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Managing Pulse Top Decay Improves Accuracy Of Current Sensing Circuits

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When a current transformer senses current in switched-mode power supplies (SMPSs) and other applications, the current transformer and the associated burden resistor can alter the shape of the switching waveform as seen by the current sensing circuit. This change in the shape of the switching pulse can be described as pulse top decay and it degrades the accuracy of the current sense signal. In cases where the design requires that the pulse top slope stay within certain limits, pulse top decay can be minimized by sacrificing the output pulse amplitude. This article quantifies the effect of the current sensing circuit on the pulse decay, providing guidance on how to properly determine the value of the burden resistor.

While previously developed techniques have attempted to quantify this effect of pulse top decay, I have found that they are not sufficiently accurate. So the analysis presented here overcomes that limitation. Besides being useful in designing current sensing circuits for conventional SMPS designs, this analysis can also be applied to sensing of any switching devices that need current scaling such as those used in large particle physics experiments.

This article also aims to show that the secondary side of a current transformer should provide the inductance energy discharge through a controllable voltage limiter, which ensures the zero volt-second integral of the current transformer.

Modeling The Current Sensing Circuit

Fig. 1 shows a current sense circuit based on a current transformer. When the primary current I_p flows as shown by the arrow in Fig. 1, the current transformer voltage goes positive on the anode of D215 and cathode of D250. D215 conducts, and practically the whole secondary current I_S flows through the burden resistor R229.

The primary current's pulse top usually has some slope, which is reflected by the secondary current but not with complete fidelity: the secondary winding current decays while the primary current does not change at the pulse top, and thus the secondary voltage drop across the burden resistor R229 decays as well. The decay time constant depends on the inductance of the current transformer's secondary winding and the value of the burden resistor.

In addition to the issue of pulse top decay, proper design of the current sense circuit requires attention to core demagnetization. In order to ensure the core demagnetization, it is desirable to use diodes D250 and D251, which limit voltage on the D215 anode during the core demagnetization (flyback) process. The flyback voltage should be limited to the value that does not hurt fulfillment of the volt-second integral condition. A method for doing this will be discussed in a subsequent section. But first we will model the current sense circuit with the objective of deriving an expression for Vout that allows us to predict pulse top decay and make adjustments to correct it.

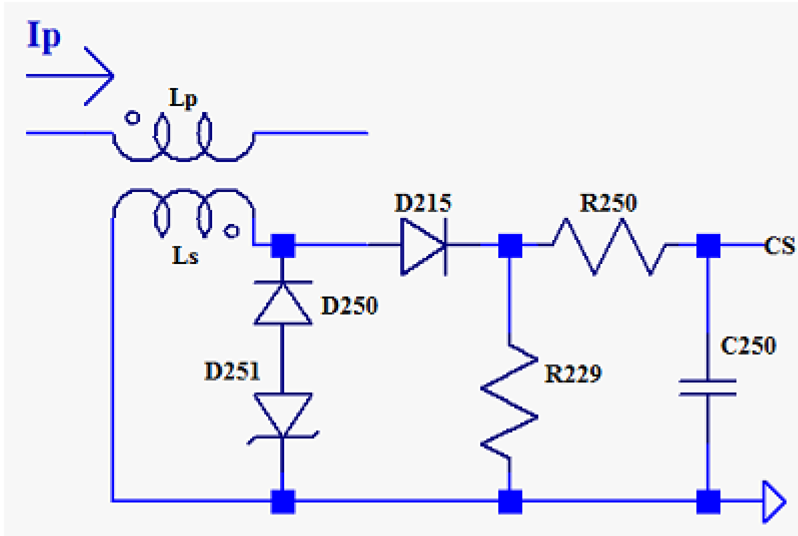

Fig. 1. Current sensing circuit based on current transformer.

Primary current I_p builds the current pulse top.

Let's define the current in the secondary winding as I_S and designate the number of turns in the primary and secondary windings as N_1 and N_2, respectively. Also, we'll designate the coupling coefficient showing the portion of the primary side energy that is transferred to the secondary side as k.

$$I_s(s) = I_p(s) \cdot \frac{N_1}{N_2} \cdot k \quad (1)$$

The secondary side always has a burden resistor that converts current into voltage V_{out}. So, the secondary impedance is actually a parallel connection of the secondary inductor and burden resistor. We can neglect the diode D215 action because the feedback system of the power supply will compensate for all voltage drop deviations across D215, and the voltage pulse shape across R229 should be sustained.

