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Debunking The Gapped Inductor Myth

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Many power converters operate on the principle that electromagnetic energy is stored during a portion of the switching period, and then released during the rest of the period. This energy is stored in inductors, which often are subject to a substantial dc current flow. This mode of operation requires taking measures to avoid saturation of the inductor's core, which may often be detrimental to the device it is used in.

The reference explains how to design such an inductor based on a distributed-gap magnetic. Usage of distributed-gap magnetics is preferable versus using magnetics with a physical air gap since the former store energy in the whole core, which has a much larger volume than the air gap. Therefore, with energy stored in a distributed gap, there is much lower power density, reducing the core operating temperature.

Building an inductor with a distributed gap requires use of powdered or amorphous materials in the core as these materials have a permeability that is controllable during manufacturing. However, the cost of cores made with such materials may be a few times that of a ferrite, which has low saturation flux density but high permeability. Adding a controlled air gap to a ferrite core lowers its permeability, reducing the effective permeability of the whole core (i.e. core plus gap). Doing this is necessary to expand the current range over which the ferrite core does not saturate. Therefore, the requirement for lower cost design often leads to use of a ferrite with an air gap.

While browsing the Internet, I recently discovered that many engineers do not treat gapped magnetics correctly: some state that an air gap increases the saturation flux density, others say that inductors based on an air gap in the magnetic core store energy in the gap only. Neither statement is true because the gap in the core just lowers the core's effective permeability, leaving the core material saturation flux density intact. And the energy is stored in the whole gapped magnetic, with the energies stored in the gap and the rest of the core in reverse proportion to the permeabilities of air and the magnetic material. Sometimes, the energy stored in the core may be substantial. Anyway, we have to deal with all possible cases of magnetic energy storage distribution over the gapped core.

This article proposes a new approach to designing gapped-core inductors. It recommends that first, designers determine the permeability that would be needed in the application if a distributed-gap core was used. Then designers calculate the real physical air gap that would be required for the equivalent air-gaped core. In addition to simplifying the calculations versus the usual design procedures, this proposed procedure allows for more-accurate sizing of the gap. At the same time, this procedure defines the parameters for a distributed-gap core, which may be a real option for the design.

We will initially define the necessary volume for the distributed-gap core and then its magnetic path length. Finally, we will define the air gap length in a real core with a high permeability.

After presenting this procedure for determining the size of the physical air gap, we'll use the equations derived here to obtain an expression for the ratio of power stored in the gap to power stored in the rest of the core. This ratio is the same as the ratio of energy stored in each part. Then we'll look at a typical inductor design example to see how much of the energy is stored in the gap.

Defining The Core Size For A Distributed Gap Inductor

The following parameters will be central to our calculations:

D_C = duty-cycle of the switching process

P_{OUT} = output power of the converter

μ_{rdg} = relative permeability of the distributed-gap core material

B_{sat} = saturation flux density for the material of the core having an air gap. This will make the core bigger but will avoid its saturation.

B_m = magnetic flux density value in a magnetic core

H_m = magnetic field strength in the core

η = power converter efficiency

f_{SW} = switching frequency

S_{core} = cross-sectional area of the core

t_{on} = time when the switch is on, and energy is being stored by the magnetic core per Fig. 1

l_{mag} = median circumference length or median magnetic line length of the magnetic core

