Designing Low-Power Flyback Inductors Using Tiny Toroids (Part 2): Calculating Losses

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Part 1 of this article described the design of a low-power flyback converter inductor using three stacked T20-26 cores—the smallest core in the Micrometals catalog.[1] Here in part 2 we present further details of the inductor design, specifically the additional design formula required for determining the winding turn length of the unibundle-constructed 6-strand winding previously specified. This part also explains how to determine winding resistance and both core and winding power loss. Using this loss information, the effect of the inductor on converter efficiency, based on ideal output power, can be found.

Does this design approximately balance core and winding losses to achieve optimum efficiency? If not, which loss dominates and why? The factors at play are explained and some conclusions are drawn about the usefulness of small cores in low-power converter designs.

Toroid Winding Length

The inductor design was simplified in part 1 by requiring only one winding operation, that of the *unibundle*: both primary and secondary windings together comprised of a total of six hexafilar strands. Three strands are paralleled for the primary winding and the remaining three connected in series externally for the secondary winding.

A small transductor with six wires protruding from each end with three pairs in parallel can be soldered directly to eight circuit-board pads or to an 8-pin DIP header. Then the secondary winding is configured with external pin-to-pin connections, plugging the DIP-mounted inductor into an 8-pin DIP socket on the prototype board.

The length of the unibundle can be found from the toroid length formulas (found in reference [2]), given here by example as a design template. The T20-26 geometric parameters for a core stack of three cores are

$$r_i = 1.12 \text{ mm};\ w = r_o - r_i = 1.42 \text{ mm};\ 3\cdot h = 3\cdot(1.78 \text{ mm}) = 5.34 \text{ mm} \Rightarrow\ r_i + w/2 = 1.83 \text{ mm}$$

where r_i s the interior radius of the toroid, r_o is the exterior radius of the toroid, and h is its height or thickness. The following intermediate parameters are entered or calculated; on a calculator, they can be stored in the designated memory locations (→ #).

$$\text{full-window layers} = \hat{M} = \frac{r_i}{(1.866)\cdot r_{bw}'} = 3.469 \to 1$$

$$\text{full-window bundle turns} = N_w = \pi\cdot\hat{M}^2 = 37.82 \to 2$$

$$\text{layers} = M = \hat{M}\cdot(1-\sqrt{1-N/N_w}) = (3.469)\cdot(1-\sqrt{1-21/37.82}) = 1.156 \to 3$$

$$r_{bw}' = 0.173 \text{ mm} \to 4$$

$$w + 3\cdot h = 6.46 \text{ mm} \to 5$$

where r_{bw}' is the outer radius of the twisted bundle.

Then the toroid winding length (l_w) formula is applied:

$$l_w = 2\cdot\pi\cdot M\cdot[(2\cdot(w+3\cdot h)+8\cdot r_{bw}'\cdot M)\cdot(\hat{M}-M/2)+\tfrac{4}{3}\cdot r_{bw}'\cdot(1-M^2)+(r_i+w/2)]$$

$$l_w = 2\cdot\pi\cdot(1.156)\cdot[(14.52\text{ mm})\cdot(2.892) - 0.0775\text{ mm} + 1.83\text{ mm}] = 2\cdot\pi\cdot(1.156)\cdot[43.74] = 31.76\text{ cm} \rightarrow 36\text{ cm}$$

The additional length is for 2 cm of bundle ends for connections and rounded up as error margin. The l_w value was accurate to within a centimeter for the prototype unit, though the layer count was somewhat less than one instead of 1.156, caused by some overlapping of strands from adjacent turns during winding construction. In effect, *M* was made greater than one, though not sequentially in the winding of turns.

