Interbundle Penetration Of Wire Bundles Improves Their Packing Factor

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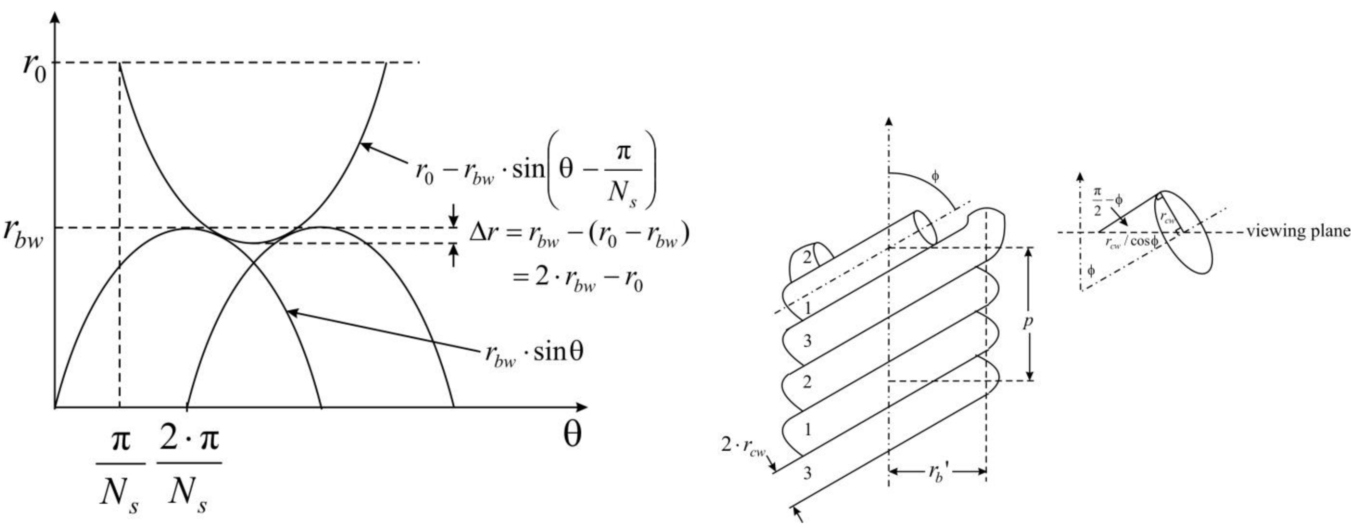
When winding transformers or coupled inductors with twisted wiring bundles, turns of adjacent wire bundles are typically wound close-packed to maximize packing factor (and in particular, porosity packing) and ultimately maximize winding current density. Bundles of a small number of strands can pack together more closely than those with a large number of strands because of the sinusoidal variation in their outside diameter. This article examines the extent to which twisted wire bundles can penetrate each other by fitting into the dips in adjacent windings and thereby increase packing factor. This analysis quantifies the relationship between the number of strands in a twisted wire bundle and the extent to which one wiring bundle penetrates or “meshes with” the other.

Modeling Adjacent Wire Bundles

The shape of a bundle of N_s strands when viewed from the side of the bundle has a characteristic ripple along its outer edges. The individual strands in the bundle can be described mathematically as a helix, and for each bundle pitch length, p, they rotate a full turn, a revolution or cycle of $2\cdot\pi$ radians around the center-line of the bundle. Viewed from the side (at any helix angle, θ), the strands follow a sinusoidal curve. In a bundle of N_s strands, each strand proceeds through one revolution of θ. Like polyphase electric power, there are thus N_s “phases” of strands, separated by $2\cdot\pi/N_s$ of angle and symmetrically spaced.

A graph of the strands viewed horizontally as functions of two adjacent bundles is shown in Fig. 1.



(a) (b)

Fig. 1. Side view of interpenetration of top bundle strand into bottom bundle, penetrating between two adjacent strands in the lower bundle (a). For reference, the construction of a single bundle is also shown (b). Bundle radius is r_{bw}, and the upper bundle is offset from the lower by π/N_s for maximum penetration of Δr. The spatial phase offset of N_s strands from each other in a bundle is $2\cdot\pi/N_s$.

To mathematically construct what is shown in the Fig. 1 graph, I will add more than the usual number of steps in the derivation (which is mainly trigonometry) to encourage you to think through it to the resulting design formulas. Knowing how formulas are derived reveals their assumptions and makes their applicability better understood.

