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Another Look At The Refined Model Of Current-Mode Control

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The analysis of current-model control in power conversion has a long history, going back over four decades, encompassing different approaches (waveform-based versus circuit-based models, averaged versus sampled waveforms) that capture different aspects of converter behavior. These different forms of current-loop models have evolved over time as their developers have attempted to improve their accuracy or completeness. First generation models by Middlebrook, Macsimović and Erickson were followed by Ridley's sampled-loop model and then Middlebrook and Tan's unified model. In a previous article series on peak-current control (see the reference), I described the development and evolution of these different current-loop models and how they led to my development of a new model I call the *refined model* of current-loop control.

For those not yet familiar with the refined model and those generally curious about modeling of current-mode control, this article offers a summary of the earlier seven-part series. It gives a broad overview of how a seemingly simple circuit (Fig. 1)—the peak (or valley) current-mode controller of switching converters—has been analyzed, or modeled, through four generations of development, culminating in what is probably the final generation, the refined model.

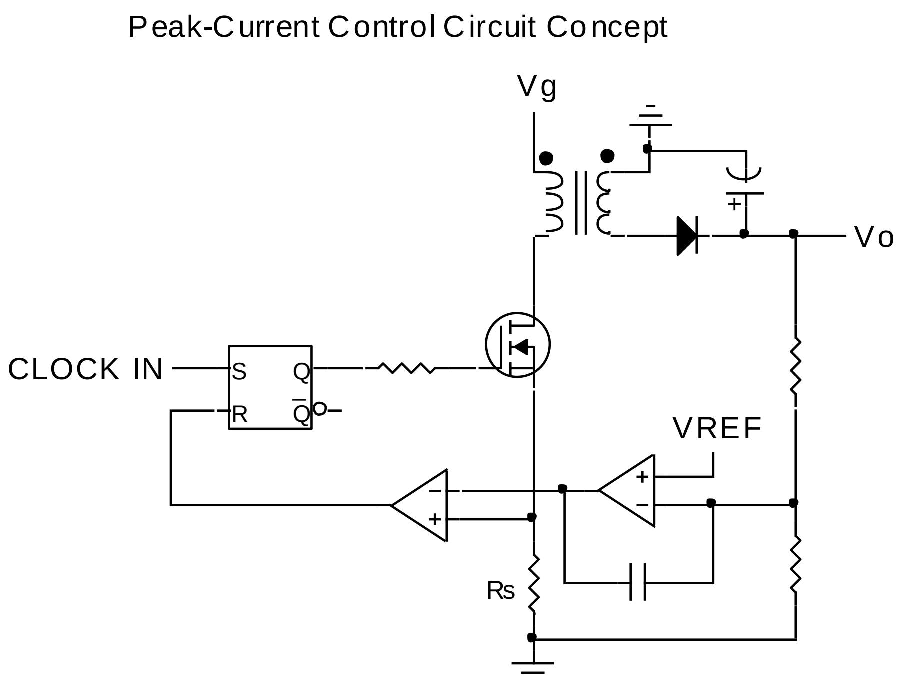

Fig. 1. The basic peak-current controller circuit—a simple circuit with complicated behavior.

Waveform Versus Circuit-Based Modeling

The historical development of current-loop models started from the inductor triangle-wave current waveform and has resulted in various *waveform-based models*. In contrast, models that begin with circuits instead of waveforms result in *circuit-based* modeling.

Waveform-based models are *behavioral* models (as waveforms express circuit behavior) that apply to any circuit for which the waveform is an approximation; circuit-based models are *structural* models from which specific behaviors are determined by circuit analysis. One scheme starts with waveforms, the other with circuits.

The inductor current, i_L waveforms are shown in Fig. 2. All waveform-based models are derived, in one way or another, from them.

