Square Vs. Round Core Legs—Balancing Core And Winding Losses

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When designing transformers and inductors for power electronics, engineers have a choice of magnetic core shapes. Newer core shapes such as EC, ER, and ETD have round center legs whereas older shapes such as EE and EI have square center legs. For toroids, both round and square or rectangular rings prevail. This article analyzes the performance differences between round and square shapes. Round center-posts are compared to square ones according to two criteria, winding length—hence resistance—and thermal performance.

Understanding the impact of core center-post shapes on these two factors can help designers optimize magnetics' performance in the application in terms of balancing core and winding losses. This article quantifies the differences in winding length and thermal shape factor (a measure of a core's ability to dissipate heat versus a sphere) for round versus square center posts.

Winding Length

Round inner legs reduce winding length over square legs and have a winding-loss advantage. Square center legs of the same area, *A*, as round legs have a side dimension, *s*;

$$A = \frac{\pi}{4} \cdot d^2 = s^2$$

The square side length associated with area *A* is thus

$$s = \frac{\sqrt{\pi}}{2} \cdot d = \sqrt{\pi} \cdot r$$

The average length of one turn around a square-leg core is

$$\bar{l}_{cs} = 4 \cdot s = 2 \cdot \sqrt{\pi} \cdot d$$

And around a round-leg core, it is

$$\bar{l}_{cr} = \pi \cdot d$$

Then the ratio of the lengths is

$$\frac{\bar{l}_{cs}}{\bar{l}_{cr}} = \frac{2}{\sqrt{\pi}} \approx 1.128$$

Therefore, a turn around a square leg is about 12.8% longer than around a round leg, and the winding is thus about 12.8% longer and winding resistance that much greater. In practice, it is worse than this because the sharp corners cause wire to bend around them in arcs that do not perfectly follow the square shape, thus adding length.

Core Thermal Performance

The comparison for round and square legs for core thermal performance has been carried out in reference [1], and adapted here. Thermal performance is expressed through the *thermal shape factor*, Ξ_θ, defined in [1] as the ratio of $A_s/V^{2/3}$ for a given core shape, normalized to the worst-case thermal core shape, that of a sphere. The thermal shape factor is how much better a shape is thermally than a sphere, and is defined as

$$\Xi_\theta \equiv \frac{\left.\frac{A_s}{V^{2/3}}\right|_{\text{shape}}}{\left.\frac{A_s}{V^{2/3}}\right|_{\text{sphere}}} = \frac{\left.\frac{A_s}{V^{2/3}}\right|_{\text{shape}}}{(36\cdot\pi)^{1/3}} \approx \frac{\left.\frac{A_s}{V^{2/3}}\right|_{\text{shape}}}{4.836} \geq 1$$

where A_s is the surface area of the core from which heat is convected away and V is core volume. For a well-designed core, there are no hot-spots and power loss density is essentially uniform throughout its volume. Ξ_θ varies with core shape but not size. For complicated shapes with multiple dimensions, the above formula gives Ξ_θ for a given core from A_s and V. Then the allowable core power-loss density, $\bar{p}_c$ is calculated from $\bar{p}_c(\text{sphere})$ and Ξ_θ as

$$\bar{p}_c = \Xi_\theta \cdot \bar{p}_c(\text{sphere})$$

A simple shape example, a cube of side length, s has

$$\frac{A_s}{V^{2/3}} = \frac{6\cdot s^2}{(s^3)^{2/3}} = 6$$

Then the thermal shape factor is

$$\Xi_\theta(\text{cube}) \approx \frac{A_s/V^{2/3}}{4.836} = \frac{6}{4.836} \approx 1.241$$

This says simply that the cube surface area is greater than that of a sphere of the same volume by about 24%.

