How To Thermally Model Magnetic Components

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The most difficult aspect of magnetic component design is the quantification of thermal behavior. Core and winding temperatures are the ultimate limitation on how much power can be transferred through a magnetic device, yet to calculate allowable power transfer at maximum design temperatures is challenging. This article surveys some of what is involved in thermal modeling and how to conceptualize it.

First, we introduce a simplified version of Fourier's heat-rate equation, and explain how from this equation, a simple network model is derived that describes the flow of thermal power (i.e. the heating rate) via conduction and convection from a magnetic component's core and windings to the air. From this simple model of a thermal network, more complex models are derived that account for the core and winding configurations associated with the different types of cores that are popular in power electronics. Next, a thermal model flow graph is used to explain how thermal power flows through the core and windings in the different configurations.

With that as background, we discuss how assumptions about the conditions for max efficiency lead to design rules of thumb for determining maximum allowable temperature rises for the core and windings. This discussion of maximum temperature rise leads to two possible methods of determining current density and thus maximum winding loss. One is based on the condition of max efficiency where core loss and winding loss are equal, while the other method is based on thermal network analysis. The designer is advised to use both methods of determining winding power loss and to conservatively go with the higher value.

However, since thermal network analysis is made difficult by the estimation of thermal resistances, the article concludes by presenting a simpler, shape-based scheme for thermal analysis. This method determines allowable core loss from the core shape and then is refined by taking into account the effect of the core-winding configuration.

Thermal Network Models

Heat is transferred by three possible mechanisms: conduction through a material, convection of heat from a surface to air or another fluid, and radiation. All three are involved with magnetic components, though radiative transfer at component temperatures is only a few percent (46 mW/cm^2) and is often ignored as padding for the thermal safety margin. Heat flow is in many ways analogous to current in electric circuits. Fourier's heat-rate equation is the "thermal Ohm's Law". It is a differential equation that, like Maxwell's equations, can be simplified for simple geometries to

$$\Delta T = \overline{P} \cdot R_\theta$$

where ΔT is the temperature rise above the temperature of the ambient surroundings, T_A. The air is considered an infinite heat sink at constant temperature, and is analogous to a voltage source. $\overline{P}$ is the average power loss and for magnetic components, total loss also consists of average winding and core losses;

$$\overline{P}_t = \overline{P}_w + \overline{P}_c$$

Heat is thermal energy and thermal power is the heating rate. The thermal resistance through which heat flows is R_θ, with units of K/W or °C/W. In a thermal network model, power is analogous to current and is modeled by a current source. Thermal resistance, R_θ is analogous to electrical resistance, and T is analogous to voltage. A simple model is shown in Fig. 1.

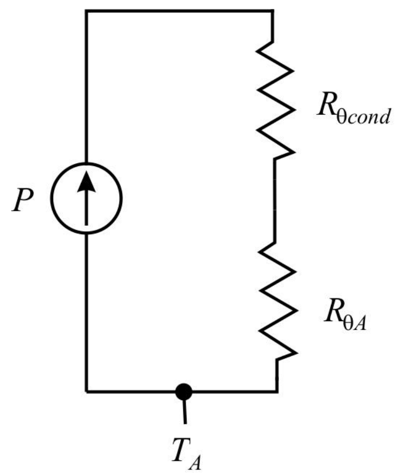

Fig. 1. Basic thermal model: thermal power, P, flows through conductive thermal resistance, $R_{\theta cond}$ to air surface and then through convective thermal resistance, $R_{\theta A}$ to the surrounding air at a temperature of T_A.

Heat flows through both the conductive core and winding material to the component surface then convects away into the air. Conduction and convection are modeled as separate resistances in series.

Core types can be categorized as having one of two possible configurations: core-winding-core (C-W-C) such as pot cores, and winding-core-winding (W-C-W) such as toroids, EE, EC, ETD, EFD, and EER cores. PQ and RM cores are somewhere in-between. These two configurations have the same form of equivalent thermal network, but they are not the same, as shown in Fig. 2. Winding heat of C-W-C cores must travel through the core to reach the air. In W-C-W cores, core heat must exit through the windings. The convection resistance, $R_{\theta A}$, depends mostly on component surface area and not on configuration.