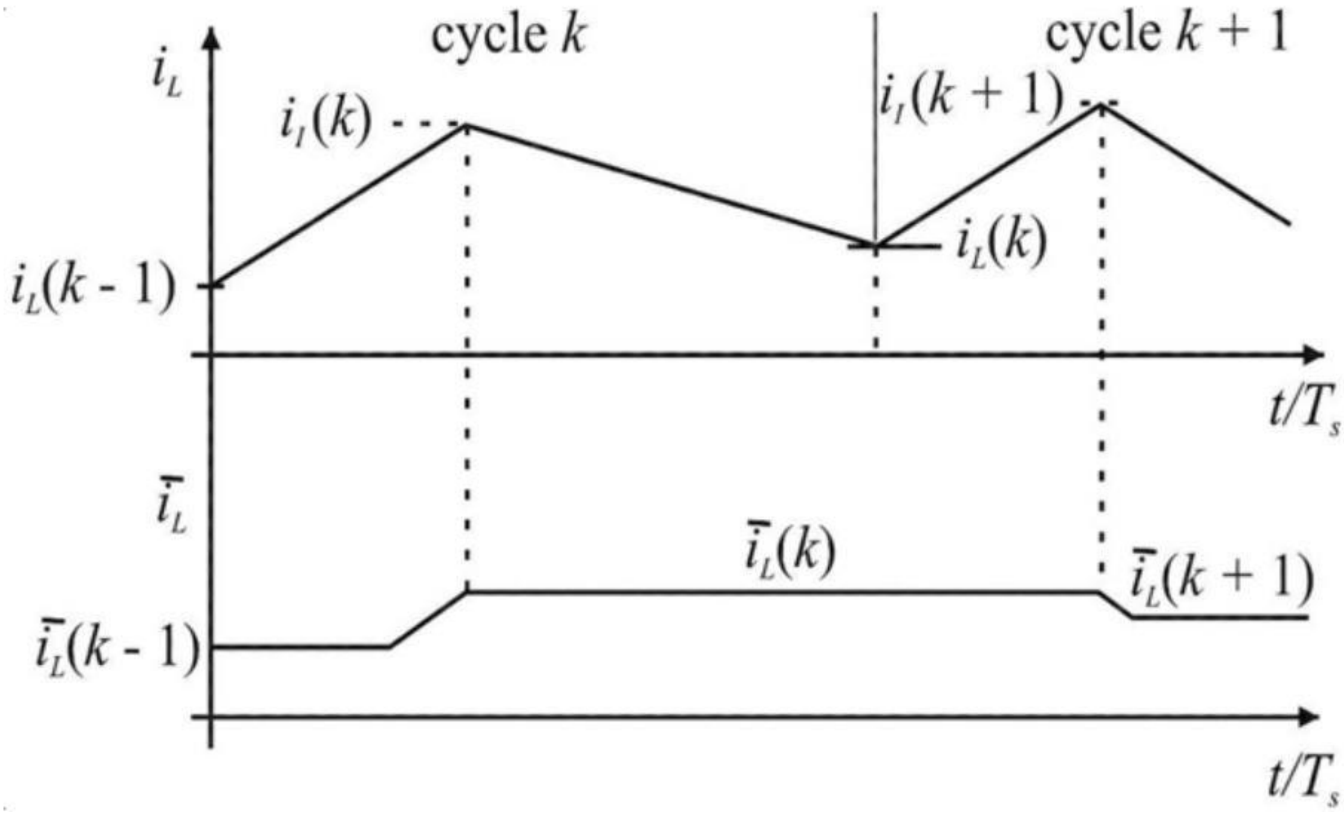

Fig. 2. The inductor current waveform of switching converters, i_L, where k is the cycle count. The lower waveform is that of the cycle-average current, $\bar{i}_L$ during the interval between peaks. The slopes in transition are those of i_L.

Models Based On Averaged Voltages And Currents

The first generation of current-loop analysis goes back to R.D. Middlebrook and CalTech students, Dragan Macsimović and Robert Erickson (both now professors at the U. of Colorado Boulder), as given in their classic power electronics book, *Fundamentals of Power Electronics, Second Edition* (KAP). They averaged voltages and currents between on and off states of the converter. This worked for loop bandwidths much less than the loop switching frequency, where approximately continuous waveforms can be analyzed with continuous control theory. Yet it did not explain an instability in which alternating cycles have different duty ratios, *D*, called *subharmonic oscillation*.

