pi \cdot f_{res1} \cdot C_{repl1}}$$

$$N_C = round\left(\frac{C_{repl}}{C_{repl1}}, 0\right) = 6$$

Assuming aging, dc biasing and other factors affecting the capacitance loss, take

N_c = 8.

Therefore, the resulting nominal parameters of the capacitors' array will be:

$$C_{repl\_tot} = N_C \cdot C_{repl1} = 37.6 \, x10^{-6} \, \text{F}$$

$$X_{repl\_tot} = \frac{X_{repl1}}{N_C} = 1.058 \, x10^{-3} \Omega$$

$$L_{repl\_tot} = \frac{L_{repl1}}{N_C} = 1.375 \, x10^{-9} H$$

$$R_{repl\_tot} = \frac{R_{Crepl1}}{N_C} = 10.9 \, x10^{-6} \Omega$$

This impedance and its components are much less than those of the original ones, which is beneficial.

This method allows for defining parameters of replacement capacitors that provide the same or better voltage ripple across the load.

Reference

"Determine Equivalent ESR, Ripple Voltage and Currents for Unequal Capacitors in Parallel" by A. Asinovski of Murata Power Solutions, Ideas for Design. Electronic Design, September 05, 2013.

About The Author

Gregory Mirsky is a senior electrical engineer with Continental Automotive Systems in Deer Park, Ill., which he joined in March 2015. In his current role, Gregory performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification.

He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

For more information on capacitors, see the How2Power's Design Guide, locate the Component category and select "Capacitors". For more information on power supply filtering, locate the Power Supply Function category and select "Filters and Snubbers".