Core Loss

The thermal design (in part 1) resulted in an allowable core loss density of 1338 mW/cm^3. The core volume with three stacked T20 cores is 0.078 cm^3. Core power loss is calculated from the allowable core loss density (from part 1) and core volume:

$$\overline{P}_c = \overline{p}_c \cdot V = (1338\text{ mW/cm}^3)\cdot(0.078\text{ cm}^3) = 104\text{ mW}$$

Winding Loss

Winding loss consists of static and dynamic losses. The total (static + dynamic) resistance is first found and used to calculate the total winding loss from Watt's law. With twisted strands, the interbundle (external) proximity effect is essentially nulled out by induced voltage on opposing sides of the half-turns. The strand proximity effect within the bundle for a small number ($N \leq 5$) of bundle strands (N_s = 6 in this plan) is small, and the strand and bundle skin effects remain. The eddy-current constant-frequency resistance ratio for round wire, F_r, is the multiplier of the resistance,

$$R_w(\xi_r) = F_r(\xi_r)\cdot R_{\delta r}\,,\; R_{\delta r} = \frac{\rho_{\text{Cu}}}{\pi\cdot\delta^2}\cdot l_w = \left(1.256\,\frac{\mu\Omega}{\text{m}\cdot\text{Hz}}\cdot f\right)\cdot l_w \text{ , round wire, Cu, 80 °C}$$

where δ = skin depth, $\xi_r = r_c/\delta$ = penetration ratio, and ρ_{Cu}= copper wire resistivity. $R_{\delta r}$ is constant for a given frequency, and the magnetic frequency is optimized in the magnetic design. No frequency optimization occurred in this design example; 250 kHz was chosen and is less than f_{MAX}, the frequency at which an increase would decrease transfer power. A 2N7000 MOSFET or larger can easily switch at this frequency with minor switching losses.

The usable core frequency, f_{MAX}, increases with core power-loss density, and small cores can withstand the higher density, making them better for higher frequency operation. (See [3] for frequency optimization.)

The previous equation is taken a step further by expressing winding resistance, with all windings referred to the primary and their combined resistances equal to R_{wp}. Total primary-referred winding resistance is thus $R_{wp} + R_{ws}' = 2\cdot R_{wp}$, where the prime (′) is winding referral through the turns ratio, n: $R_{ws}' = n^2\cdot R_{ws}$. Now collect factors in R_{wp} as follows:

$$R_{wp} = F_r\cdot\left(\frac{R_{\delta r}}{l_w}\right)\cdot\left(\frac{l_w}{N_s}\right) = \frac{F_r}{N_s}\cdot\left[\left(\frac{R_{\delta r}}{l_w}\right)\cdot l_w\right] = f_r\cdot\left[\left(\frac{R_{\delta r}}{l_w}\right)\cdot l_w\right]$$

where N_s is the number of winding-bundle strands and $f_r = F_r/N_s$ is the *bundle* F_r. The expression in square brackets is nearly constant for a fixed frequency because l_w is nearly constant over a wide range of strand sizes. Thus, to minimize R_{wp}, f_r is the parameter to minimize, and the entire eddy-current effect is contained within it.

The eddy-current effects are the strand skin effect, f_{rS} and the bundle skin effect, f_{rSb};

$$f_r = \frac{f_{rS}}{N} + f_{rSb}$$

With N strands in parallel, the skin effect resistance reduces by N. There is only a single bundle, so f_{rSb} is (divided by one and) added to the strand skin effect. Not only is current crowded to the periphery of strands by the skin effect, it also is crowded to the outside of the bundle as the bundle skin effect.

To find R_{wp}, the following parameters are calculated for operation at f = 250 kHz:

$$\text{round-wire strand skin depth, } \delta_r \approx \frac{73.5\text{ mm}}{\sqrt{f/\text{Hz}}} = 0.147\text{ mm , Cu wire, 80 °C}$$

$$\xi_r = r_c(\#\,39)/\delta = 0.046\text{ mm}/0.147\text{ mm} = 0.313$$

where r_c is the conductive radius of the round wire. Additionally,

$$\frac{R_{\delta r}}{l_w} = \left(1.256\frac{\mu\Omega}{\text{m}\cdot\text{Hz}}\cdot(250\text{ kHz})\right) = 314\text{ m}\Omega/\text{m}$$

The approximate strand skin-effect equation[4] is

$$f_{rS} \approx \frac{1}{2\cdot\xi_r - 1}\text{ , } \xi_r > 0.5\text{, else } f_{rS} \approx 0$$

