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Designing Low-Power Flyback Inductors Using Tiny Toroids (Part 1): A Boost Converter Application

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Power-converter design requiring 0.2 W to 2 W of power can most easily be solved with a linear supply. But in power-critical applications, if the input and output port voltages are significantly different, the result can be excessive or unnecessary power loss. The three configurations of PWM-switch converters can deliver the required power at high efficiency, though they involve the design of a small inductor. This article offers some insight into how core sizes relate to circuit requirements and presents a design example.

Can toroid cores, such as iron-powder (Fe-pwd) cores that are as small as magnetic beads, be used for power conversion? The answer is affirmative. Though some agility is required in building the prototypes, it is less than required for working with TSSOP-size components on boards.

The smallest Micrometals Fe-pwd toroid core in the low-cost 26 material (color coded white on yellow) is the T20-26, 0.2 inches in diameter (2.54-mm outside radius), with a ring width of 1.42 mm. This particular core will be used in an example procedure developed here as a *design template*. Along with its analytical details, this procedure includes numerous tips to simplify wire and core selection and to ease building of prototypes. But before explaining the design procedure, it would be good to establish how core geometry relates to circuit voltages and currents.

Core Size Versus Electrical Parameters

The heuristic rules in relating core geometry to circuit voltage and current are

1. Maximum flux (voltage times time, or $\Delta\lambda = V_L \cdot \Delta t$) scales up with core cross-sectional area.
2. Maximum field current (turns times circuit current, $N\bar{i} = N \cdot I$) scales up with magnetic path length (toroid ring circumference).

The justification for these rules follows from simplified Maxwell's equations. The voltage rule is based on the minimum allowable turns,

$$N_\lambda = \frac{\text{circuit flux}}{\text{field flux}} = \frac{\Delta\lambda}{\Delta\phi(\bar{p}_c)} = \frac{V_L \cdot \Delta t}{\Delta B(\bar{p}_c) \cdot A}$$

where V_L is the average voltage across the inductor during the on-time of the converter switching cycle (or off-time; $|\Delta\lambda|$ refers to the on- and off-time magnitudes (which are the same in a stable lossless converter) and $\Delta\phi(\bar{p}_c)$ is the allowable core power-loss density field flux. Average core power loss, $\bar{p}_c$, is related to the magnetic switching frequency and the field-density ripple, $\Delta B = 2 \cdot \hat{B}_\sim$, where $\hat{B}_\sim$ is the ripple amplitude of B; and A is the magnetic-path cross-sectional area. Then N_λ is the minimum number of allowable flux-limited turns for the inductor; any fewer turns will overheat the core.

Solving the above equation for the maximum average on-time inductor voltage per turn (which is the *field-referred voltage*),

$$\frac{V_L}{N_\lambda} = \frac{2 \cdot \hat{B}_\sim(\bar{p}_c) \cdot A}{t_{on}}$$

The frequency of the design example is f = 250 kHz. For a given size of core with power-loss density, $\bar{p}_c$, the catalog power-loss curves, plotted at various frequencies, will give $\hat{B}_\sim(\bar{p}_c, f)$ on the horizontal axis of the graph. The nominal duty-ratio, D, for a PWM-switch converter is optimal in minimizing switch conduction loss at

D = 0.5. Substitute the core data into the above volts-per-turn formula and the following table results for two switching frequencies,

$$t_{on}(100\text{ kHz}) = 5\ \mu\text{s}\ ;\ t_{on}(250\text{ kHz}) = 2\ \mu\text{s}$$

The table expands beyond small cores to be more complete, and also includes some E cores.

Table. Maximum average on-time inductor voltage per turn for various iron-powder toroid cores and some E cores.