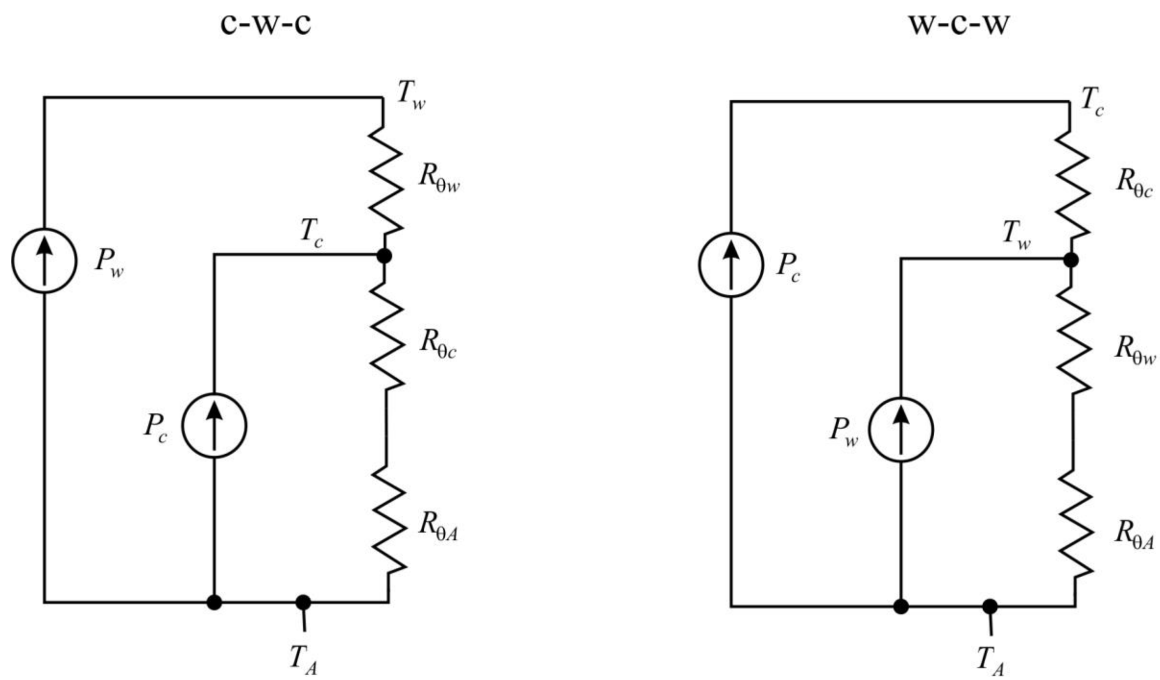

Fig. 2. Component configurations core-winding-core (left) and winding-core-winding (right) have different thermal network models. For C-W-C cores such as pot cores, all the winding heat goes through the core. For W-C-W cores such as toroids, the core heat goes through the winding.

A more general model is shown in Fig. 3. Core and winding heat flow through separate paths to the air and the heat common to both flows through $R_{\theta wc}$. In this model, $R_{\theta w}$ and $R_{\theta c}$ are configuration-dependent and are not approximately equal between configurations as in Fig. 2.

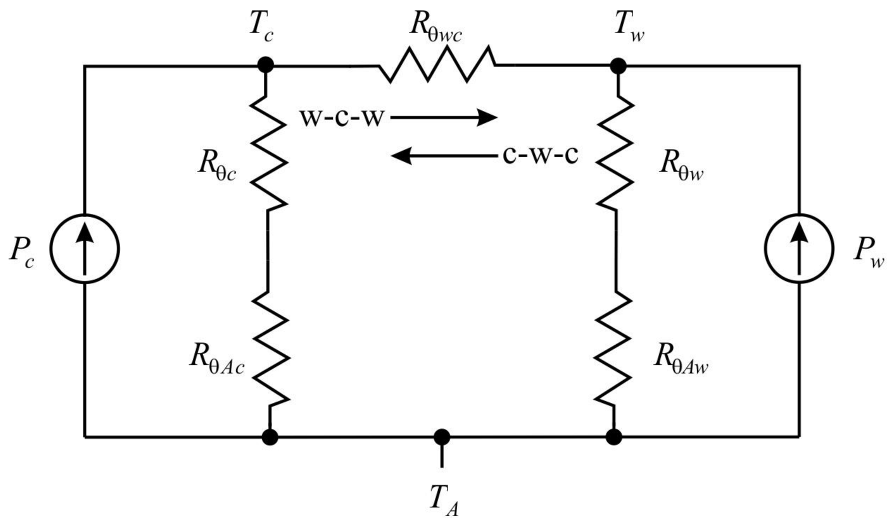

Fig. 3. A more general magnetic-component thermal model. $R_{\theta wc}$ allows winding heat to flow through the core or core heat to flow through the winding, though $R_{\theta c}$ and $R_{\theta w}$ depend on core-winding configuration.

