Designing Energy Storing Inductors Properly

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It is hard to find any electronic power device that does not utilize at least one inductor that stores magnetic energy for a while and then releases it when required. Usually, these inductors operate at a high frequency of tens to thousands of kilohertz and create a current ripple that depends on the inductors' parameters. This current ripple is created by the process of the inductor's magnetic core magnetizing and demagnetizing, which is necessary for efficient usage of these inductors and not allowing their cores to saturate.

In many implementations, like boost converters, inductors operate with significant dc bias. This bias brings the operating point on the core's B-H curve closer to the saturation area. This factor is often overlooked, and inductors in energy-storing implementations are being designed without taking into account the bias. This effect actually limits available flux density in the core and to compensate for this drawback, a larger core size should be selected to permit the required current ripple in the inductor.

Another factor that is often disregarded, relates to the fact that magnetic energy requires some physical volume to be stored. This volume is inherently tied to the inductor magnetic core properties. The physical volume of a magnetic core also depends on the power the inductor is required to transfer. Thus, magnetic core size depends strictly on how much power should be stored by the magnetic core, with the assumption that the same amount of power should be released to the load. The time interval, intended for storing the magnetic energy, and time interval, dedicated to the magnetic energy release to the load, define the power device operating duty-cycle.

It is conventional to make a gap in high-permeability ferrite magnetic cores to prevent them from saturating by reducing the effective magnetic core material permeability. This leads to an uneven power density distribution over the core, so that the small gap volume takes the whole power density. As a result, the core and wire heat up tremendously. Therefore, usage of the "distributed gap" material for the inductors that store energy, is a must because this material stores the magnetic energy in the whole volume.

This article attempts to show that when designing an energy-storing inductor, one should consider not just the current ripple in the coil and filter capacitors but also the dc biasing current and power that the inductor under design should operate at. It may be useful for those who design diverse types of power converters, filters subject to dc biasing, solenoids and other electromagnetic components such as electromagnetic accelerators and weapons.

DC Bias Limits Inductance

Very often inductors operate at significant dc biasing. Such is the case with boost converter inductors. To understand the impact of the dc bias, we need to determine its effect on permissible magnetic flux density in the core, which defines the permissible number of turns. Let's begin by noting that I_{DC} is the dc current through the inductor.

Magnetic flux density B_{DC} created in the core, having the magnetic path length l_m, by the magnetic field strength H_{DC} is defined as

$$B_{DC} = H_{DC} \cdot \mu_0 \cdot \mu_1 \qquad (1)$$

But per Ampere's Law, the field strength H_{DC} depends on the magnetizing current I_{DC} and the number of winding turns N_W as follows:

$$H_{DC} \cdot l_m = I_{DC} \cdot N_W \qquad (2)$$

Deriving H_{DC} from (2), we obtain:

$$H_{DC} = \frac{I_{DC} \cdot N_W}{l_m} \tag{3}$$

Then, plugging in equation (3) into (1), we get an expression for magnetic flux density that includes dc current through the inductor:

$$B_{DC} = \frac{I_{DC} \cdot N_W}{l_m} \cdot \mu_0 \cdot \mu_1 \tag{4}$$

Equation (4) shows that, for a given dc current, in order to keep the flux density in the core lower, the core should have a longer magnetic path length or fewer number of turns. The same expression (4) defines the position of the initial operating point on the B-H curve, which should be selected with respect to the core saturation flux density B_{sat}. The difference ΔB between the B_{sat} and B_{DC} defines the headroom for the core flux density change during the inductor operation (see the figure.)