Core Type	V_L/N, mV		Core Type	V_L/N, mV	
	100 kHz	250 kHz		100 kHz	250 kHz
T20-26	58.0	78.2	T157-26	975	1378
T26-26	155	207	T184-26	1504	2068
T37-26	123	166	T201-26	2023	2670
T37-52	149	205	T250-26	2458	3456
T44-26	166	228	T300D-26	2434	3211
T50-26	184	258	E49026	162	222
T50B-26	124	225	E100-26	451	605
T50D-26	312	424	E137-26	834	1088
T68-26	258	340	E162-26	1224	1610
T80-26	259	347	E187-26	1587	2232
T106-26	685	923	E220-26	1702	2269
T130-26	726	977	E305A-26	2985	3940
T131-26	850	1151	E450-26	5842	7874
T150-26	852	1153			

For a T20-26 inductor voltage of 5 V at 100 kHz, the required number of turns is N = 5 V/58.0 mV = 86. This number of turns can be manually wound on a small core, but as N increases, wire size decreases. A heuristic rule in working with small wire is that manual prototyping becomes increasingly difficult above 40 AWG.

For 42 AWG wire, the ampacity is only 15 mA. This small wire is usually bundled as Litz wire to reduce eddy-current losses and increase ampacity. It is easy to break wire above 40 AWG and careful handling is required when building bundles or winding it onto bobbins.

The allowable $\overline{p}_c$ is the thermal aspect of magnetic design and it decreases with core size. A T130-26 core has a volume of V = 5.78 cm^3, and for equal core and winding loss, maximum $\overline{p}_c$ = 306 mW/cm^3. The much smaller T20-26, for which V = 0.026 cm^3, has a maximum $\overline{p}_c$ = 1.866 W/cm^3. Smaller cores expel heat better because their surface area-to-volume ratio is larger than larger cores. Distributed magnetics uses multiple smaller cores to achieve a thermal advantage, and with the low cost of small cores, can be a feasible design solution. Our design example will follow this multi-core approach.

The T20-26 core operated at 250 kHz can be used to design a 5-V to 12-V isolated flyback converter. To reduce turns to a more convenient number for manual winding, three T20-26 cores were stacked, tripling the magnetic volume of a single core. The magnetic cross-section, *A*, triples, yet the circumference and hence the field current, $N\bar{i}$, remains the same, at $N\bar{i}$ = 35 A for saturation factor, k_{sat} = 0.6. ($L(I) = 0.6 \cdot L(0\text{ A}) = k_{sat} \cdot L_0$).

The larger volume of the stacked core reduces the maximum $\overline{p}_c$. Using the thermal shape-based method[1] of calculating $\overline{p}_c$ and consequently $\hat{B}_{\sim}$, the shape-based design formulas are invoked for $V = 3 \cdot (0.026\text{ cm}^3) = 0.078\text{ cm}^3$:

$$\text{sphere thermal } r = \left(\frac{3 \cdot V}{4 \cdot \pi}\right)^{1/3} \approx (0.6204) \cdot V^{1/3} = 0.265\text{ cm}$$

The allowable $\overline{p}_c$ for the worst-case core shape (a sphere) and a maximum temperature rise of Δ40°C (= Δ40 K) is

$$\overline{p}_c(\text{sphere}) = \frac{\Delta T}{(8.33\,\text{cm} \cdot \text{K/W}) \cdot r^2 + (167\,\text{cm}^2 \cdot \text{K/W}) \cdot r} = 892\text{ mW/cm}^3$$

The thermal advantage of the toroidal shape over that of a sphere is the *thermal shape factor*, Ξ_θ = 1.5 for round toroids. (Square ring cross-sections have Ξ_θ = 1.8, but the more conservative value is being used here for a general toroid design template.). Then the allowable

$$\overline{p}_c = \Xi_\theta \cdot \overline{p}_c(\text{sphere}) = (892\,\text{mW/cm}^3) \cdot (1.5) = 1338\text{ mW/cm}^3$$

(In the more refined shape-based thermal model, another factor is included that takes into account the fraction of winding-generated heat that exits through the core, but this is already included in Ξ_θ in the above value.) The Micrometals catalog power-loss graph for 26 material at 1338 mW/cm³ and 250 kHz shows a value on the horizontal axis of $\hat{B}_{\sim} \approx$ 28 mT. Then $\Delta B = 2 \cdot \hat{B}_{\sim} \approx$ 56 mT.