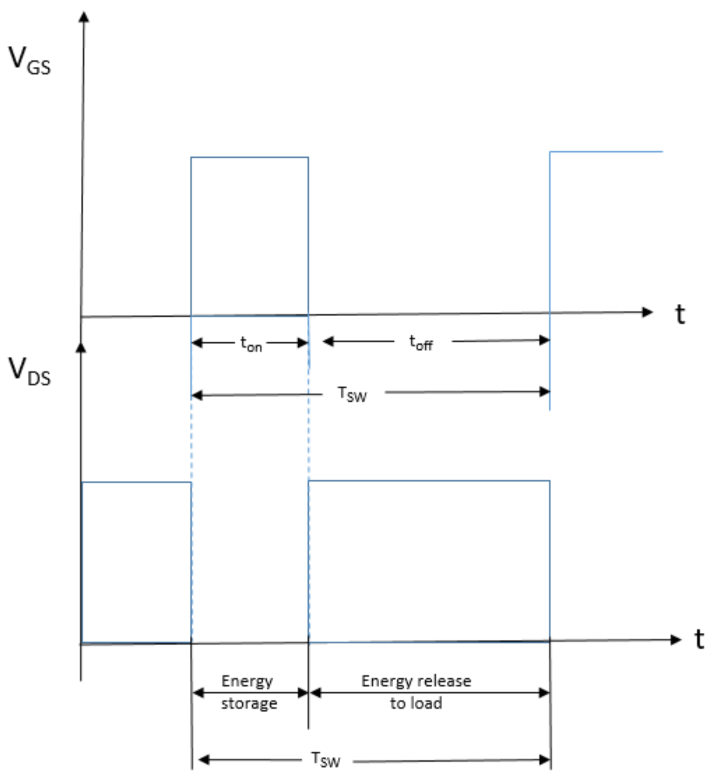

Fig. 1. Power switch waveforms contain the timing parameters needed to calculate core size for the distributed-gap inductor.

It is known that energy E_m stored in a magnetic core is proportional to its physical volume (Vol):

$$E_m = \frac{B_m \cdot H_m}{2} \cdot Vol \tag{1}$$

If this energy was stored for t_{on} we can talk about power P_{max} stored by the core.

$$P_{max} = \frac{E_m}{t_{on}} \tag{2}$$

Plugging equation (1) into (2), we obtain:

$$P_{max} = \frac{B_m \cdot H_m \cdot Vol}{2 \cdot t_{on}} \tag{3}$$

The parameter that is given at the start of the design, is the output power P_{out} which converts into the average energy E_{out} for the period of the switching process:

$$E_{out} = P_{out} \cdot T_{sw} \tag{4}$$

Due to the energy loss of the energy conversion process, the average energy E_m stored in the core during the storage process, should be higher than that provided to the output, as determined by the efficiency, η:

$$E_m = \frac{E_{out}}{\eta} \tag{5}$$

Plugging equation (4) into (5), we get:

$$E_m = \frac{P_{out} \cdot T_{sw}}{\eta} \tag{6}$$

This energy is created by power P_{max} acting for t_{on}:

$$P_{max} = \frac{\frac{P_{out} \cdot T_{sw}}{\eta}}{t_{on}}$$

which is

$$P_{max} = \frac{P_{out}}{D_c \cdot \eta} \tag{7}$$

Substituting the P_{max} from equation (3) into (7) we get:

$$\frac{\frac{B_m \cdot H_m}{2} \cdot Vol}{t_{on}} = \frac{P_{out}}{D_c \cdot \eta} \qquad (8)$$

Presuming we are dealing with a distributed-gap core having permeability μ_{rdg}

$$H_m = \frac{B_m}{\mu_0 \cdot \mu_{rdg}} \qquad (9)$$

and

$$t_{on} = T_{sw} \cdot D_c = \frac{D_c}{f_{sw}} \qquad (10)$$

Substituting these expressions for H_m and t_{on} into (8), we can define the distributed-gap magnetic core necessary volume as:

$$Vol = \frac{2 \cdot P_{out} \cdot \mu_0 \cdot \mu_{rdg}}{B_m^{\ 2} \cdot \eta \cdot f_{sw}} \qquad (11)$$

Defining The Size Of The Air Gap In A Ferrite Core

Assuming

$$Vol = l_{mag} \cdot S_{core}$$

we can define the permeability of the distributed-gap core, having selected some adequate core and assigned the saturation flux density value B_{sat} to the B_m. A few iterations may be required to define a permeability and/or core size, which are close to standard values:

$$\mu_{rdg} = \frac{B_m^{\ 2} \cdot S_{core} \cdot \eta \cdot f_{sw} \cdot l_{mag}}{2 \cdot P_{out} \cdot \mu_0} \qquad (12)$$

This distributed-gap-core permeability ensures non-saturable operation of the core up to the maximum output power.