$$\Delta B = B_{sat} - B_{DC} \tag{5}$$

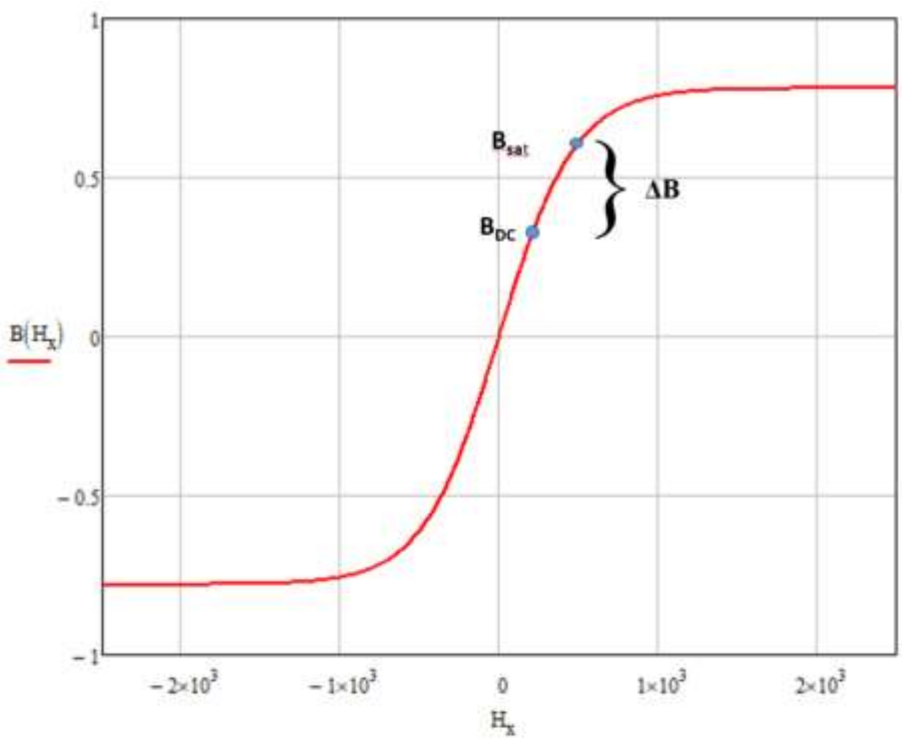

Figure. Since BDC varies directly with inductor current, IDC, as per equation (4), it helps to define the headroom for the core flux density change (ΔB) during inductor operation.

Let's designate a portion of the B-H loop dedicated to the ac flux density change ΔB as α

$$\alpha = \frac{\Delta B}{B_{DC}}$$

which means

$$B_{DC} = \frac{B_{sat}}{\alpha + 1} \tag{6}$$

Usually the given parameters are output power P_{OUT} and supply voltage V_{in} along with the converter efficiency estimate η. They define the inductor current RMS value:

$$I_{RMS} = \frac{P_{out}}{\eta \cdot V_{in}} \tag{7}$$

The RMS value is composed of two values—the I_{DC} and the ripple component amplitude, defined by the inductor—I_{rip}. This value is a half of the ripple current swing

$$I_{RMS} = \sqrt{\frac{{I_{DC}}^2 + {I_{rip}}^2}{2}} \tag{8}$$

Solving (7) with respect to I_{DC}, we get:

$$I_{DC} = \sqrt{2 \cdot {I_{RMS}}^2 - {I_{rip}}^2} \tag{9}$$

Substituting (7) into (9), we obtain:

$$I_{DC} = \sqrt{2 \cdot \left(\frac{P_{out}}{\eta \cdot V_{in}}\right)^2 - {I_{rip}}^2} \tag{10}$$

It is reasonable to have the ripple as a portion of the RMS value, which can be expressed by the same α value:

$$I_{rip} = \alpha \cdot I_{DC} \tag{11}$$

Then

$$I_{DC} = \frac{\sqrt{2} \cdot P_{out} \cdot \sqrt{\alpha^2 + 1}}{V_{in} \cdot \eta \cdot \alpha^2 + V_{in} \cdot \eta} \tag{12}$$

obtained after substitution and simplification.

Formula (12) allows us to define the core flux density that occurs due to the dc current through this core by plugging it in into equation (4) for the I_{DC}.

$$B_{DC} = \frac{\frac{\sqrt{2} \cdot P_{out} \cdot \sqrt{\alpha^2 + 1}}{V_{in} \cdot \eta \cdot \alpha^2 + V_{in} \cdot \eta} \cdot N_W}{l_m} \cdot \mu_0 \cdot \mu_1 \qquad (13)$$

Formula (6) limits the value of the flux density in the core generated by the dc component of the input current or limits the number of turns for this specific inductor:

$$B_{DC} = \frac{B_{sat}}{\alpha + 1}$$

Therefore from (6) and (13), we get:

$$\frac{B_{sat}}{\alpha + 1} = \frac{\frac{\sqrt{2} \cdot P_{out} \cdot \sqrt{\alpha^2 + 1}}{V_{in} \cdot \eta \cdot \alpha^2 + V_{in} \cdot \eta} \cdot N_W}{l_m} \cdot \mu_0 \cdot \mu_1 \qquad (14)$$