The secondary side impedance is:

$$Z(s) = \frac{s \cdot L_s}{1 + s \cdot \tau} \quad (2)$$

where

$$\tau = {}^{L_s}/_{R229}.$$

The output voltage looks this way:

$$V_{out}(s) = I_s(s) \cdot Z(s)$$

Taking the expression for $I_S(s)$ from equation (1) and the expression for Z(s) from equation (2), we get:

$$V_{out}(s) = I_p(s) \cdot \frac{N_1}{N_2} \cdot k \cdot \frac{s \cdot L_s}{1 + s \cdot \tau} \tag{3}$$

Defining the pulse duration in the primary winding as t_d, we obtain an expression for the primary current in the time domain:

$$I_p(t) = I_p \cdot \left(\Phi(t) - \Phi(t - t_d)\right) \tag{4}$$

To simplify this expression, apply the LaPlace transform:

$$I_p \cdot \left(\Phi(t) - \Phi(t - t_d)\right) \,|_{assume, t_d > 0}^{Laplace} \rightarrow -\frac{I_p \cdot (e^{-s \cdot t_d} - 1)}{s}$$

which leads to the following LaPlace expression for the output voltage:

$$V_{out}(s) = -\frac{I_p \cdot (e^{-s \cdot t_d} - 1)}{s} \cdot \frac{N_1}{N_2} \cdot k \cdot \frac{s \cdot L_s}{1 + s \cdot \tau} \tag{5}$$

Going back to the time domain, we obtain

$$-\frac{I_p \cdot L_s \cdot N_1 \cdot k \cdot (e^{-s \cdot t_d} - 1)}{N_2 \cdot (\tau \cdot s + 1)} \,|_{assume, t_d > 0}^{invLaplace} \rightarrow -\frac{I_p \cdot L_s \cdot N_1 \cdot k \cdot e^{-\frac{t}{\tau}} - I_p \cdot L_s \cdot N_1 \cdot k \cdot e^{-\frac{t - t_d}{\tau}} \cdot \Phi(t - t_d)}{N_2 \cdot \tau}$$

By simplifying the above expression, we obtain the following expression for Vout in the time domain:

$$V_{out}(t) = \frac{I_p \cdot L_s \cdot N_1 \cdot k \cdot \left(e^{-\frac{t}{\tau}} - e^{-\frac{t - t_d}{\tau}} \cdot \Phi(t - t_d)\right)}{N_2 \cdot \tau} \tag{6}$$

Equation (6) describes the output voltage of a current transformer. It has a Heaviside function that stems from the pulsating input current as expressed in equation (4).

From (6), we can derive an expression for the current transformer output slope that accounts for the influence of the burden resistor, which then enables us to determine a burden resistor value that ensures the necessary slope. A thorough analysis of this function makes us introduce a very small variation Δt of the time in the t_d area, otherwise the integral in the LaPlace transform will not converge:

$$V_{out}(s) = \frac{I_p \cdot L_s \cdot N_1 \cdot k \cdot \left(e^{-\frac{t}{\tau}} - e^{-\frac{t - t_d}{\tau}} \cdot \Phi(t - t_d)\right)}{N_2 \cdot \tau}$$

substitute,

$$t = t_d - \Delta t \rightarrow$$

$$V_{out}(t_d - \Delta t) = \frac{I_p \cdot L_s \cdot N_1 \cdot k \cdot e^{-\frac{t_d - \Delta t}{\tau}} - I_p \cdot L_s \cdot N_1 \cdot k \cdot e^{\frac{\Delta t}{\tau}} \cdot \Phi(-\Delta t)}{N_2 \cdot \tau}$$

Therefore,

$$V_{out}(t_d - \Delta t) = \frac{I_p \cdot L_s \cdot N_1 \cdot k \cdot e^{-\frac{t_d - \Delta t}{\tau}} - I_p \cdot L_s \cdot N_1 \cdot k \cdot e^{\frac{\Delta t}{\tau}} \cdot \Phi(-\Delta t)}{N_2 \cdot \tau} \qquad (7)$$