The lower-bundle left strand is $r_{bw}\cdot\sin\theta$; the right strand, offset by $\theta = 2\cdot\pi/N_s$ is $r_{bw}\cdot\sin(\theta - 2\cdot\pi/N_s)$. The upper strand reaching down also has a bundle radius of r_{bw} and is $r_0 - r_{bw}\cdot\sin(\theta - \pi/N_s)$. It is offset in phase to center its peak between the two bottom strand peaks. Then the first equation that constrains where the strands are positioned is

$$r_{bw} \cdot \sin\theta = r_0 - r_{bw} \cdot \sin\left(\theta - \frac{\pi}{N_s}\right)$$

In equating the two functions and solving, their intersections are found. Imagine lowering the upper strand by decreasing r_0 until it touches the lower strands. The goal is to find r_0, for then the penetration depth, Δr is easily determined as the valley, $r_0 - r_{bw}$ of the upper strand subtracted from the peak, r_{bw}, of the lower strand;

$$\Delta r = r_{bw} - (r_0 - r_{bw}) = 2 \cdot r_{bw} - r_0$$

Using the sine sum-of-angles formula, the above constraint equation can be put in the form,

$$\frac{r_0}{r_{bw}} = \sin\theta + \sin\left(\theta - \frac{\pi}{N_s}\right) = \sin\theta + \left(\sin\theta \cdot \cos\frac{\pi}{N_s} - \cos\theta \cdot \sin\frac{\pi}{N_s}\right) = \left(1 + \cos\frac{\pi}{N_s}\right) \cdot \sin\theta - \left(\sin\frac{\pi}{N_s}\right) \cdot \cos\theta$$

With this one constraint, the upper and lower functions can intersect in two points. To constrain them to a single tangential point, their derivatives are equated:

$$\frac{d}{d\theta}\left(r_{bw} \cdot \sin\theta\right) = \frac{d}{d\theta}\left[r_0 - r_{bw} \cdot \sin\left(\theta - \frac{\pi}{N_s}\right)\right] \Rightarrow r_{bw} \cdot \cos\theta = -r_{bw} \cdot \cos\left(\theta - \frac{\pi}{N_s}\right)$$

This additional constraint requires that the functions at their intersection also have the same slopes, which constrains them to be tangent to each other. Applying the cosine sum-of-angles trig identity, the equation reduces to

$$[1 + \cos(\pi/N_s)] \cdot \cos\theta + [\sin(\pi/N_s)] \cdot \sin\theta = 0$$

Solving for $\cos\theta$ using the identity, $\sin^2\theta + \cos^2\theta = 1$,

$$\cos\theta = -\frac{\sin(\pi/N_s)}{1 + \cos(\pi/N_s)} \cdot \sin\theta \Rightarrow \sqrt{1 - \sin^2\theta} = -\frac{\sin(\pi/N_s)}{1 + \cos(\pi/N_s)} \cdot \sin\theta \Rightarrow 1 - \sin^2\theta = \left(-\frac{\sin(\pi/N_s)}{1 + \cos(\pi/N_s)}\right)^2 \cdot \sin^2\theta$$

Solving for $\sin^2\theta$ and simplifying,

$$\sin\theta = \sqrt{\frac{1}{1 + \left(\frac{\sin(\pi/N_s)}{1 + \cos(\pi/N_s)}\right)^2}} = \pm\frac{1 + \cos(\pi/N_s)}{\sqrt{(1 + \cos(\pi/N_s))^2 + \sin^2(\pi/N_s)}} = \pm\sqrt{\frac{1 + \cos(\pi/N_s)}{2}}$$

The ± sign is from the numerator, a result of $\sqrt{x^2} = |x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases} = \pm x$. Two values for θ are possible, but the one that applies here is the one that places θ in the second quadrant: $\sin(\pi - \theta) = -\sin\theta$. As can be seen in Fig. 1, the intersection of strands is at an angle of $\theta > \pi/2$, past its peak.

