How To Optimize Turns For Maximum Inductance With Core Saturation

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Some inductor applications require a minimum inductance over the full current range of inductor operation. As current increases, saturation increases and the *saturation factor*,

$$k_{sat} = \frac{\mu}{\mu_i} = \frac{\mathcal{L}}{\mathcal{L}_0} = \frac{L(i)}{L(0\,\text{A})}$$

decreases from 1 at zero current; μ is permeability, L is circuit inductance, and $\mathcal{L}$ is L referred to the magnetic field as *field inductance*, $\mathcal{L} = L/N^2$ where N is the number of turns. The quantities in the denominators are the zero-current (no saturation) values.

The typical inductor operating range of k_{sat} in power applications is 0.8 to 0.5. As turns are increased, inductance, L, increases by N^2, but so does saturation, causing k_{sat} to decrease. The circuit current, i, referred to the field as *field current, Ni* = $N \cdot i$, increases with N causing field intensity, H, to increase proportionally. Circuit inductance is related to k_{sat} by

$$L = k_{sat} \cdot L(0\,\text{A}) = k_{sat} \cdot L_0 = N^2 \cdot (k_{sat} \cdot \mathcal{L}_0)$$

N has an optimal value, N_{opt}, at maximum L because increasing saturation progressively decreases L with increasing turns (as Ni increases) while the turns themselves increase L. If the saturation curves decrease faster than N^2, then adding more turns decreases inductance. So in essence, finding the optimum number of turns (and achieving max inductance) means finding the maximum number of turns that can be wound on a core before inductance starts to decrease due to saturation. Therefore to solve for the optimal operating-point, we need an equation that models core saturation.

We begin by modeling the saturation characteristics of different cores, which allows us to compare the relative rates at which they saturate versus current. And it leads to a design formula for how much inductance will decrease for a given increase in current at a given number of turns and a given zero-current inductance.

We then derive a formula for the optimal number of turns to achieve maximum inductance at a given operating current and for a given core, followed by an equation that determines the maximum inductance value that will be obtained. (Designers can use these last two equations to evaluate the number of turns and maximum inductances for different cores.)

By deriving an equation for k_{sat} at maximum inductance, we see that k_{sat} is solely determined by core properties, and we derive some k_{sat}-related factors that will simplify the application of the formulas derived for N_{opt} and L_{max} as illustrated with a simple design example.

Approximate Magnetic Saturation Equation

Saturation as $k_{sat}(H)$ can be approximated as a mathematical function. A typical iron-powder (Fe-pwd) graph of saturation is shown in Fig. 1 from the Micrometals catalog 4G, page 18 (or catalog issue J, page 26).