The slope begins at t = 0 and ends at $t = t_d - \Delta t$

At the beginning of the slope at t = 0

$$V_{out}(0) = \frac{I_p \cdot L_s \cdot N_1 \cdot k}{N_2 \cdot \tau} \qquad (8)$$

At the end of the slope

$$V_{out}(t_d - \Delta t) = \frac{I_p \cdot L_s \cdot N_1 \cdot k \cdot e^{-\frac{t_d - \Delta t}{\tau}}}{N_2 \cdot \tau} \qquad (9)$$

Defining the voltage decay during the pulse duration as:

$$\Delta V_{out} = V_{out}(0) - V_{out}(t_d - \Delta t)$$

and plugging in equations (8) and (9) for $V_{out}(0)$ and $V_{out}(t_d - \Delta t)$, we get:

$$\Delta V_{out} = \frac{I_p \cdot L_s \cdot N_1 \cdot k \cdot \left(e^{-\frac{t_d - \Delta t}{\tau}} - 1\right)}{N_2 \cdot \tau} \qquad (10)$$

The slope as the relative pulse top decline is:

$$\frac{\Delta V_{out}}{V_{out}(0)} = \frac{\frac{I_p \cdot L_s \cdot N_1 \cdot k \cdot \left(e^{-\frac{t_d - \Delta t}{\tau}} - 1\right)}{N_2 \cdot \tau}}{\frac{I_p \cdot L_s \cdot N_1 \cdot k}{N_2 \cdot \tau}} = \delta$$

, which after simplifying gives

$$\delta = 1 - e^{-\frac{1}{\tau}(t_d - \Delta t)} \tag{11}$$

Solving equation (11) with respect to τ, we obtain:

$$\tau = -\frac{t_d - \Delta t}{ln(1-\delta)} \tag{12}$$

Hence

$$\frac{L_s}{R229} = -\frac{t_d - \Delta t}{ln(1-\delta)}$$

and

$$R229 = -\frac{L_s \cdot ln(1-\delta)}{t_d - \Delta t}$$

As $\Delta t << t_d$, the following holds true:

$$R229 = -\frac{L_s \cdot ln(1-\delta)}{t_d} \tag{13}$$

A Design Example

To demonstrate the application of the equations derived above, we'll present a numerical example.

We assume the following values for the current sensing circuit:

L_s = 60 mH

δ = 0.002

Dmax =0.98

N_1 = 1

N_2 = 200

k = 0.95

f_{sw} = 125 kHz

We can then calculate the following parameters:

I_p = (2500 W/42 V)• D_{max} =58.333 A

t_d = (D_{max}/f_{SW}) =7.84 x 10^{-6} s

R229 = (L_s•ln(1- δ))/t_d = 15.321 Ω

τ = t_d/(ln (1- δ)) = 3.916 x 10^{-3} s

τ = L_s/R229 = 3.916 x 10^{-3} s

Then, with these values we can use equations (4) and (6), which are repeated here, to plot the primary pulse current $I_p(t)$ and output voltage as shown in Fig. 2.

$$I_p(t) = I_p \cdot (\Phi(t) - \Phi(t - t_d))$$

$$V_{out}(t) = \frac{I_p(t) \cdot L_s \cdot N_1 \cdot k \cdot \left(e^{-\frac{t}{\tau}} - e^{-\frac{t-t_d}{\tau}} \cdot \Phi(t - t_d)\right)}{N_2 \cdot \tau}$$