The shape of a round center-leg is cylindrical, having

$$A_s = 2\cdot\pi\cdot r\cdot l\ ;\ \ V = \pi\cdot r^2\cdot l \ \Rightarrow \frac{A_s}{V^{2/3}} = \frac{2\cdot\pi\cdot r\cdot l}{(\pi\cdot r^2\cdot l)^{2/3}} = 2\cdot\left(\frac{\pi\cdot l}{r}\right)^{1/3} \Rightarrow$$

$$\Xi_\theta(\text{cylinder}) \approx \frac{2\cdot\left(\frac{\pi\cdot l}{r}\right)^{1/3}}{(36\cdot\pi)^{1/3}} = \frac{2}{36^{1/3}}\cdot\left(\frac{l}{r}\right)^{1/3} \approx 0.6057\cdot\left(\frac{l}{r}\right)^{1/3}$$

A prismatic or "square" leg having the same volume as the round leg has

$$A_s = 4\cdot s\cdot l\ ;\ V = s^2\cdot l \Rightarrow \frac{A_s}{V^{2/3}} = \frac{4\cdot s\cdot l}{(s^2\cdot l)^{2/3}} = 4\cdot\left(\frac{l}{s}\right)^{1/3} \Rightarrow$$

$$\Xi_\theta(\text{square - leg}) \approx \frac{4\cdot\left(\frac{l}{s}\right)^{1/3}}{(36\cdot\pi)^{1/3}} \approx 0.8271\cdot\left(\frac{l}{s}\right)^{1/3}$$

Solve for *s* as a function of *r* of the round leg that makes the volumes equal;

$$V = \pi\cdot r^2\cdot l = s^2\cdot l \Rightarrow s = \sqrt{\pi}\cdot r$$

Substituting for *s*,

$$\Xi_\theta(\text{square - leg}) \approx \frac{4\cdot\left(\frac{l}{\sqrt{\pi}\cdot r}\right)^{1/3}}{(36\cdot\pi)^{1/3}} = \left(\frac{16}{9}\right)^{1/3}\cdot\frac{1}{\sqrt{\pi}}\cdot\left(\frac{l}{r}\right)^{1/3} \approx 0.6835\cdot\left(\frac{l}{r}\right)^{1/3}$$

Then the relative advantage of the square leg to the round leg is

$$\frac{\Xi_\theta(\text{square - leg})}{\Xi_\theta(\text{cylinder})} \approx \frac{0.6835}{0.6057} = 1.128$$

The square shape of the center leg can rid it of heat 13% better than the round leg because it has more surface area for the same volume, all else being equal. Thus the square center-leg is thermally superior to the round center-leg.

Closure

Thermal and winding resistance design considerations for inner-leg cores are a tradeoff between core and winding loss. The power-loss ratio,

$$\psi = \frac{P_w}{P_c}$$

for maximum interwinding power transfer works out to be slightly greater than one as described in reference [2]. This seems to favor slightly the square legs. However, ψ is dominated in typical magnetics design by other factors than the center-post of the core, and no general conclusions can be drawn. Yet the analysis is illuminating; if a square-post design has ψ too large, switching to a round-post core could be advantageous. And if a design with a round-post core has a suboptimally low ψ, a square center-post might be better.

From the above design derivations, numbers can be put to how much suboptimality square or round inner-legs bring, and it can be of some significance. The equations show that the newer round center-leg cores are not always better than the older square-leg cores. It depends on whether winding or core loss needs to be minimized.

Toroids and EF and EFD cores require further analysis (a subject for another article). While the results here can be readily applied to toroids in that square- or rectangular-ring toroids have longer windings and better heat expulsion, EF and EFD cores have rounded rectangular center-posts for good thermal performance while the rounded ends reduce winding length.

References

1. *Power Magnetics Design Optimization* (*PMDO*), see chapter 3, "Magnetic Design". This work is available in laminated-paper book form at www.innovatia.com or contact the author for a free PDF copy at www.innovatia.com/Inquiry.htm.

2. It is commonly thought that maximum power transfer across windings occurs whenever $\overline{P}_w = \overline{P}_c$, but that is only approximately true and at high efficiency. An in-depth analysis of power transfer is given in "Magnetics Optimization (Part 1): Equal Core And Winding Losses Do Not Maximize Power Transfer," How2Power Today, September 2015, and in *Power Magnetics Design Optimization*, D. Feucht, www.innovatia.com, chapter 5, "Optimization", pages 270 – 308.

About The Author

Dennis Feucht has been involved in power electronics for 30 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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