By the heuristic rules of core geometry, the maximum inductor on-time voltage (for a given t_{on}) triples with three stacked cores. The window area (and hence maximum field current) remains the same, but the minimum number of turns [2] is reduced by one third to

$$N_\lambda = \frac{(4.1\,\text{V}) \cdot (2\,\mu\text{s})}{(56\,\text{mT}) \cdot (3 \cdot 2.3\,\text{mm}^2)} = \frac{8.2\,\mu\text{V} \cdot \text{s}}{0.386\,\mu\text{V} \cdot \text{s}} = 21.24 \rightarrow 21$$

For a small converter, a small MOSFET switch such as a 2N7000 is used. It has an on-resistance of a few ohms. The total voltage drop across MOSFET and winding resistance (the voltage drop that is not applied across the winding magnetizing inductance) is estimated as 0.9 V, leaving 4.1 V across the winding inductance.

To stack the three cores in alignment, they were inserted onto a workbench scribe, one at a time, with cyanoacrylate glue applied to adjacent core cross-sections. Toroids can be difficult to wind for a full winding window. Thus the manual prototype fractional winding area, A_{ww} to window area, A_w = 3.94 mm² is chosen to be

$$k_{ww} = A_{ww}/A_w = 0.5$$

At first glance, wasting half the window area might seem excessive. However, a half-full window in a toroid has a winding inner radius of $r_{ww} \approx 0.707 \cdot r_i$, where r_i = toroid inner radius = 1.12 mm. This leaves an open center radius of r_{ww} = 0.792 mm, not much when trying to poke a final winding turn through the center of the core. Consequently, a 21-turn primary winding and equal-area secondary each have a winding area of (3.94 mm²)/2 = 1.97 mm².

Equal primary and secondary winding areas are optimum for single-ended (not push-pull) PWM-switch converters; each winding is allotted $A_{ww}/2$ for an equal winding power-loss density. Then the wire area, with packing factor taken into account, allows each strand to have an area of

$$A_{cwp} = \frac{A_{ww}/2}{N} = \frac{1.97\,\text{mm}^2/2}{21} = 0.047\,\text{mm}^2,\; k_{ww} = 0.5$$

The packed conductor area, A_{cwp} is the insulated-wire area, A_{cw}, increased by the wire-packing fill factor, k_{pf}, which has been derived (in a previous *How2Power Today* article,[3]) for round wire as $k_{pf} \approx 1/1.26 \approx 0.794$;

$$A_{cwp} = \frac{A_{cw}}{k_{pf}} \approx (1.26)\cdot A_{cw} = (1.26)\cdot(\pi \cdot r_{cw}^2)\;\;, k_{pf} \approx 1/1.26$$

Then the allowable insulated-wire radius is

$$r_{cw} = \sqrt{\frac{k_{pf}\cdot(k_{ww}\cdot A_w)}{N\cdot\pi}} = \sqrt{\frac{(0.794)\cdot A_{cwp}}{\pi}} = 0.109\text{ mm} \Rightarrow \#33\ (0.108\text{ mm})$$

From the AWG wire table, the closest wire size that fits is #33 which has an ampacity of 118 mA. However, wire ampacity depends on the winding size. The reference current density is $\tilde{J}_0$ = 4.5 A/mm^2 for a copper winding on a core having an area-product, $A{\cdot}A_w$ = 1 cm^4. The formula for ampacity resizing from this reference-size area-product is[4]

$$\frac{\tilde{J}}{\tilde{J}_0} = \left(\frac{A\cdot A_w}{\text{cm}^4}\right)^{-\frac{1}{8}}$$

For the T20 size, the ampacity multiplier is $\tilde{J}/\tilde{J}_0$ = 2.4. Thus, #33 copper wire wound on a T20 core has an ampacity of (118 mA)·(2.4) = 283 mA. Smaller cores have larger power densities and increased allowable current.

The next problem is how to actually construct the windings. For a CL (flyback) PWM-switch converter with duty-ratio, D = 0.5, the voltage transfer function is $n{\cdot}V_s/V_g = V_s'/V_g = D/D' = 1$. The voltage input across the primary inductance of V_p = 4.1 V, and on the output side of the inductor, with a series Schottky diode, $V_o + V_D$ = 12 V + 0.3 V = 12.3 V.