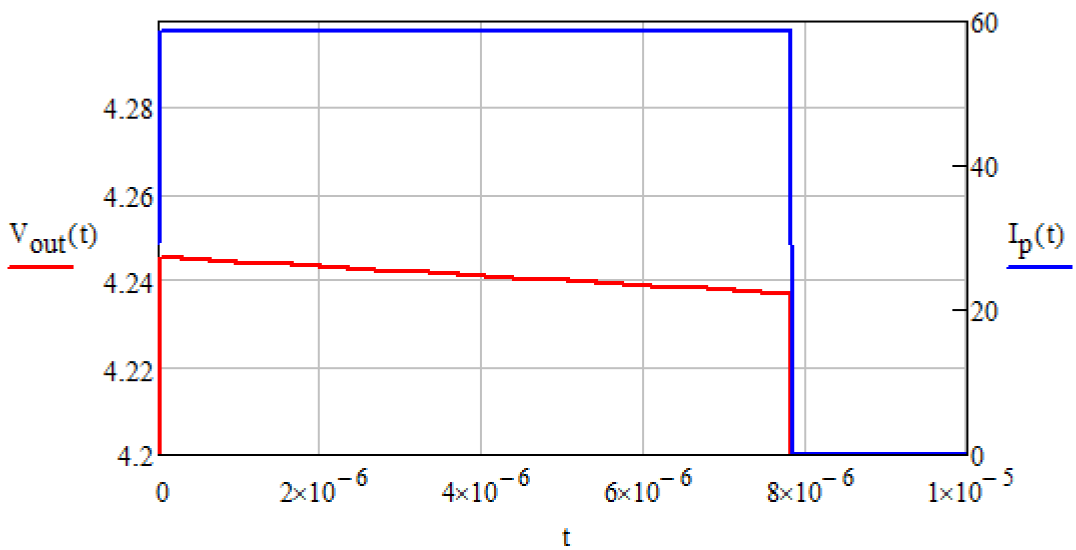

Fig. 2. The primary current pulse $I_p(t)$ and the output voltage $V_{out}(t)$ produced by the current sensing circuit in the design example.

In this example, $V_{out}(0)$ = 4.246 V. This initial output voltage may be too high for the current sense input of a PWM controller like the UCC2895, whose spec is shown in the table. This threshold on this pin to invoke current limiting is 2 V typical.

Table. Voltage limit for current sense input of UCC2895 PWM controller.

Cycle-by-cycle current limiting					
VCC_LIM, CS pin cycle-by-cycle threshold		1.94	2	2.06	V

However, it's not just the voltage level we care about, it's also the decay of the output voltage pulse as shown in Fig. 2. We can address both problems by adjusting the value of the burden resistor.

If we reduce R229 by a factor of 4 we get:

R229 = R229/4 = 15.321 Ω /4 = 3.83 Ω. To go with a standard resistor, let's assume R229 = 4.02 Ω. In that case, we recalculate the value for τ as follows:

$$\tau = {}^{L_s}/_{R229} = {}^{60\,mH}/_{4.02\Omega} = 0.015\text{ s}$$

and then use equation (6) again to replot V_{out}:

$$V_{out}(t) = \frac{I_p(t)\cdot L_s \cdot N_1 \cdot k \cdot \left(e^{-\frac{t}{\tau}} - e^{-\frac{t-t_d}{\tau}} \cdot \Phi(t-t_d)\right)}{N_2 \cdot \tau}$$

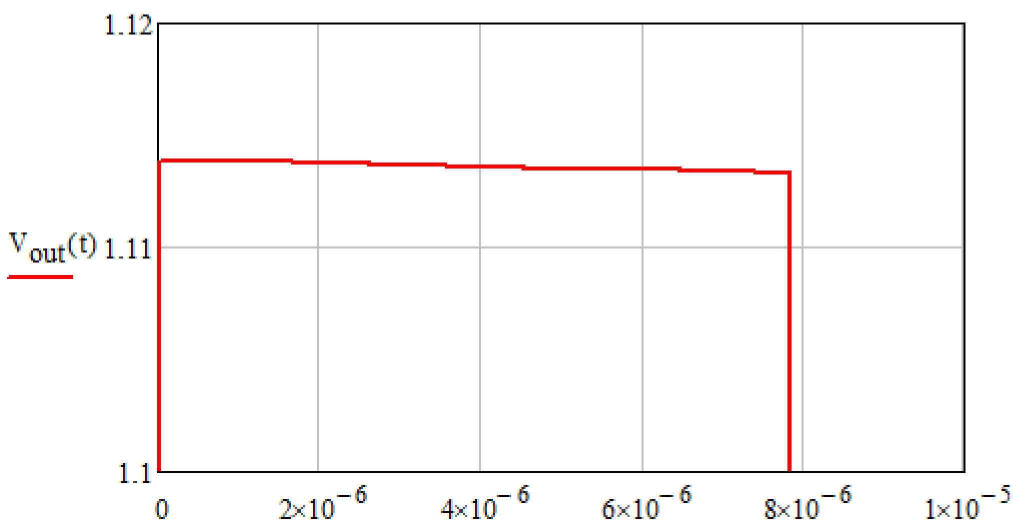

Fig. 3. Output voltage of the current sensing circuit after reducing the value of the burden resistor by a factor of 4. The amplitude of Vout has been reduced to accommodate the current sense input of the PWM controller IC and as a secondary benefit the decay of the pulse top has been reduced.