If the distributed-gap core is unavailable, it is possible to calculate the air gap size for an equivalent ferrite core. We can calculate the ferrite core gap that makes the resulting permeability of the gapped core equal to the permeability of the distributed-gap core. In order to do that we apply the full-current theorem to this case.

Designating magnetic field strength in the ferrite material as H_m, in the air gap as H_g and in the distributed-gap core as H_{dg} we can write the following equation using the gap and core lengths identified in Fig. 2:

$$H_m \cdot \left(l_{mag} - l_{gap}\right) + H_g \cdot l_{gap} = H_{dg} \cdot l_{mag}$$

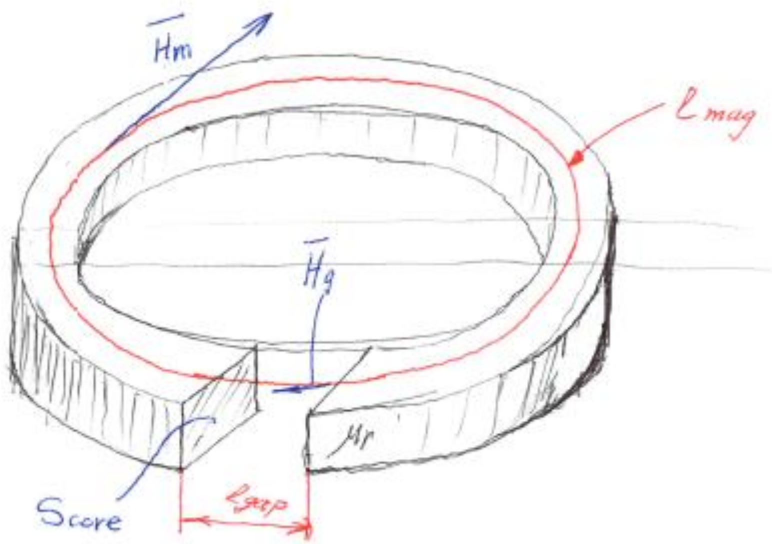

Fig. 2. Key parameters for calculating the size of the gap in a gapped-core ferrite inductor.

Since

$$H_m = \frac{B_m}{\mu_0 \cdot \mu_r} \tag{14}$$

$$H_g = \frac{B_g}{\mu_0} \tag{15}$$

$$H_{dg} = \frac{B_{dg}}{\mu_0 \cdot \mu_{rdg}} \tag{16}$$

where B_m, B_g and B_{dg} are magnetic flux densities in the ferrite core, air gap and distributed-gap core respectively, we can rewrite (13) as

$$\frac{B_m}{\mu_0 \cdot \mu_r} \cdot \left(l_{mag} - l_{gap}\right) + \frac{B_g}{\mu_0} \cdot l_{gap} = \frac{B_{dg}}{\mu_0 \cdot \mu_{rdg}} \cdot l_{mag} \tag{17}$$

But the magnetic flux lines are uninterruptible, which means the magnetic flux density is uniform along the whole core length, so

$$B_m = B_g = B_{dg} \tag{18}$$

and therefore (17) reduces to

$$\frac{1}{\mu_0 \cdot \mu_r} \cdot \left(l_{mag} - l_{gap}\right) + \frac{1}{\mu_0} \cdot l_{gap} = \frac{1}{\mu_0 \cdot \mu_{rdg}} \cdot l_{mag} \tag{19}$$

Rearranging equation (19) gives us an expression for l_{gap}:

$$l_{gap} = \frac{\left(\mu_{rdg} - \mu_r\right) \cdot l_{mag}}{\left(1 - \mu_r\right) \cdot \mu_{rdg}} \tag{20}$$

We have to keep in mind that the ferrite core and distributed-gap core should have the same size (or close to the same size) and configuration. Or, if we are considering a virtual distributed-gap core, it should have the same size and configuration as the real ferrite core. (We're describing the distributed-gap core as "virtual" for the cases where we don't intend to use a distributed-gap core and are simply calculating its values for the purpose of designing a gapped-core inductor with an air gap.)