Now, it is reasonable to define, what maximum number of winding turns is allowed to keep the core beyond saturation, by using formula (14):

$$N_W = \frac{\sqrt{2} \cdot B_{sat} \cdot V_{in} \cdot \eta \cdot l_m \cdot \sqrt{\alpha^2 + 1}}{2 \cdot P_{out} \cdot \mu_0 \cdot \mu_r \cdot (\alpha + 1)} \qquad (15)$$

We have got the inductor number of turns limitation but the core geometry is still unknown. This limit on the number of turns is a very important result since it places an upper limit on the inductor's maximum achievable inductance.

Energy Storage Requirement Dictates The Core's Minimum Volume

The inductor being designed is intended to store energy for the time defined by the duty-cycle, and release it over the rest of the period. Energy may be stored in the core volume only, provided there is no gap in the core.

The magnetic energy W_m at flux density B_m and magnetic field strength H_m stored in the magnetic core for the time interval, defined by the duty-cycle D_C, having volume Vol is

$$W_m = \frac{B_m \cdot H_m}{2} \cdot \mathrm{Vol} \qquad (16)$$

Assuming the duty-cycle is unity (i.e., 100%), we can calculate the power P_m the core can handle as

$$P_m = \frac{B_m \cdot H_m \cdot Vol}{2} \cdot \frac{f_{SW}}{D_c} \qquad (17)$$

Hence, from 17 we can determine the core volume at this maximum power level:

$$Vol = \frac{2P_m \cdot D_c}{\mu_0 \cdot \mu_r \cdot {H_m}^2 \cdot f_{SW}} \qquad (18)$$

Now, from Ampere's Law we know that

$$H_m = \frac{I_m \cdot N_W}{l_m} \tag{19}$$

So, if we plug this expression for H_m into (18), we obtain:

$$Vol = \frac{2 \cdot P_m \cdot {l_m}^2 \cdot D_c}{{I_m}^2 \cdot {N_W}^2 \cdot f_{SW} \cdot \mu_0 \cdot \mu_r} \tag{20}$$

If we then plug in equation (15) for N_W and simplify, we get an expression for the minimum core volume that accounts for both the maximum power level the core must handle and the turns limitation:

$$Vol = \frac{4 \cdot D_c \cdot P_{out} \cdot \mu_0 \cdot \mu_r \cdot (\alpha + 1)^2}{{B_{sat}}^2 \cdot \eta \cdot f_{SW} \cdot (\alpha^2 + 1)} \tag{21}$$

Therefore, the magnetic core volume, required to handle output power P_{out}, is directly proportional to the core material permeability and cannot be smaller than the value calculated using (21). But it can be *greater* than this value if necessary to provide higher inductance.

With regards to the fact that

$$Vol = l_m \cdot S_m \tag{22}$$

where S_m is the core cross-sectional area, we can use a core of any shape that satisfies equation (21).

Now, we can define the inductor's inductance:

$$L_m = \frac{\mu_0 \cdot \mu_r \cdot S_m}{l_m} \cdot {N_W}^2 \tag{23}$$

Equating (22) and (21), we get:

$$l_m = \frac{4 \cdot D_c \cdot P_m \cdot {P_{out}}^2 \cdot \mu_0 \cdot \mu_r \cdot (\alpha + 1)^2}{{B_{sat}}^2 \cdot {I_m}^2 \cdot S_m \cdot {V_{in}}^2 \cdot \eta^2 \cdot f_{SW} \cdot (\alpha^2 + 1)} \tag{24}$$

and define inductance from (23):

$$L_m = \frac{{B_{sat}}^2 \cdot {I_m}^2 \cdot {N_W}^2 \cdot {S_m}^2 \cdot {V_{in}}^2 \cdot \eta^2 \cdot f_{SW} \cdot (\alpha^2 + 1)}{4 \cdot D_c \cdot P_m \cdot {P_{out}}^2 \cdot (\alpha + 1)^2} \tag{25}$$

Assuming that

$$I_m{}^2 \cdot V_{in}{}^2 = \frac{P_{out}{}^2}{\eta^2} \qquad (26)$$

we can re-write equation (25) as

$$L_m = \frac{B_{sat}{}^2 \cdot N_W{}^2 \cdot S_m{}^2 \cdot f_{SW} \cdot (\alpha^2 + 1)}{4 \cdot D_c \cdot P_m \cdot (\alpha + 1)^2} \qquad (27)$$

Expression (27) gives us the value for the inductance L_m, which fully accounts for the effect of dc bias.