Substituting $\cos\theta$ into

$$\frac{r_0}{r_{bw}} = \sin\theta + \sin\left(\theta - \frac{\pi}{N_s}\right) = \sin\theta + \left(\sin\theta \cdot \cos\frac{\pi}{N_s} - \cos\theta \cdot \sin\frac{\pi}{N_s}\right) = \left(1 + \cos\frac{\pi}{N_s}\right) \cdot \sin\theta - \left(\sin\frac{\pi}{N_s}\right) \cdot \cos\theta \Rightarrow$$

$$\frac{r_0}{r_{bw}} = \left(1+\cos\frac{\pi}{N_s}\right)\cdot\sin\theta - \left(\sin\frac{\pi}{N_s}\right)\cdot\left(-\frac{\sin(\pi/N_s)}{1+\cos(\pi/N_s)}\cdot\sin\theta\right) = 2\cdot\sin(\pi-\theta) = \sqrt{2\cdot(1+\cos(\pi/N_s))}$$

Then substituting into

$$\frac{\Delta r}{r_{bw}} = 2 - \frac{r_0}{r_{bw}} = 2 - \sqrt{2\cdot(1+\cos(\pi/N_s))}$$

Bundle penetration reduces the spacing between bundles and in effect reduces their radius to

$$r_{bw}'' = r_{bw} - \frac{\Delta r}{2} = \frac{r_0}{2}$$

Finally, the *bundle interpenetration* (not to be confused with the eddy-current penetration factor, $\xi_r = r_c/\delta$) is defined as

$$\frac{r_{bw}''}{r_{bw}} = \frac{1}{2}\cdot\frac{r_0}{r_{bw}} = 1 - \frac{1}{2}\cdot\frac{\Delta r}{r_{bw}}$$

Some values are listed in the following table. The right column gives the fractional decrease in radius caused by penetrating bundles.

Table. The difference in radius between penetrated and unpenetrated radii normalized to the unpenetrated radius ($\Delta r/r_{bw}$) and the bundle interpenetration ratio (r_{bw}''/r_{bw}), both as a function of N_s in two adjacent wire bundles.

N_s	$\frac{\Delta r}{r_{bw}}$	$\frac{r_{bw}''}{r_{bw}}$
1	2	0
2	$2-\sqrt{2}\approx 0.586$	$\sqrt{2}/2 \approx 0.7071$
3	$2-\sqrt{3} \approx 0.268$	$\sqrt{3}/2 \approx 0.866$
4	$2-\sqrt{2+\sqrt{2}} \approx 0.152$	0.924
4.946	0.1	0.95
5	0.0979	0.951
6	$2-\sqrt{2+\sqrt{3}} \approx 0.0682$	0.966
7	0.0501	0.975
15.7	0.01	0.995

The case of one strand is degenerate but included to show that packing bundles as single strands does not let the strands penetrate at all. Two-strand bundles do not have a high bundle fill factor, k_{pb}, compared with N_s = 3 or 5 but this is compensated somewhat by the high penetration. When phased to penetrate maximally, the bundles, do so for almost 30% of the bundle radius.

As strands are added to bundles, their penetration decreases (as does ripple in a polyphase rectifier) until for about 16 strands, the penetration is less than 1%. Thus, for $N_s >> 1$, penetration becomes negligible. However, bundles with few strands have proximity-effect advantages over bundles with many strands, and the above table can be used to assess their benefits. Additionally, the outer sub-bundles of Litz wire are optimally of a low number of sub-bundles to minimize eddy currents, and the above table can apply to the sub-bundles.

The assumption of the table is that the turn length of a bundle is divisible by bundle pitch—that is, the turn length is an integer multiple of the pitch so that the adjacent bundle "waves" are in-phase and hence the maximum protrusion of one bundle fits into the minimum of another, as shown in Fig. 1. However, with existing winding methods, achieving the in-phase arrangement of adjacent wire bundles is unfeasible. So, in the general case, adjacent bundles will be "out of sync" and an average penetration might more accurately apply in practice by reducing penetration by 2, an average of r_{bw}" and zero.

As a consequence of bundle penetration, and given a previous article's coverage of the twist factor (see the reference) and the effects of twisting (resulting in r_{bw}'), both factors affect bundle packing factor. The twisted r_{bw}, which is r_{bw}', is used in the above formulas for r_{bw} so that twisted bundles that penetrate each other can also be accounted for in the winding model. While twisting increases bundle radius (to the disadvantage of reducing packing factor), the penetration factor in part compensates for it by increasing packing for bundles with a smaller (typically 5 or less) number of strands.

Reference

"The Geometry of Twisted Wire Bundles" by Dennis Feucht, How2Power Today, July 2018.

About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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