Then D can be adjusted slightly from ½ to accommodate variation in the diode voltage, V_D, so that the turns ratio of the primary to secondary windings, $n = N_p/N_s$, can be an integer, n =1/3. Next the transductor is specified as having 21 primary turns of #33 wire and 63 turns of secondary wire with A_{cwp} being one third that of the primary, or 0.047 mm^2/3 = 0.016 mm, which is #38 wire.

To insure a fit, one wire size less is chosen, that of #39, which has a T20 ampacity of 70 mA. At 12-V out, this is an output power of 0.84 W. With the more difficult winding of #38 wire instead, secondary current increases to 89 mA and output power increases to slightly over a watt.

An integer turns ratio of 3 also achieves a simplification in transductor construction by using *unibundle*[5] construction: a single bundle of multiple strands connected in series or parallel to achieve the correct n. If six strands of #39 wire are used in a bundle, then three strands each are allotted to the two windings. The primary strands are connected in parallel for the additional current and the secondary strands are connected in series to effect n = 1/3 with 63 turns.

This design works out optimally in that each winding has the same power capability, with the primary operating at three times the current and a third the voltage of the secondary. A manually-wound prototype of the transductor is shown in the figure (left) along with the core stack and bundle wire before winding (right).

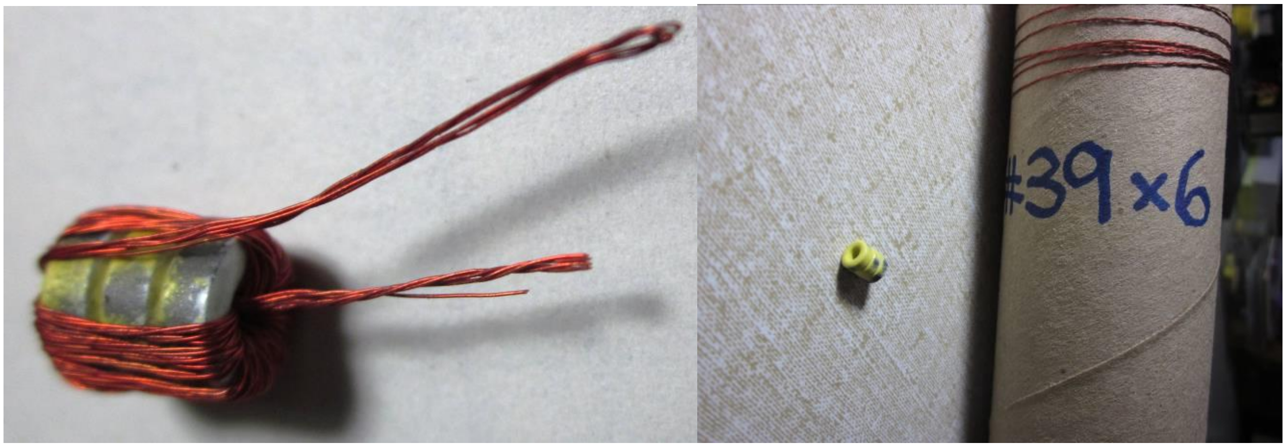

Figure. Close view (left) of a stack of three T20-26 iron-powder cores with 21 turns of a twisted bundle of #39 wire by six strands; view of core stack (right) next to a paper-towel tube, used to conveniently store the #39 × 6 winding bundle.