The more general model of Fig. 3, however, cannot represent both C-W-C and W-C-W configurations without a change in the meaning of $R_{\theta w}$ and $R_{\theta c}$ between them. In the W-C-W case, $T_c > T_w$ and some of P_c flows from left to right through $R_{\theta wc}$. In the C-W-C configuration, some of P_w flows from right to left through $R_{\theta wc}$. For C-W-C, P_w flows through $R_{\theta wc}$, which equals $R_{\theta w}$ in the Fig. 2 C-W-C model, and $R_{\theta w}$ of Fig. 3 is open-circuited so that all of P_w goes through $R_{\theta wc}$.

In the W-C-W configuration, P_c flows through $R_{\theta wc} = R_{\theta c}$ of Fig. 2, and $R_{\theta c}$ of Fig. 3 is open-circuited. For RM and PQ cores, $R_{\theta c}$ and $R_{\theta w}$ both have finite resistances, and some of P_c and P_w go through both core and winding. Then the fraction of power that flows through $R_{\theta wc}$ is designated by the parameters f_w and f_c.

Thermal Model Flow-Graph

A thermal model can be expressed as a flow-graph, as shown in Fig. 4. The fraction of winding power that exits through the core is f_w, and the fraction of core power that exits through the winding is f_c.

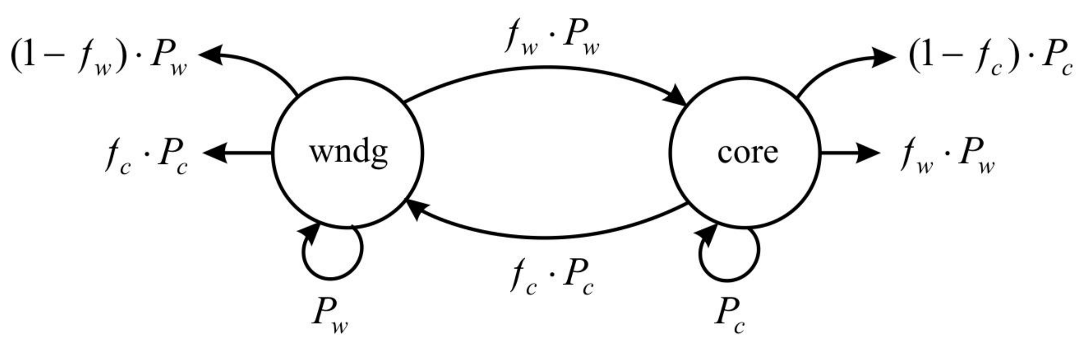

Fig. 4. Thermal flow-graph showing thermal power flows of winding and core; f_w is the fraction of winding power loss that exits to the surroundings through the core, and f_c is the fraction of core power loss that exits to the surroundings through the winding.

P_c is the power loss generated in the core and P_w is the winding loss in the winding. The flow-graph power exchange between the core and winding is structurally symmetrical though behaviorally, the flow parameter values result in asymmetrical flows, as given in Table 1 with extreme configuration values. With a C-W-C configuration of cores and windings, in the extreme case, it's possible for all of the heat generated in the windings to flow out to ambient through the cores, while none of the heat generated in the cores flows out through the windings. With a W-C-W configuration, the opposite extreme case is possible.

Table 1. Extreme cases of thermal power flow in a magnetic component as a function of core-winding configuration. 1 represents 100% of thermal power flow.

Configuration	f_w	f_c
C-W-C	1	0
W-C-W	0	1

The flows are asymmetrical not only because of structural differences in core-winding thermal paths but also because of thermal resistance values. The geometric formula for thermal resistance has the same form as that of electrical resistance;

$$R_\theta = \frac{l}{\sigma_\theta \cdot A}$$

where l is the thermal path length, A is the cross-sectional area of the path, and σ_θ is the thermal conductivity. Copper wire σ_θ is about 95 times that of ferrite core material. Wire insulation decreases overall winding thermal conductivity yet typically $R_{\theta w} << R_{\theta c}$ for comparable l and A. Thus, core heat raises the winding temperature to be comparable to that of the core in both C-W-C and W-C-W configurations. Because $R_{\theta w} << R_{\theta c}$, $\Delta T_w << \Delta T_c$ for winding and core in the direction of heat flow.