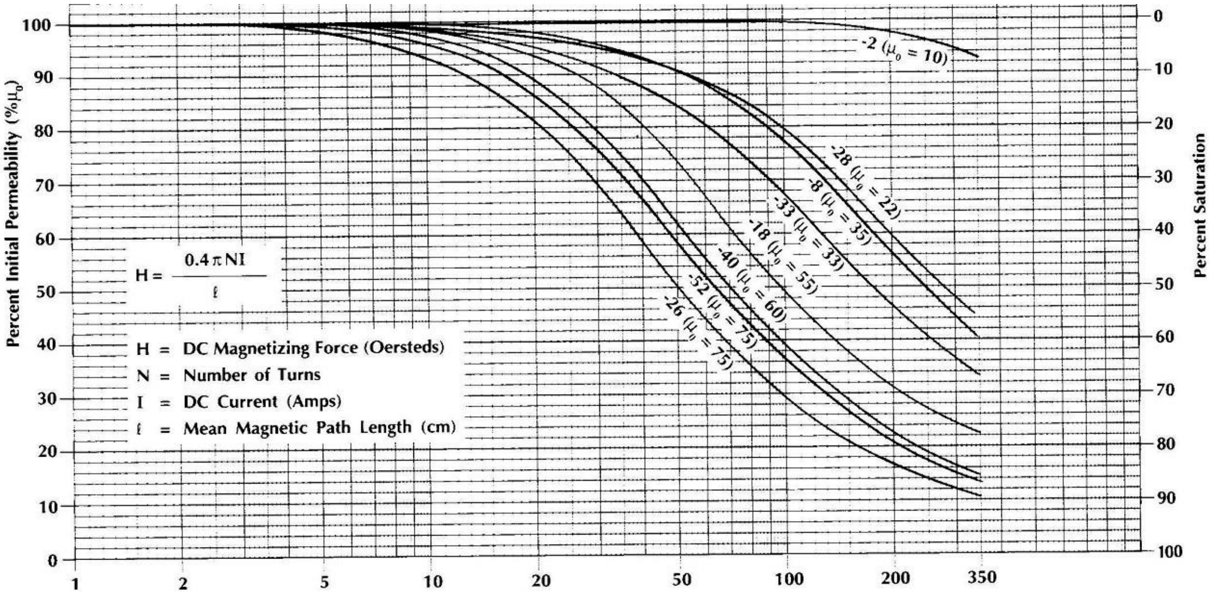

Fig. 1. Graph from Micrometals catalog showing saturation factor, $k_{sat}(H)$ for Fe-pwd core materials with different values of μ_0.

This graph plots $k_{sat}(H)$ for various core materials of Fe-pwd (in H units of Oersteds, 79.6 A/m per Oe: multiply the horizontal-axis values by 80 for approximate A/m values).

Manufacturers nowadays use exponentiated rational functions with multiple parameters and fit the saturation curves using numerical analysis. The curves can be approximated more simply over the typical design range of k_{sat} using line segments on a semi-log graph, as shown in Fig. 2. This type of plot allows us to more easily compare the saturation characteristics of cores from different manufacturers. But more importantly, this plot gives us a simple mathematical model that can be used to find the optimum operating points.

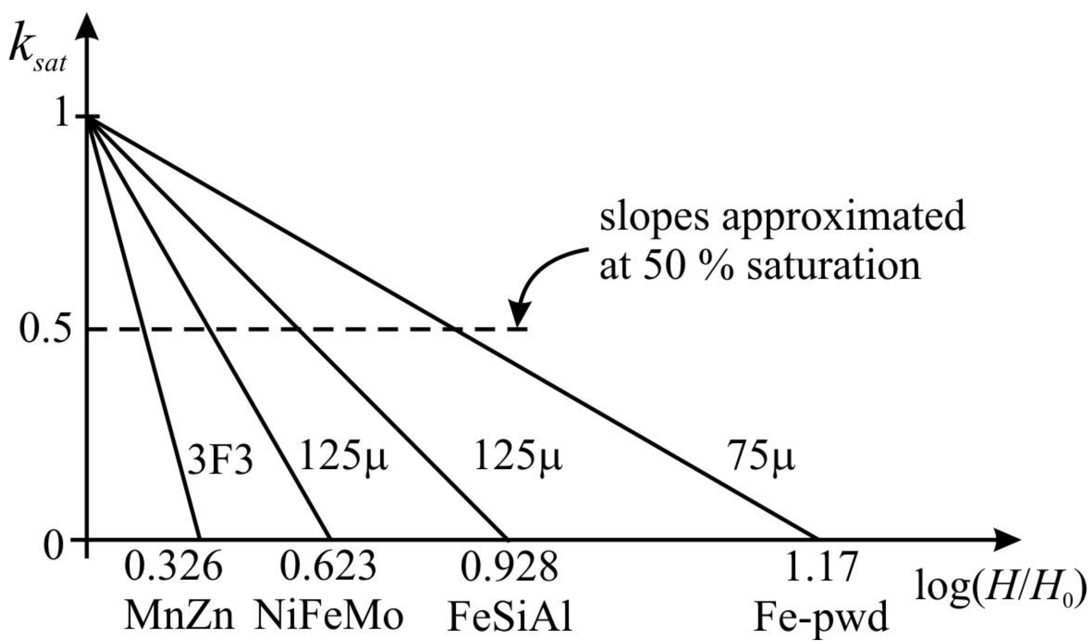

Fig. 2. Semi-log asymptotic approximation of saturation region of $k_{sat}(H)$, normalized to H_0 for various materials.