The bundle skin depth, δ_b is different than δ_r for strands because not all of the area within the bundle is conductive. It is reduced to

$$\delta_b \approx \frac{\delta_r}{\sqrt{k_{pb}\cdot k_{pw}}} > \delta_r$$

where k_{pw} is the strand porosity (wire insulation spacing) and k_{pb} is the fill factor within the bundle. The bundle penetration ratio (wire conductive radius in skin depths) is thus

$$\xi_{rb} = \xi_r\cdot\sqrt{N_s} = (0.313)\cdot(2.449) \approx 0.767\text{ ; } f_{rSb} \approx \frac{1}{2\cdot\xi_{rb} - 1} - \frac{1}{\xi_{rb}^2} = 0.174 \Rightarrow$$

$$f_r = \frac{f_{rS}}{N_s} + f_{rSb} \approx f_{rSb} = 0.174$$

We can now find winding resistance;

$$R_{wp} = f_r\cdot\left[\left(\frac{R_{\delta r}}{l_w}\right)\cdot l_w\right] = (0.174)\cdot[(0.314\ \Omega/\text{m})\cdot(36\text{ cm})] = 19.7\text{ m}\Omega$$

Primary winding power loss is $\overline{P}_{wp} = R_{wp}\cdot\tilde{i}_p^{\,2}$ = (19.7 mΩ)·(283 mA)² = 1.58 mW

Secondary loss is the same from R_{ws};

$$\overline{P}_{ws} = R_{ws}\cdot\tilde{i}_s^{\,2} = \frac{R_{ws}'}{n^2}\cdot\left(n\cdot\tilde{i}_s'\right)^2 = R_{ws}'\cdot\tilde{i}_s'^{\,2} = 1.58\text{ mW}$$

where the primes (′) are the secondary winding quantities referred to the primary winding and are primary-side quantities. In the above equation, they are referred back to the secondary winding. Therefore, the total winding loss is

$$\overline{P}_w = \overline{P}_{wp} + \overline{P}_{ws} = 3.16 \text{ mW}$$

Combined with the core loss, the total transductor loss is

$$\overline{P}_t = \overline{P}_c + \overline{P}_w = 104 \text{ mW} + 3.16 \text{ mW} = 107 \text{ mW}$$

Efficiency

The core loss dominates over the winding loss. This is unusual because the core is so small. It can sustain a much higher loss density than larger cores and results in a higher core loss. Winding loss is not commensurate with core loss because, although the wire size is small, its length is also short and this constrains winding resistance to a value typical of larger cores with larger wire. Relative to output power, the output-power efficiency can be defined by

$$\eta_o = \frac{\overline{P}_o - \overline{P}_t}{\overline{P}_o} = 1 - \frac{107 \text{ mW}}{840 \text{ mW}} = 87.3\,\%$$

where the no-loss output power, $\overline{P}_o$ = 840 mW, is from part 1. The calculated output power with losses is 733 mW.

The wide disparity of core and winding loss also affects the *power-transfer efficiency*. Ideally, efficiency peaks near equal core and winding losses for high η. The *power-loss ratio*, $\psi = \overline{P}_w / \overline{P}_c \approx 0.030 << 1$. Maximum transfer efficiency occurs at $\psi_{max} \approx 1$, but for this transductor, $\psi \approx 0$ because of the high core loss. More turns would reduce core loss by reducing Δ*B*, and winding loss would increase because of smaller and longer wire required to accommodate more turns. The small core allows it to be operated at a very high loss density, and this results in a core-loss-dominated transductor.

In closing, this design exercise shows that it is sometimes practical to use small cores—essentially beads—for low-power converter magnetic design at an acceptable efficiency and with no extraordinary construction difficulties. Though small cores are not designed to be optimum for power transfer, as this example shows, they can be used in designs that for some applications are quite acceptable.

References

All of the referenced materials below, unless otherwise noted, are by the author and are also found in the author's magnetics book, *Power Magnetics Design Optimization* (*PMDO*), in laminated-paper book form at www.innovatia.com Contact the author for a free PDF copy at www.innovatia.com/Inquiry.htm.

1. "Designing Low-Power Flyback Inductors Using Tiny Toroids (Part 1): A Boost Converter Application," How2Power Today, June 2019 (not included in PMDO).

2. Derived in "How To Calculate Toroid Winding Length," How2Power Today, September 2013.

3. Derived in "Determining Maximum Usable Switching Frequency For Magnetics In CCM-Operated Converters," How2Power Today, April 2015.

4. Adapted from derivations in "Eddy-Current Effects In Magnetic Design (Part 1): The Skin Effect" How2Power Today, August 2016.

About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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