Maximum Power Transfer

Power transfer between windings is maximized whenever $\overline{P}_w \approx \overline{P}_c$ and this leads to some rules of thumb about allowable temperature rises for the windings and cores (ΔT_w and ΔT_c). Then the operating-point is at η_{max}, the peak of the transfer efficiency curve. For cores operating at η_{max} for $\eta_{max} \approx 1$, $\overline{P}_w \approx \overline{P}_c$. Thus,

$$\overline{P}_w = \frac{\Delta T_w}{R_{\theta w}} \approx \overline{P}_c = \frac{\Delta T_c}{R_{\theta c}} \Rightarrow \frac{R_{\theta c}}{R_{\theta w}} = \frac{\Delta T_c}{\Delta T_w}$$

In both C-W-C and W-C-W configurations, ΔT_w is given more design margin, set at ΔT_w = 30K, the temperature rise for typical wire table ampacity. For C-W-C, the inner winding is hotter than the core, and a reduced ΔT_w reduces T_w. For W-C-W, the core power also heats the core, and the lower ΔT_w mitigates this somewhat. The ΔT_c value of ΔT_c = 40K should, at a maximum ambient temperature of 50°C, operate typical ferrite cores at the lowest loss and yet have a negative temperature coefficient. Thus the minimum-loss temperature is typically around 90°C.

Two Thermal Analysis Schemes

The winding ampacity for a given temperature rise as given in wire tables is based on the current-density formula,

$$\frac{\tilde{J}_c(\Delta 30\,\mathrm{K})}{\tilde{J}_0} = \left(\frac{A \cdot A_w}{\mathrm{cm}^4}\right)^{-1/8}, \ \tilde{J}_0 = 4.5\ \mathrm{A/mm^2, Cu}$$

Ampacity method of determining winding current density and thus max winding power. This is based on the max-η condition and wire-table values.

The derivation of this formula[1] includes the max-η condition of $\overline{P}_w \approx \overline{P}_c$. The area product, $A{\cdot}A_w$—the magnetic cross-sectional area and window area—scales the result so that current density is reduced for higher core volume. Thermal network analysis is only loosely linked to this ampacity formula and results in another estimate of maximum power transfer. In this case, the ampacity value depends on area product, $A{\cdot}A_w$, as determined by the primary power, $\overline{P}_p$ as

$$A \cdot A_w = \left(\frac{\kappa_p \cdot D}{k_p \cdot \tilde{j}_c \cdot (2 \cdot \hat{B}_{\sim}) \cdot f_s} \right) \cdot \overline{P}_p$$

Area-product method of determining winding current density and max winding power. This is the basis for the ampacity equation.

where the area product is proportional to $\overline{P}_p$ if the parameters in the coefficient of the area-product expression are constant. However, k_p, the wire packing factor, varies with wire size, $\tilde{j}_c$ varies with core size, $\hat{B}_{\sim} \cdot f_s$ varies with core material, the current form factor κ_p varies with current ripple waveshape, and the duty ratio D varies with converter operating-point. Thus the area-product method of determining current density is a largely independent method for estimating transfer power because both core and circuit parameters assumed by it are independently chosen.

The minimum of the two power values obtained with the two methods described here is the conservative rating, and it is often the case that the ampacity-based power exceeds that of network analysis when eddy-current effects on winding resistance are included. The ampacity rating is based on static current and can be expected to be higher, though a well-designed winding might have a resistance lower than the "optimal" primary-referred ampacity-based, η_{max} winding resistance,

$$R_{wpopt} = \frac{\overline{P}_{wp}}{\tilde{i}_p^{\,2}} = \frac{\overline{P}_c / 2}{\tilde{i}_p^{\,2}} \;, \eta_{\max} \;\Rightarrow\; \overline{P}_c = \overline{P}_w = \overline{P}_{wp} + \overline{P}_{ws} = 2 \cdot \overline{P}_{wp} \Rightarrow \overline{P}_{wp} = \overline{P}_c / 2$$

where $\tilde{i}_p = I_{max}$ = primary winding ampacity. If $R_{wp} < R_{wpopt}$, then the allowable $\tilde{i}_p > I_{max}$.

