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The Op Amp-Based Differential Amplifier: Not As Easy As It Looks

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When an inexpensive differential amplifier is needed, designers use the schematic shown in Fig. 1 (see the reference). They often just choose resistor values to achieve equality of gains in both inverting and non-inverting branches, forgetting that the sources of the signals V1 and V2 may have different output impedances. This impedance difference may completely destroy the differential amplifier operation, causing an output offset and compromising the common-mode rejection ratio (CMRR). Often, it is important that input impedances for signals V1 and V2 be equal to maintain acceptable values for CMRR and Vout offset.

Given a value of R1 and a required gain, we can select the values of the remaining resistors, R3-R4, with the goal of setting the input impedances equal. This article explains how to calculate those required resistor values, while also accounting for the variability of these values to initial tolerance, thermal coefficient of resistance, and aging. After deriving the necessary formulas, we present a design example that illustrates their use in designing a differential amplifier with equal impedances on each input and acceptable CMRR.

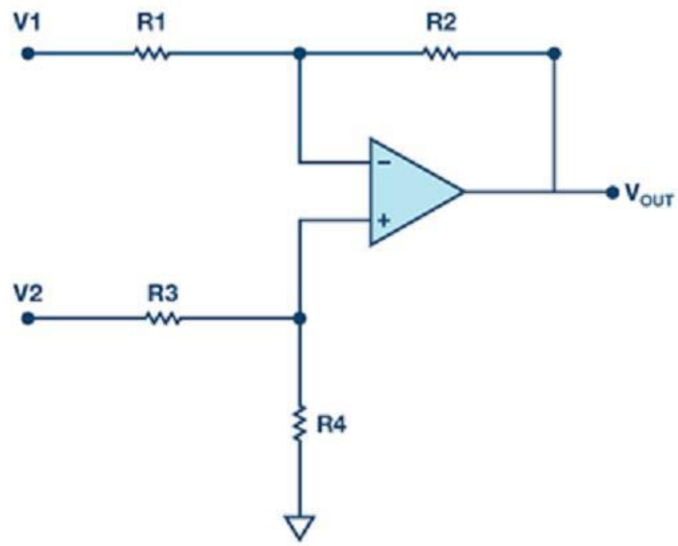

Fig. 1. The commonly used differential amplifier.

Defining Resistor Values

It is easy to obtain an expression for the output voltage V_{out} in the schematic shown above:

$$V_{out} = \left(1 + \frac{R2}{R1}\right) \cdot \frac{R4}{R3 + R4} \cdot V2 - \frac{R2}{R1} \cdot V1 \qquad (1)$$

The input impedance for the non-inverting input V2 is

$$R_{in\_p} = R3 + R4 \quad (2)$$

The input impedance for the inverting input V1 is

$$R_{in\_n} = R1 \quad (3)$$

We have to keep in mind that the connection of R1 and R2 is a virtual ground.

To keep input impedances equal, this condition should be observed:

$$R_{in\_p} = R_{in\_n} \quad (4)$$

Therefore

$$R1 = R3 + R4 \quad (5)$$

In expression (1),

$$\left(1 + \frac{R2}{R1}\right) \cdot \frac{R4}{R3 + R4}$$

is the gain of the non-inverting branch of the differential amplifier and R2/R1 is the gain modulus of the inverting branch. We assume that the differential amplifier is a linear structure, and the superposition principle applied holds true.

Hence, after applying (5), we get:

$$\left(1 + \frac{R2}{R3 + R4}\right) \cdot \frac{R4}{R3 + R4} = \frac{R2}{R3 + R4} \quad (6)$$

This gives the expression for R2 as

$$R2 = \frac{R4^2 + R3 \cdot R4}{R3} \quad (7)$$

Defining the modulus of the inverting branch gain as G_0, we can write down:

$$G_0 = \frac{R2}{R1} \tag{8}$$

which evokes

$$R2 = G_0 \cdot R1 \tag{9}$$

Therefore equation (7) may be written as

$$\frac{R4^2 + R3 \cdot R4}{R3} = G_0 \cdot R1 \tag{10}$$

From equation (5) we get

$$R4 = R1 - R3 \tag{11}$$

and equation (10) turns into

$$\frac{(R1 - R3)^2 + R3 \cdot (R1 - R3)}{R3} = G_0 \cdot R1 \tag{12}$$

Solving for R3 we obtain:

$$R3 = \frac{R1}{G_0 + 1} \tag{13}$$

This means we cannot just assign a value for R3, but rather have to find it.