The plots are based on

$$k_{sat} = 1 - \frac{\log H - \log H_0}{\log H_T - \log H_0} = 1 - \frac{\log(H/H_0)}{\log(H_T/H_0)} = \frac{\log(H_T/H)}{\log(H_T/H_0)} \;, H \geq H_0$$

where H_T is the value of H at which the line segment intersects zero (k_{sat} = 0), and H_0 is the asymptotic breakpoint at the onset of saturation. H_0 is found on the graph by linearly extrapolating a line tangent to the plot at k_{sat} = 0.5 upward to where it intersects k_{sat} = 1, at H_0, as shown in Fig. 3 on the catalog curve for 26 material. For $H < H_0$, k_{sat} = 1

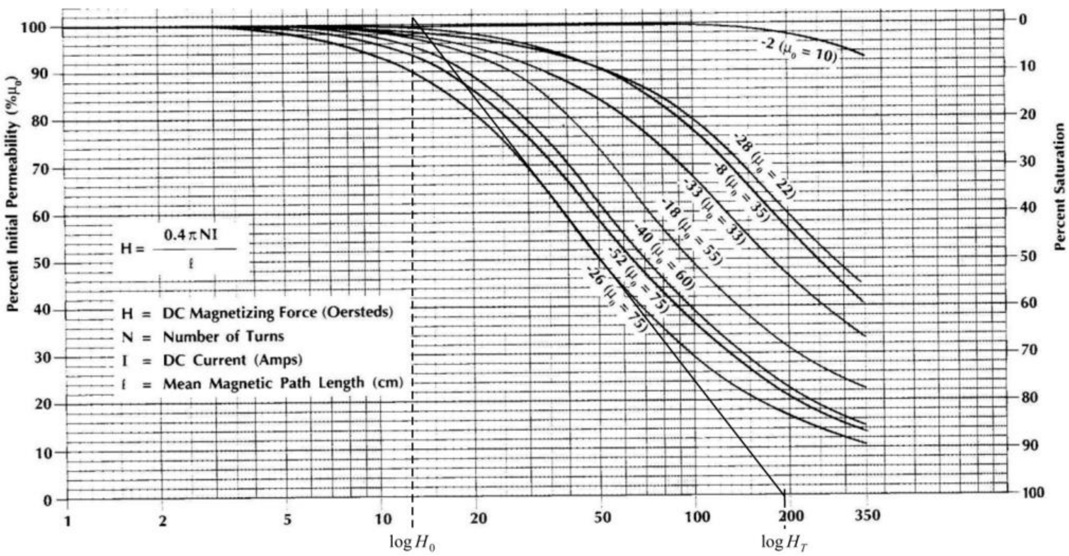

Fig. 3. Linearized approximation at k_{sat} = 0.5 of saturation region, drawn on catalog curve for 26 material.

Magnetic field intensity, H, is related to the field current, Ni, by Ampere's Law. In simplified form, it is

$$H \cdot l = Ni = N \cdot i$$

where l is the magnetic path length of the core. For a given core, l is constant and Ni scales with H. Ratios of two different Ni are equal to ratios of corresponding H. Thus we can express $k_{sat}(H)$ as $k_{sat}(Ni)$ and relate saturation to the field current. This is more useful for optimization because both N and the circuit current, i, are included in it. Thus

$$k_{sat} = \frac{\log(Ni_T / Ni)}{\log(H_T / H_0)}$$

For a given $k_{sat}(H)$, H_T and H_0 are parameters of k_{sat}, and $\log(H_T/H_0)$ is constant for a given curve.