Shape-Based Thermal Model

Thermal network analysis requires estimation of thermal resistances and this can be an arduous task. A different thermal design scheme (in *PMDO*, pages 75 – 90, see reference 2) avoids this problem by basing thermal analysis on shape-based reasoning and is an alternative to the network model. The worst shape for a core is a sphere because it has the lowest surface area to volume ratio and thus the highest thermal resistance for a given material. Core power loss,

$$\overline{P}_c = \overline{p}_c \cdot V$$

where $\overline{p}_c$ = core power-loss density and V = core volume. A well-designed core has uniform power loss over its volume, and the core heat leaves the core from its surface area. Commercial core shapes all have lower thermal resistance than a sphere and the extent to which their thermal resistance is lower is quantified by a size-independent factor, the *thermal shape factor*, Ξ_θ. An expression for this factor can be derived from core volume and surface area (see *PMDO*[2]). Then the improvement in allowable core power density over that of a sphere is

$$\overline{p}_c = \Xi_\theta \cdot \overline{p}_c(\text{sphere})$$

Given the magnetic frequency and the maximum $\overline{p}_c$, a maximum ΔB can then be determined from catalog core-loss graphs.

An additional refinement of this thermal model accounts for the core-winding configuration, the additional core heating from the winding, by parameter f_w. If all the winding heat goes through the core ($f_w = 1$), then under the η_{max} condition of $\overline{P}_c \approx \overline{P}_w$, the core has twice the heat, and $\overline{p}_c$ must be reduced to half. If instead the core heats only itself ($f_w = 0$), then the full value of $\overline{p}_c$ can be used for design. For estimates of f_w in-between—and *PMDO* gives PQ and RM cores a value of ⅓—then an additional factor is multiplied to the $\overline{p}_c$ formula of

$$\frac{1}{1+f_w} \approx 1 - \tfrac{1}{2} \cdot f_w \in [\,½,\, 1]$$

The full shape-based thermal formula is thus

$$\overline{p}_c = \left(\frac{1}{1+f_w}\right) \cdot \Xi_\theta \cdot \overline{p}_c(\text{sphere}) \approx (1 - \tfrac{1}{2} \cdot f_w) \cdot \Xi_\theta \cdot \overline{p}_c(\text{sphere})$$

Table 2 gives values of the different f_w factors used to calculate $\overline{p}_c$ for different values of f_w. The last column shows the error in the approximation of the f_w factor.

Table 2. f_w factors for shape-based thermal formula for core power loss.

f_w	$\left(\frac{1}{1+f_w}\right)$	$(1-\frac{1}{2}\cdot f_w)$	$(1-\frac{1}{2}\cdot f_w)$ – $\left(\frac{1}{1+f_w}\right)$
0	1	1	
0.1	0.909	0.95	
0.25	0.8	0.875	
0.333	0.75	0.833	Δ0.0833
0.414	0.707	0.793	max: Δ0.0858 (12.1%)
0.5	0.667	0.75	Δ0.0833
0.667	0.6	0.667	
0.75	0.571	0.625	
0.9	0.526	0.55	
1	0.5	0.5	

The penalty for cores having f_w = ⅓ reduces allowable $\overline{p}_c$ by × 0.833, a 16.7% reduction over f_w = 0. Only f_w is used in the shape-based formula because core heating of the winding, with its lower thermal resistance, will not raise ΔT_w by much. In setting f_c = 0, the model is simplified, though for aluminum or other windings with higher thermal resistance, it can be included in the model.

The rough approximation of f_w shows that the model is intended to be approximate, yet accurate enough to be useful for design. In the comparisons made with core manufacturer data, the curve fit is well within that required for magnetic component design.

References

1. A derivation of the ampacity formula is found in *Transformers and Inductors for Power Electronics*, W. G. Hurley and W. H. Wölfle, Wiley, 2013. A review of this book is available here.
2. *Power Magnetics Design Optimization* by D.L. Feucht, Innovatia Laboratories.
3. A general magnetics book that has more thermal modeling than most is *Inductors and Transformers for Power Electronics* by Alex Van den Bossche and Venislav C. Valchev, CRC press, Taylor & Francis Group, 2005, available from www.taylorandfrancis.com. A review of this book is available here.

About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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