Now, we can find R4 using (11):

$$R4 = R1 - R3 = R1 - \frac{R1}{G_0 + 1} = R1 \cdot \left(1 - \frac{1}{1 + G_0}\right) = R1 \cdot \frac{G_0}{1 + G_0} \tag{14}$$

So, as we have seen, we just have to set two parameters: input impedance R1 and necessary gain G_0 to obtain values for R2, R3 and R4.

Gain Accuracy And Stability

In order to make computations concise the following method of parameter notation and resistance calculation is proposed. Here we define resistance as a function of a few parameters. As we note each source of error in the resistor value, we'll assign a value taken from a resistor data sheet and we'll use these same values when working through the design example that follows.

First, there is resistor tolerance, which is provided by the manufacturer. It may have both plus and minus signs.

$$\delta_{Rtol} := 0.01$$

Next is the thermal coefficient of resistance. Its sign is never specified by manufacturer and may be arbitrary.

$$\delta_{Rth} := \frac{10^{-4}}{K}$$

Another measure of resistor variability is aging coefficient, which may have a positive sign only. Its minimal value is 0, which corresponds to the new product.

$$\delta_{Rage} := \begin{pmatrix} 0 \\ 0.02 \end{pmatrix}$$

To apply the temperature coefficient, we'll need to specify the largest temperature range to use. Specifically, we are concerned with the maximum deviation in temperature from room temperature (25°C) that produces the most extreme change in resistance. In this case, going from 25°C to 125°C has this effect.

$T_{max} - T_{nom}$

where T_{max} = 125°C and T_{nom} = 25°C.

Using the MathCAD 15 features, we can write down:

$$Rn\left(Rx, \delta_{Rtol}, \delta_{Rth}, \delta_{Rage}, T_{nom}, T_{max}\right) := Rx \cdot \begin{bmatrix} \left(1 - \delta_{Rtol}\right) \cdot \left[1 - \left(T_{max} - T_{nom}\right) \cdot \delta_{Rth}\right] \cdot \left(1 + \delta_{Rage_0}\right) \\ 1 \\ \left(1 + \delta_{Rtol}\right) \cdot \left[1 + \left(T_{max} - T_{nom}\right) \cdot \delta_{Rth}\right] \cdot \left(1 + \delta_{Rage_1}\right) \end{bmatrix} \quad (15)$$

where R_x is the data sheet value at 25°C.

The universal expression in (15) can be applied to find the variations in all four resistor values, which in turn, enables us to calculate the variations in gain and CMRR.

Now that we have formulas for R2, R3, R4 in terms of R1, and for the variability of these resistances, we can illustrate their use by assigning example values for R1 and gain:

R1 = 100 kΩ

G_0 = 20

Next, we'll calculate the variations in R1 due to the tolerance, aging and temperature per equation (15). Here, we assume the values of resistor tolerance, thermal coefficient of resistance, aging coefficient, and the temperature values noted above. We'll use these same parameters when calculating the variability of R2, R3 and R4.

$$R1 := Rn\left(R1, \delta_{Rtol}, \delta_{Rth}, \delta_{Rage}, T_{nom}, T_{max}\right) = \begin{pmatrix} 9.801 \times 10^4 \\ 1 \times 10^5 \\ 1.041 \times 10^5 \end{pmatrix} \Omega$$

With max, nominal and minimum values of R1 defined, we can now find the variations of R2:

$$R2 := G_0 \cdot R1 = \begin{pmatrix} 1.96 \times 10^6 \\ 2 \times 10^6 \\ 2.081 \times 10^6 \end{pmatrix} \Omega$$

This allows us to select a standard 1% scale value for R2 based on the nominal value just calculated for R2. It just so happens that the calculated nominal value is available as a standard value and may be slightly different. So, we select

R2 = $R2_{1,0}$ = 2 x 10^6 Ω where $R2_{1,0}$ refers to the second value down in the matrix above for R2.