The saturation model parameters are found by solving the k_{sat} equation above;

$$H_T = \left(\frac{H}{H_0^{k_{sat}}}\right)^{\frac{1}{1-k_{sat}}}$$

At k_{sat} = 0.5, where iron powder (Fe-pwd) curves typically inflect,

$$H_T = \left(\frac{H(0.5)}{\sqrt{H_0}}\right)^2 = \frac{[H(0.5)]^2}{H_0} \Rightarrow \log\left(\frac{H_T}{H_0}\right) = \log\left(\frac{H(0.5)}{H_0}\right)^2 = 2 \cdot \log\left(\frac{H(0.5)}{H_0}\right)$$

Values for various core materials, including the Fe-pwd material used for the asymptotic plots, are given in the following table with the H values derived from the catalog plots. The $\log(H_T/H_0)$ parameter is the decades of range of transition of the curve, a measure of how gradually it saturates. The larger the value, the more range over which saturation occurs. Fe-pwd saturates over a 1.17 decade range, the most gradual transition. Ferrites are at the other extreme and saturate abruptly to a low k_{sat} value at $H_{sat} \approx H_0$.

Table 1. Values of saturation model parameters for various core materials.

Material	μ_r	H_0, A/m	$H(0.5)$, A/m	H_T, A/m	$\log\left(\frac{H_T}{H_0}\right)$
Fe-pwd (26)	75	1035	3980	15305	1.17
FeSiAl	125	1100	3200	9309	0.928
NiFeMo	300	800	1740	3785	0.675
	125	2100	4300	8805	0.623
MnZn 3F3	2000	24.5	35.7	52.02	0.326

One useful design formula we can derive is the value of L_2 at a current I_2 given L_0 and another value, of L_1 at I_1;

$$\Delta k_{sat} = k_{sat2} - k_{sat1} = \frac{L_2}{L_0} - \frac{L_1}{L_0} = \frac{\log(I_T/I_2)}{\log(H_T/H_0)} - \frac{\log(I_T/I_1)}{\log(H_T/H_0)} = \frac{-\log(I_2/I_1)}{\log(H_T/H_0)}$$

Corresponding current parameters to those of H are

$$I_T = \frac{H_T \cdot l}{N}\ ,\ I_0 = \frac{H_0 \cdot l}{N}$$

and depend on core size for l. Then simplifying the Δk_{sat} equation,

$$\boxed{L_2 = L_1 - \frac{L_0}{\log(H_T/H_0)} \cdot \log\left(\frac{I_2}{I_1}\right)}$$

This design formula can be used to determine how much L will decrease (to L_2) if current is increased by a factor of I_2/I_1. N is constant as is L_0.

Optimal Number Of Turns

The goal is to find N that maximizes L at an operating-point, I, where

$$L = N^2 \cdot (k_{sat} \cdot \mathscr{L}_0) = N^2 \cdot \mathscr{L}_0 \cdot \frac{\log(Ni_T / N \cdot I)}{\log(H_T/H_0)}$$

Noting that $\log(x) = \ln(x)/\ln(10)$ and that

$$\frac{d}{dN}\ln[f(N)] = \frac{1}{f(N)} \cdot \frac{d}{dN} f(N)$$

then $N = N_{opt}$ at maximum L is found from $dL/dN = 0$. Taking the derivative,

$$\frac{dL}{dN} = 2 \cdot N \cdot \mathcal{L}_0 \cdot \frac{\log(Ni_T / N \cdot I)}{\log(H_T / H_0)} + N^2 \cdot \frac{\mathcal{L}_0}{\log(H_T / H_0) \cdot \ln 10} \cdot \left[\left(\frac{N \cdot I}{Ni_T} \right) \cdot \left(\frac{Ni_T}{I} \right) \cdot \left(-\frac{1}{N^2} \right) \right]$$

The optimization condition simplifies to

$$2 \cdot N \cdot \mathcal{L}_0 \cdot \frac{\log(Ni_T / N \cdot I)}{\log(H_T / H_0)} = N^2 \cdot \frac{\mathcal{L}_0}{\log(H_T / H_0) \cdot \ln 10} \cdot \left[\left(\frac{N \cdot I}{Ni_T} \right) \cdot \left(\frac{Ni_T}{I} \right) \cdot \left(\frac{1}{N^2} \right) \right] = \frac{N \cdot \mathcal{L}_0}{\log(H_T / H_0) \cdot \ln 10} \Rightarrow$$

$$2 \cdot \log(Ni_T / N \cdot I) = \frac{1}{\ln 10} \Rightarrow \ln(Ni_T / N \cdot I) = \frac{1}{2} \Rightarrow \frac{Ni_T}{N \cdot I} = \sqrt{e}$$