The strand wire size in the six-strand bundle is calculated from bundle geometry formulas[6] which take into account the extent to which a bundle expands from twisting. The increase in bundle radius depends on how much it is twisted. This is quantified by the pitch (length of one twist) to bundle radius ratio, p/r_b' which is typically about 30. The bundle radius, r_b' = the radius to the center of the outermost ring strands and is

$$r_b' = r_{bw}' - r_{cw}$$

where r_{bw}' is the bundle radius as measured by calipers, to the outside of the bundle. The formula for the twisted bundle radius as a ratio of insulated-strand wire radius, r_{cw}, is

$$\frac{r_{bw}'}{r_{cw}} = \sqrt{\frac{N_s}{k_{pb} \cdot k_{tw}}}$$

where N_s = number of strands in bundle = 6; $k_{pb} \approx 1/1.265 = 0.7905$ is the average fill factor within a six-strand bundle for round wire, and k_{tw} is the *twist factor,*

$$k_{tw} = \frac{p}{\sqrt{p^2 + (2 \cdot \pi \cdot r_b')^2}} = \frac{p / r_b'}{\sqrt{(2 \cdot \pi)^2 + (p / r_b')^2}}$$

For $p/r_b' = 30$, $k_{tw} \approx 0.9788$. The bundle includes both primary and secondary windings and has a twisted radius of

$$r_{bw}' = \sqrt{\frac{A_{bw}}{\pi}} = \sqrt{\frac{k_{ww} \cdot A_w}{N \cdot \pi}} = \sqrt{\frac{(0.5) \cdot (3.94\,\text{mm}^2)}{21 \cdot \pi}} = 0.173\,\text{mm}$$

The strand radius can be calculated and found in the wire table from

$$r_{cw} = \frac{r_{bw}'}{\sqrt{\frac{N_s}{k_{pb} \cdot k_{tw}}}} = \frac{0.173\,\text{mm}}{\sqrt{\frac{6}{(0.7905) \cdot (0.9788)}}} = \frac{0.173\,\text{mm}}{2.785} = 0.062\text{ mm} \rightarrow 39\text{ AWG}$$

The allowed radius is somewhat less than #38 wire because of the twisting and bundling effects, and #39 wire will fit easily with k_{ww} = 0.5.

As was the case for the prototype unit in Fig. 1, the inductor is easy to wind. During winding, a scribe is poked into the core to compress the winding away from the center for threading through the final turns. The threading end of the bundle should be given a solid tip by twisting it tightly. When counting turns, increment the count only at the moment that the bundle is grasped on the other side of the core as it is threaded through. Then repeat this number to yourself until the next increment occurs.

The primary winding is finished by first finding the corresponding start and finish ends of each strand with an ohmmeter, then soldering the start and finish ends of three strands in parallel to form the primary winding. For the secondary, starts are connected to finishes twice, thereby connecting three strands in series for the secondary winding.

The 70-mA ampacity of #39 wire at 12-V out results in 0.84 W of power, sufficient for some low-power applications. Subtracted from this ideal power value are the core and winding losses.

One benefit of the unibundle construction is that with primary and secondary windings twisted together, not only is the coupling "tight" (coupling coefficient, $k \approx 1$), eddy-current losses are reduced to only the strand and bundle skin effects by twisting a small number of strands. The core and winding losses will take additional calculations, which are left for part 2 of this article. And to wind toroids, the winding length must also be known and the bundle cut to length before winding can proceed. An accurate toroid winding length formula is derived in the references and will be given in part 2.

References

All of the referenced materials below, unless otherwise noted, are by the author and are also found in the author's magnetics book, *Power Magnetics Design Optimization* (*PMDO*), in laminated-paper book form at www.innovatia.com Contact the author for a free PDF copy at www.innovatia.com/Inquiry.htm.

1. Derived in "How to Choose Magnetic Core Size," How2Power Today, May 2013.
2. Derived in "How to Choose Magnetic Core Size" and in "Optimal Turns", chapter 5, in *PMDO*, www.innovatia.com
3. Derived in "How To Calculate Winding Packing Factor," How2Power Today, November 2016.
4. Derived in *Transformers and Inductors for Power Electronics* by W. G. Hurley and W. H. Wölfle, Wiley, 2013, page 125, eqn (5.4).
5. Unibundle design is developed in "Eddy-Current Effects in Magnetic Design: Part 6: Winding Bundles", How2Power Today, March 2017 and in "Single-Bundle Windings Make It Easier To Build Custom Magnetics In-House," How2Power Today, March 2014.
6. Bundle geometry formulas are found in "The Geometry of Twisted Wire Bundles," How2Power Today, July 2018.

About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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