Now that we have a value for R2, we'll want to calculate the variations in its value using (15).

$$R2 := Rn\left(R2, \delta_{Rtol}, \delta_{Rth}, \delta_{Rage}, T_{nom}, T_{max}\right) = \begin{pmatrix} 1.96 \times 10^6 \\ 2 \times 10^6 \\ 2.081 \times 10^6 \end{pmatrix} \Omega$$

Since our selected resistor value for R2 was the same as the nominal value originally calculated and since the values for tolerance, temp coefficient and other factors were the same as for R1, the calculated max and min values for R2 are also the same in both calculations of R2. But this won't always be the case as we'll see with R3.

Next, we find variations of R3, based on the range of values for R1:

$$R3 := \frac{R1}{G_0 + 1} = \begin{pmatrix} 4.667 \times 10^3 \\ 4.762 \times 10^3 \\ 4.955 \times 10^3 \end{pmatrix} \Omega$$

The closest standard value with 1% tolerance is:

R3 = 4.75 kΩ

So that is the actual value we select and it allows us to calculate the maximum and minimum variations in R3.

$$R3 := Rn\left(R3, \delta_{Rtol}, \delta_{Rth}, \delta_{Rage}, T_{nom}, T_{max}\right) = \begin{pmatrix} 4.655 \times 10^{3} \\ 4.75 \times 10^{3} \\ 4.942 \times 10^{3} \end{pmatrix} \Omega$$

Notice that since the selected value for R3 was slightly different from the originally calculated nominal value, the minimum and maximum variations calculated for the selected value are also different from those initially calculated.

Then, we find variations of R4:

$$R4 := R1 \cdot \frac{G_0}{1 + G_0} = \begin{pmatrix} 9.334 \times 10^{4} \\ 9.524 \times 10^{4} \\ 9.91 \times 10^{4} \end{pmatrix} \Omega$$

Given the nominal for R4 just calculated, we choose the closest standard value in 1%:

R4 = 95.3 kΩ

and then calculate its variations:

$$R4 := Rn\left(R4, \delta_{Rtol}, \delta_{Rth}, \delta_{Rage}, T_{nom}, T_{max}\right) = \begin{pmatrix} 9.34 \times 10^{4} \\ 9.53 \times 10^{4} \\ 9.916 \times 10^{4} \end{pmatrix} \Omega$$

Armed with the four resistor values, we can determine the resulting gain variations for each branch.

For the non-inverting branch:

$$G_{01} := \overrightarrow{\left[\left(1 + \frac{R2}{\mathrm{reverse}(R1)}\right) \cdot \frac{R4}{\mathrm{reverse}(R3) + R4}\right]} = \begin{pmatrix} 18.842 \\ 20.003 \\ 21.236 \end{pmatrix}$$

And for the inverting branch:

$$G_{02} := \overrightarrow{\frac{R2}{\mathrm{reverse}(R1)}} = \begin{pmatrix} 18.839 \\ 20 \\ 21.233 \end{pmatrix}$$

where values of R1-R4 are selected to obtain the largest variations in the gain values. The "reverse" designation indicates that the value of that particular resistance was chosen to be going to the opposite extreme of the other resistance values in the expression, in order to obtain the widest swings in the min and max values.

Finally, we calculate CMRR:

$$20 \cdot \log\left(\overrightarrow{\frac{G_0}{G_{01} - G_{02}}}\right) = \begin{pmatrix} 76.508 \\ 76.483 \\ 76.459 \end{pmatrix} \quad \text{dB}$$

As we see in this example, proper calculations of the simple differential amplifier components allow us to obtain good enough CMRR using regular 1% resistors.

Reference

"Rarely Asked Questions—Issue 165 Discrete Difference Amplifier vs. an Integrated Solution" by Jordyn Ansari and Chau Tran, Analog Dialogue 53-05, May 2019.

About The Author

Gregory Mirsky is a senior electrical engineer with Continental Automotive Systems in Deer Park, Ill., which he joined in March 2015. In his current role, Gregory performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification.

He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.