The optimal-turns equation reduces to

$$\boxed{N_{opt} = \frac{Ni_T}{I \cdot \sqrt{e}} \approx 0.6065 \cdot \frac{Ni_T}{I} = 0.6065 \cdot \frac{H_T \cdot l}{I}}$$

For a given core, $Ni_T = H_T \cdot l$ is constant, and the optimal turns varies inversely with current, I.

The maximum inductance is found by substituting N_{opt} into the inductance equation;

$$L(I) = N^2 \cdot \mathcal{L}_0 \cdot \frac{\log(Ni_T / N \cdot I)}{\log(H_T / H_0)} = \left(\frac{Ni_T}{I \cdot \sqrt{e}} \right)^2 \cdot \frac{\mathcal{L}_0}{\log(H_T / H_0)} \cdot \log \left(\frac{Ni_T}{\frac{Ni_T}{I \cdot \sqrt{e}} \cdot I} \right) \Rightarrow L_{\max} = N_{opt}^2 \cdot \frac{\mathcal{L}_0}{\log(H_T / H_0)} \cdot \log \sqrt{e}$$

Then

$$\boxed{L_{\max} = \left(\frac{H_T \cdot l}{I} \right)^2 \cdot \frac{\mathcal{L}_0}{\log(H_T / H_0)} \cdot \frac{\log \sqrt{e}}{e} \approx \left(\frac{H_T \cdot l}{I} \right)^2 \cdot \frac{\mathcal{L}_0}{\log(H_T / H_0)} \cdot (0.07988)}$$

For a fixed N, $L_{\max}$ is inversely proportional to I^2. Thus inductor energy, $L_{\max} \cdot I^2$ is constant. $L(N, I)$ is plotted in Fig. 4 for parameter I = 15 A and 30 A on a semi-log graph. The dotted plot is the inductor without saturation.

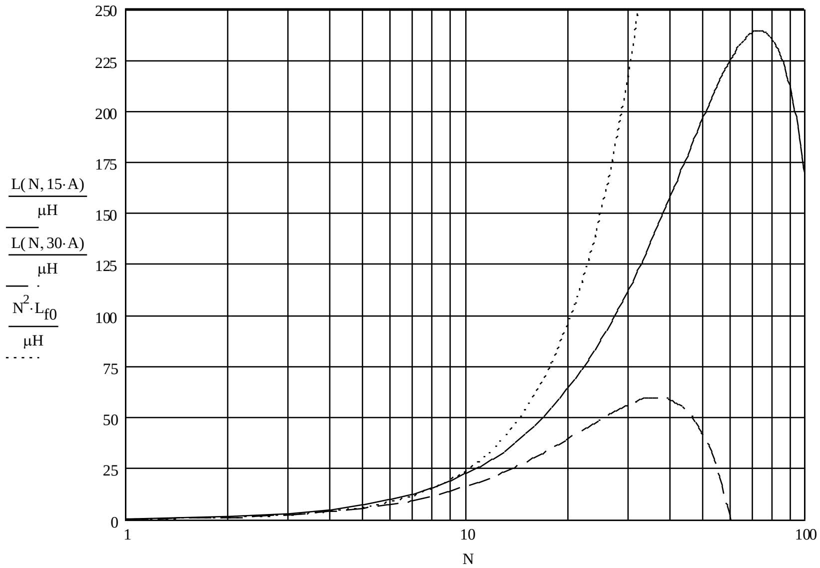

Fig. 4. Plots of inductance, L(N, I) with current as parameter. At half current, N_{opt} is twice and L_{max} is four times larger.