Example Of A Boost Inductor Design

Now let's apply the equations derived above to specify the key parameters for a boost inductor. The following parameters are determined by constraints of the boost converter design:

P_{out} = 2400 W

μ_r = 60

B_{sat} = 0.5 T

f_{SW} = 125 kHz

l_m = 109 mm

V_{in} = 48 V

D_c = 0.8

η = 0.98

$P_m = P_{out}/\eta = 2.449 \times 10^3$ W

α= 0.2

Plugging these parameters into equation (21) for core volume gives

$$Vol = \frac{4 \cdot D_c \cdot P_{out} \cdot \mu_0 \cdot \mu_r \cdot (\alpha+1)^2}{B_{sat}{}^2 \cdot \eta \cdot f_{SW} \cdot (\alpha^2+1)} = 0.026 \text{ L}$$

Again, this is the minimum magnetic volume that ensures the inductor's operability. Any bigger size is beneficial until the core loss issue kicks in.

Next we apply this value for core volume to determine the necessary inductance value. First, we calculate magnetic cross-sectional area:

$$S_m = \frac{Vol}{l_m} = 240.187 \text{ x } 10^{-6} m^{2x10^0}$$

Then, determine the turns limit:

$$N_W = round\left[\frac{\sqrt{2}\cdot B_{sat}\cdot V_{in}\cdot \eta \cdot l_m \cdot \sqrt{\alpha^2+1}}{2\cdot P_{out}\cdot \mu_0 \cdot \mu_r \cdot (\alpha+1)}\right] = 9$$

So, finally we can calculate inductance:

$$L_m = \frac{{B_{sat}}^2 \cdot {N_W}^2 \cdot {S_m}^2 \cdot f_{SW} \cdot (\alpha^2+1)}{4\cdot D_c \cdot P_m \cdot (\alpha+1)^2} = 13.458 \text{ x } 10^{-6}\ H$$

From the inductance value, we determine a value of input current ripple:

$$\Delta I = \frac{D_c \cdot V_{in}}{L_m \cdot f_{SW}} = 22.827\ A$$

And compare it against the input current value

$$I_{in} = \frac{P_{out}}{\eta \cdot V_{in}} = 51.02\ A$$

If we see that the current ripple is too high, we can increase the inductor volume a little:

In this case, let's select Vol = 0.05 L, which is approximately double our calculated minimum core volume. This new core has l_m = 159 mm. That leads to a magnetic cross-sectional area of

$$S_m = \frac{Vol}{l_m} = 314.465 \text{ x } 10^{-6} m^{2x10^0}$$

and, with our new value of l_m, comes a new turns limit:

$$N_W = round\left[\frac{\sqrt{2}\cdot B_{sat}\cdot V_{in}\cdot \eta \cdot l_m \cdot \sqrt{\alpha^2+1}}{2\cdot P_{out}\cdot \mu_0 \cdot \mu_r \cdot (\alpha+1)}\right] = 12$$

Then, finaly we get the new inductance value:

$$L_m = \frac{{B_{sat}}^2 \cdot {N_W}^2 \cdot {S_m}^2 \cdot f_{SW} \cdot (\alpha^2+1)}{4\cdot D_c \cdot P_m \cdot (\alpha+1)^2} = 41.01 \text{ x } 10^{-6}\ \text{H}$$

which is about 3X our initial value. For this core size, we do see a lower value of current ripple:

$$\Delta I = \frac{D_c \cdot V_{in}}{L_m \cdot f_{SW}} = 7.491\ A$$

while max input current remains:

$$I_{in} = \frac{P_{out}}{\eta \cdot V_{in}} = 51.02\ A$$

As this example demonstrates, sometimes it is good to oversize the inductor in order to reduce the current ripple.

About The Author

Gregory Mirsky is a senior electrical engineer with Continental Automotive Systems in Deer Park, Ill., which he joined in March 2015. In his current role, Gregory performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification.

He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

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