The Sampled-Loop Model

In the late 1980s, Ray Ridley discovered the "continuous time" (actually, piecewise-continuous) or *sampled-loop model*, based on the valley points of inductor current, i_L, at the end of each cycle, and not the average i_L. The sampled-loop model accounted for subharmonic oscillation by recognizing that the inductor during the off-time is a sample-and-hold device, storing the current of the cycle. As a sample-and-hold (S&H) circuit, it is the dual of capacitor S&H circuits. This S&H behavior can be seen in Fig. 2 in the lower waveform.

Ridley's model did not unify sampling behavior with the dynamics of the average inductor current. It is the average that is of interest in converter design because an average is an approximation of an ideal constant-current waveform, which is what is desired at the converter ports. Consequently, the resulting transfer function of per-cycle average current is dynamically different than Ridley's model.

The Unified Model

In the 1990s, Middlebrook and his student, F. Dong Tan introduced what they called a *unified model,* which was an attempt to achieve sampling and average current in a single model. They made some improvements to Ridley's work, for instance, by taking the sampler out of the feedback path and moving it to the front of the forward path. However, their model was not derived from a single set of basic equations but, like previous models, had independent assumptions about (and equations for) the PWM block in the loop.

In steady-state operation, the inductor current value is the same at the beginning and end of cycles. The *waveform equation* of the Fig. 2 waveform is

$$i_I(k) = i_L(k-1) + m_U \cdot \delta(k) \cdot T_s = m_D \cdot \delta'(k) \cdot T_s + i_L(k)$$

The z-domain and z transforms can be used as a shortcut between the familiar time and complex-frequency (s) domains. The delays of T_s between adjacent cycles can be expressed in the time domain with cycle count, k as $k{\cdot}T_s$. The variable, z^{-1} is a delay of one period and transforms a time-domain variable, $x((k-1){\cdot}T_s)$ into $z^{-1}{\cdot}x(z)$. It appears in the sampled s-domain as $e^{-s \cdot t}$. The sampled-loop steady-state valley-current transfer function in z of Ridley's model is

$$T_{CV}(z) = \frac{i_l(z)}{i_i(z)} = \frac{1}{D'} \cdot \frac{z}{z + \left(\frac{D}{D'}\right)} = \frac{z}{D' \cdot z + D}$$

How this is related to waveform equations is shown in the description of the final current-loop model, the *refined* model.

The Refined Model

The *refined model* derives the waveform and slope equations of the waveform-based model from the *incremental average current*, $\bar{i}_l$ instead of valley current, i_l by making the following substitution into the current waveform difference equation:

$$i_l(k) = 2 \cdot \bar{i}_l(k) - i_i(k)$$

where $i_i(k)$ is the discrete incremental input (commanded) current. The resulting average inductor-current waveform equation is

$$\bar{i}_l(k) = -\frac{D}{D'} \cdot \bar{i}_l(k-1) + \frac{1}{2} \cdot \left[\left(1 + \frac{1}{D'}\right) \cdot i_i(k) + \frac{D}{D'} \cdot i_i(k-1)\right] = -\frac{D}{D'} \cdot \left(\bar{i}_l(k-1) + \tfrac{1}{2} \cdot [i_i(k) - i_i(k-1)]\right) + \frac{1}{D'} \cdot i_i(k)$$

This results in a more complicated s-domain transfer function that has a delay within the switching cycle to account for averaging. To compare with the sampled-loop transfer function in z, the refined-model transfer function is

$$T_C(z) = \frac{\bar{i}_l(z)}{i_i(z)} = \frac{1/2}{D'} \cdot \frac{(D'+1) \cdot z + D}{z + \frac{D}{D'}} = \frac{\tfrac{1}{2} \cdot [(D'+1) \cdot z + D]}{D' \cdot z + D} = \tfrac{1}{2} \cdot (1 + T_{CV}(z))$$

In the s-domain, the resulting approximation is found by substituting the s-domain advance factor for $z = e^{s \cdot t}$;

$$T_C(s)=\frac{\bar{i}_l(s)}{i_i(s)}\cong\frac{1+\frac{D}{2}\cdot(e^{-s\cdot T_s}-1)}{\left(\frac{s}{\omega_s/2}\right)^2+\pi\cdot(\tfrac{1}{2}-D)\cdot\left(\frac{s}{\omega_s/2}\right)+1}$$