The saturation factor at L_{max} is found from

$$k_{sat} = \frac{L_{max}}{L_0} = \frac{L_{max}}{N_{opt}^2 \cdot \mathcal{L}_0} = \frac{N_{opt}^2 \cdot \mathcal{L}_0 \cdot \dfrac{\log \sqrt{e}}{\log(H_T / H_0)}}{N_{opt}^2 \cdot \mathcal{L}_0} \Rightarrow$$

$$\boxed{k_{sat} = \frac{\log \sqrt{e}}{\log(H_T / H_0)} \approx \frac{0.217}{\log(H_T / H_0)}}$$

Thus k_{sat} at L_{max} is determined only by the properties of the core material. The results can be tabulated as follows.

Table 2. k_{sat} at L_{max} for different core materials.

Material	μ_r	$\log\left(\dfrac{H_T}{H_0}\right)$	$k_{sat}(L_{max})$
Fe-pwd (26)	75	1.17	0.186
FeSiAl	125	0.928	0.234
NiFeMo	300	0.675	0.322
	125	0.623	0.349
MnZn 3F3	2000	0.326	0.666

As an example, a T201-26 Fe-pwd core has l = 118 mm and $\mathcal{L}_0$ = 242 nH. At I = 30 A of circuit current,

$$N_{opt} \approx 0.6065 \cdot \frac{H_T \cdot l}{I} = (0.6065) \cdot \frac{(15305\,\text{A/m}) \cdot (0.118\,\text{m})}{30\,\text{A}} = (0.6065) \cdot \frac{1806\,\text{A}}{30\,\text{A}} = \frac{1095\,\text{A}}{30\,\text{A}} = 36.5$$

At N = 36.5, NI = (36.5)·(30 A) = 1095 A of static field current. At this rather high value, $H = NI/l$ = 9280 A/m (117 Oe), which on the catalog saturation graph corresponds to $k_{sat} \approx 0.26$, and for which

$$L = N^2 \cdot [k_{sat} \cdot \mathcal{L}_0] = (36.5)^2 \cdot [(0.26) \cdot (242\text{ nH})] = 83.8\ \mu\text{H}$$

Using L derived from the saturation approximation,

$$L_{\max} \approx \left(\frac{H_T \cdot l}{I}\right)^2 \cdot \frac{\mathcal{L}_0}{\log(H_T / H_0)} \cdot (0.07988) = \left(\frac{1806\ \text{A}}{30\ \text{A}}\right)^2 \cdot (206.8\ \text{nH}) \cdot (0.07988) = 59.9\ \mu\text{H}$$

This lower approximated value results from linearly extending the catalog curve downward, reaching a lower value of $k_{sat} \approx 0.186$ than the actual curve value for the given $H(I)$. This error can be significant, though in practice, inductors are usually not operated at such a low k_{sat}, and typical error is much less. What this example shows is that maximum inductance is achieved for a 30-A circuit current—at least for Fe-pwd 26 material—by driving it hard with many turns. At half the current of 15 A, N_{opt} doubles to 73 turns, and

$$L_{\max}(15\ \text{A}) = (30\ \text{A}/15\ \text{A})^2 \cdot L_{\max}(30\ \text{A}) = 4 \cdot L_{\max}(30\ \text{A}) = 240\ \mu\text{H}$$

Closure

Inductance is maximum for a given current, I, at an optimum value of turns, N_{opt} that varies inversely with I. The maximum inductance, $L_{\max}$ varies inversely with I^2. As I is increased, N_{opt} decreases, as does $L_{\max}$, until $k_{sat} \to 0$, and saturation dominates over turns to decrease inductance.

At some maximum value of $N = N_w$, window area constrains N, and an acceptable $L_{\max}$ for the given core size and current is unrealizable. In power conversion, cores are not usually operated below about $k_{sat} \approx 0.5$ because of the diminishing returns for both transfer energy in a coupled inductor and inductance where it is to be maximized.

Even so, except for measurement applications where saturation degrades measurement accuracy, practical power-magnetics design operates along the approximate saturation lines near the log-center of the saturation region. These plots can show at a glance the relative tradeoffs among core materials for saturation effects and with the derived design formulas for optimizing the operating point, a value of k_{sat} can be chosen optimally instead of arbitrarily.

About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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