As Fig 2 shows, the slope of the output voltage pulse top has been noticeably reduced from that seen in Fig 1. Now, the burden resistor is not a factor in pulse shape and the current sensor output voltage decay is determined only by the transformer's inability to transfer the dc component of the primary current.

Burden Resistor Power Rating—Worst Case

In addition to determining its resistance value, we also need to know what power rating is needed for the burden resistor.

The burden resistor's power dissipation is defined this way:

$$P_{R229} = f_{SW} \cdot \int_0^{\frac{1}{f_{SW}}} \frac{\left(V_{out}(t)\right)^2}{R229} dt = 0.302 \text{ W}$$

Assuming the resistor's power rating must be derated with temperature, it is reasonable to select a 1-W resistor in this case.

Primary current I_p Interrupts—Reaction Of The Secondary Side

When the primary current interrupts, the secondary voltage V_{sec} changes polarity, and goes high so that the full volt-second integral for the period equals zero.

$$V_{sec} \cdot (1 - D_{max}) \cdot \frac{1}{f_{SW}} = D_{max} \cdot \frac{1}{f_{SW}} \cdot V_{out}(0)$$

(This satisfies the condition that the volt-second integral for a period should be zero.)

$$V_{sec} = -\frac{2D_{max} \cdot V_{out}(0)}{D_{max} - 1} = 54.58 \text{ V}$$

Here the coefficient 2 was entered just because of the Heaviside function property:

Φ(0) = 0.5

This voltage V_{sec} should be secured by a transzorb having a trip voltage of about 70 V. An example would be Vishay's SMPC64AN.

Current Sensing Circuit Filter

To complete the design of the current sensing circuit, we need to calculate values for the RC filter. In order to keep the output pulse front and rear ends intact, the RC filter for the current sensing circuit should have a cutoff frequency of about 10X the switching frequency, which leads to a value for the filter time constant:

$$\tau_f = \frac{1}{10 \cdot 2\pi \cdot f_{SW}} = \frac{1}{10 \cdot 2\pi \cdot 125 \times 10^3 Hz} = 1.273 \times 10^{-7} \text{s}$$

To obtain this value, we calculate the RC values as follows:

τ_f = R250•C250

Let's assume a resistor value of R250 = 1.0 kΩ. Then,

$$C250 = \frac{\tau_f}{R250} = 1.273 \times 10^{-10}\text{F}$$

This corresponds to 127 pF and for the sake of choosing standard values, we'll say C250 = 200 pF.

Finally, we can verify the cutoff frequency of the RC filter:

$$f_c = \frac{1}{2\pi \cdot R250 \cdot C250} = 7.958 \times 10^5 \frac{1}{s}$$

Verification By Simulation

We can check the accuracy of our circuit model by simulating the current sense circuit from our design example. Fig. 4 shows the primary current seen on the current transformer input and Fig. 5 shows the simulated output.