In comparison, the closed-loop valley-current transfer function for the sampled-loop model is

$$T_{CV}(s)\cong\frac{1}{\left(\frac{s}{\omega_s/2}\right)^2+\pi\cdot(\tfrac{1}{2}-D)\cdot\left(\frac{s}{\omega_s/2}\right)+1}$$

The quadratic poles are identical but averaging the current adds a delay in the numerator to account for the phase shift of the average from the valley current. The avg-current function can be approximated by substituting the modified Padé approximation for the *s*-domain time delay of T_s , $e^{-s\cdot T_s}$ which results in the approximated closed average-current-loop function,

$$T_C(s)=\frac{\bar{i}_l}{i_i}\cong\frac{\left(\frac{s}{\omega_s/2}\right)^2+\left(\frac{\pi}{2}\cdot D'\right)\cdot\left(\frac{s}{\omega_s/2}\right)+1}{\left[\left(\frac{s}{\omega_s/2}\right)^2+\pi\cdot(\tfrac{1}{2}-D)\cdot\left(\frac{s}{\omega_s/2}\right)+1\right]\cdot\left[\left(\frac{s}{\omega_s/2}\right)^2+\frac{\pi}{2}\cdot\left(\frac{s}{\omega_s/2}\right)+1\right]}$$

This is the transfer function of the incremental (small-signal) closed current loop. It has similarities to previous models but includes the delay of the average current, which does not occur at the end of the cycle where the valley current is, but within it.

For the blocks within the loop, the converter power-transfer circuit function, G_{id}, of the refined model is equivalent to that of the sampled-loop model. G_{id} is duty ratio in, inductor current out. Converted to the *s*-domain, it is

$$G_{id}(s)=\frac{\bar{i}_l(s)}{d(s)}=\frac{1}{2}\cdot\frac{i_l(s)}{d(s)}=\frac{\Delta I_{L0}}{2\cdot s\cdot T_s}\quad,\quad \Delta I_{L0}=\frac{V_{off}\cdot T_s}{L}=\text{constant}$$

It differs from the valley-current G_{idV} by the ½ factor which is the difference between steady-state valley and average current.

The PWM block, F_m in the forward path of the loop has been the subject of various modeling efforts. F_m of the refined $T_C(s)$ can be found by solving for it from the closed-loop feedback equation,

$$T_C(s)=\frac{\bar{i}_l(s)}{i_i(s)}=\frac{F_m(s)\cdot G_{id}}{1+F_m(s)\cdot G_{id}}=\frac{1}{1+\frac{1}{F_m(s)\cdot G_{id}}}$$

and substituting the above G_{id}. Then in normalized form, the PWM transfer function is

$$F_m(s) \cong \left(\frac{4}{\Delta I_{L0} \cdot D'}\right) \cdot \frac{\left(\frac{s}{\omega_s/2}\right)^2 + \left(\frac{\pi}{2} \cdot D'\right) \cdot \left(\frac{s}{\omega_s/2}\right) + 1}{\frac{2}{\pi \cdot D'} \cdot \left(\frac{s}{\omega_s/2}\right)^3 + 2 \cdot \left(\frac{s}{\omega_s/2}\right)^2 + \frac{2}{\pi \cdot D'} \cdot \left(1 + \frac{\pi^2}{2} \cdot \left(\tfrac{1}{2} - D\right)\right) \cdot \left(\frac{s}{\omega_s/2}\right) + 1}$$

from which the steady-state (quasistatic) F_m is

$$F_{m0} = \frac{4}{D'} \cdot \frac{L \cdot f_s}{V_{off}} = \frac{4}{\Delta I_{L0} \cdot D'}$$

For $[s/(\omega_s/2)]^2 << 1$, or low frequencies,

$$F_m(s) \cong \left(\frac{4}{\Delta I_{L0} \cdot D'}\right) \cdot \frac{1}{\frac{2}{\pi \cdot D'} \cdot \left(\frac{s}{\omega_s/2}\right) + 1}$$

The refined $F_m(s)$ has, like the unified model, a single-pole $F_m(s)$ at $\omega_s/2$ but it varies differently with D.