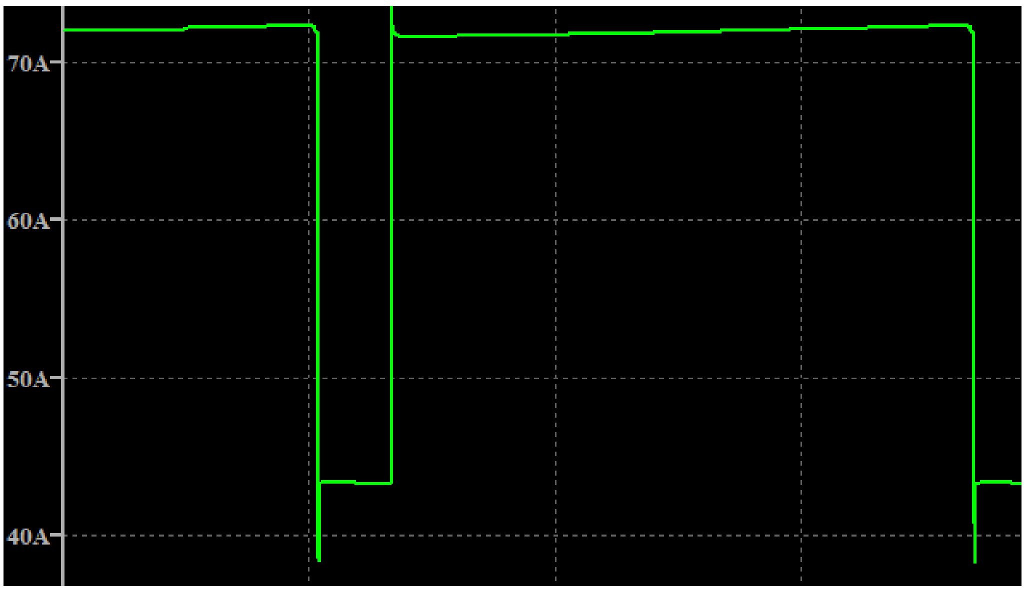

Fig. 4. Simulation of primary winding current for the example current sensing circuit.

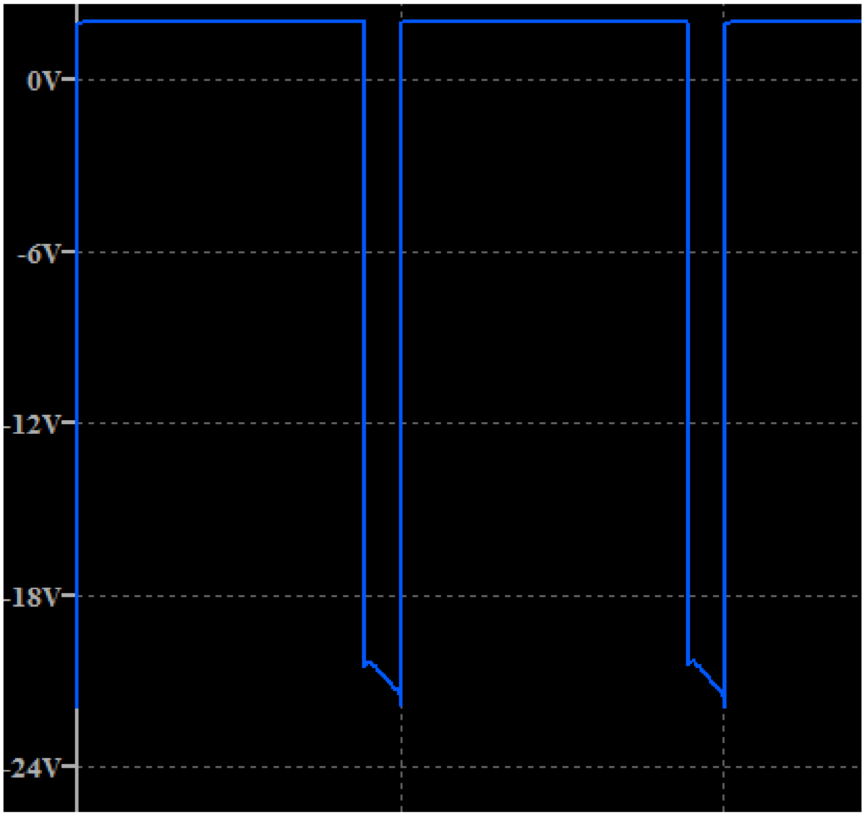

Fig. 5. Simulation of the voltage across the secondary winding of the current transformer for the example current sensing circuit. No pulse top decay is observed.

A proper design of the current transformer allows us to represent the primary winding current with the burden resistor voltage drop with any pre-determined accuracy.

About The Author

Gregory Mirsky is a senior electrical engineer with Continental Automotive Systems in Deer Park, Ill., which he joined in March 2015. In his current role, Gregory performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification.

He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

For more information on current sensing, see How2Power's Design Guide, and do a Keyword search on "current sense".