The comparison of the different F_{m0} for the various current-loop models can be simplified by excluding the effects of slope compensation that prevents subharmonic oscillation. Differences in slope compensation methods confuse the comparison. The uncompensated models must still correctly predict uncompensated behavior to be valid.

The addition of the compensating waveform at the summing block can be regarded as a modification of the input, i_i. Then the F_{m0} of the unified model of Tan and Middlebrook is essentially

$$F_{mV0} = \frac{1}{\Delta I_{L0} \cdot \left(\tfrac{1}{2} - D\right)}$$

whereas the sampled-loop model of Ridley is valley-point-based and is

$$F_{m0} = \frac{1}{\Delta I_{L0} \cdot (1 - D)}$$

The "Fairchild" model (Holloway and Eirea) and also that of Tymerski and Li are equivalent for average current:

$$F_{m0} = \frac{1}{\tfrac{1}{2} \cdot \Delta I_{L0} \cdot (1 - D)}$$

where the ½ accounts for the difference between valley and average current. In the refined model

$$F_{m0} = \frac{2}{\Delta I_{L0} \cdot D'}$$

and is equivalent to the sampled-loop and Fairchild F_{m0}. The total forward path, G, thus has a net static gain that is that of the sampled-loop and Fairchild models because G_{id} (for average i_l) is half that of G_{idV}. The cubic denominator of the refined $F_{m0}(s)$ can be simplified to

$$F_m(s) \cong \frac{2}{\Delta I_{L0} \cdot \frac{1}{2} \cdot D'} \cdot \frac{1}{\left(\frac{s}{\omega_s/2}\right) \cdot \frac{1}{2} \cdot \pi \cdot D' + 1}, \; s << \frac{\omega_s}{2}$$

If the ½ of G_{id} is brought into F_{m0},

$$F_{m0} = \frac{1}{\Delta I_{L0} \cdot \frac{1}{2} \cdot D'}$$

and is the same as the Tymerski and Fairchild F_{m0}. All equivalent models have D' in the denominator of F_{m0}. The unified model of Tan has (1/2 – D). This accounts for the $f_s/2$ resonance, but it should instead belong in the frequency-dependent factor of the transfer function, as it does in the other models.

By placing the sampling function, which Ridley called $H_e(s)$, in the feedback (H) path, the sampled-loop model of Ridley lacks a free transfer-function variable needed to form a transfer function. However, if H_e is brought forward through the summing block into the forward path (to make it more like the unified model), then sampling occurs after the error summing block and a transfer function exists. This is a justification for preferring a model like the unified model that samples the error quantity, with sampler at the input to the forward path, G, so that a transfer function can be derived. The refined model is a refinement of the unified model and retains this sampler placement in the loop.

The low-frequency-average (lf-avg) model, as presented by Erickson and Macsimović in their book, established the quasistatic value (F_{m0}) of the PWM transfer function, F_m. Middlebrook and Tan used an equivalent average inductor current expression in their "unified" model though derived a different way and not obviously equivalent.

The current-loop modeling story goes on but is reaching maturation. The two main improvements of the refined model are that average-current dynamics are included as part of the derivation of the model from basic principles, and in doing this, the PWM block function also drops out of the derivations and does not need independent assumptions about it, as in the previous models.

What is left to do for current-loop modeling is to combine waveform- and circuit-based models into one grand model. However, this poses what are probably needless challenges in that the two accomplish different goals. The waveform-based models have an ideal and simplified inductor waveform with linear slopes. Real circuits include resistance which causes (slightly) exponential slopes and are better modeled with circuit analysis. However, the waveform-based models apply to any circuits for which the waveforms are a valid approximation. At National Semiconductor (now Texas Instruments), Robert Sheehan's effort has been to generalize circuit-based analysis so that it extends toward the generality of waveform-based modeling.

For typical converter design, the waveform-based models are quite sufficient, and even Ridley's second-generation model is adequate in most cases. For those wanting a more inclusive and theoretically "cleaner" model, the refined model is preferred, though newer and without as much design experience applied to it.

References

"Special Series on Current-Mode Control," a seven-part article series by Dennis Feucht, published in the September 2011 through March 2012 issues of How2Power Today.

About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where *he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.*

For more information on current-mode control, see How2Power's Design Guide, locate the Design Area category and select Control Methods.