Where Does The Stored Energy Hide In A Gapped Core?

If we return to equation (3)

$$P_{max} = \frac{B_m \cdot H_m \cdot Vol}{2 \cdot t_{on}}$$

and use the following expression for H_m:

$$H_m = \frac{B_m}{\mu_0 \cdot \mu_r}$$

we can figure out the power stored in the magnetic core volume Vol_{mag}:

$$P_{mag} = \frac{B_m{}^2 \cdot Vol_{mag}}{2 \cdot t_{on} \cdot \mu_0 \cdot \mu_r} \tag{21}$$

Solving (21) with respect to $B_m{}^2$ gives

$$B_m{}^2 = \frac{2 \cdot P_{mag} \cdot t_{on} \cdot \mu_0 \cdot \mu_r}{Vol_{mag}} \tag{22}$$

Power, stored in the air gap, is

$$P_{gap} = \frac{B_m{}^2 \cdot Vol_{gap}}{2 \cdot t_{on} \cdot \mu_0} \tag{23}$$

which gives

$$B_m{}^2 = \frac{2 \cdot P_{gap} \cdot t_{on} \cdot \mu_0}{Vol_{gap}} \tag{24}$$

Since the flux density is uniform over the whole core

$$\frac{2 \cdot P_{mag} \cdot t_{on} \cdot \mu_0 \cdot \mu_r}{Vol_{mag}} = \frac{2 \cdot P_{gap} \cdot t_{on} \cdot \mu_0}{Vol_{gap}} \tag{25}$$

Therefore, the stored power ratio, P_{gap}/P_{mag} is

$$\frac{P_{gap}}{P_{mag}} = \frac{Vol_{gap} \cdot \mu_r}{Vol_{mag}} \tag{26}$$

which represents the ratio of power stored in the gap to the power stored in the rest of the core, which is the same as the ratio of energies stored in the respective parts of the magnetic.

Since the cross-sectional area is uniform, equation (26) reduces to:

$$\frac{P_{gap}}{P_{mag}} = \mu_r \cdot \frac{l_{gap}}{l_{mag} + l_{gap}} \tag{27}$$

This expression allows us to calculate just how much of the energy stored by the inductor is stored in the gap, as we'll demonstrate in the following section.

A Design Example

Let's begin by designating the following shorthand for the power storage ratio:

$$\beta = \frac{P_{gap}}{P_{mag}}$$

Now let's assume the following design parameters: μ_r = 2000, B_{sat} =0.35 T, D_c = 0.42, S_{core} = 6 mm x 8 mm = 48 x 10^{-6} m^2, η = 0.8, l_{mag} = 160 mm, P_{out} = 120 W and f_{SW} = 100 kHz.

Then using the equations from above, we can calculate the following value of permeability for a distributed-gap inductor:

$$\mu_{rdg} := \mathrm{round}\left(\frac{B_{sat}^{\ 2} \cdot S_{core} \cdot \eta \cdot f_{sw} \cdot l_{mag}}{2 \cdot P_{out} \cdot \mu_0}\right) = 250$$

Then, applying other equations derived above, we can determine the physical size of the gap for an equivalent air-gap inductor:

$$l_{gap} := \frac{(\mu_{rdg} - \mu_r) \cdot l_{mag}}{(1 - \mu_r) \cdot \mu_{rdg}} = 560.28 \times 10^{-6}\,\mathrm{m}$$

In this case, the power storage ratio is:

$$\beta := \mu_r \cdot \frac{l_{gap}}{l_{mag} + l_{gap}} = 6.979$$

So, in this example, the power (or energy) stored in the gap is about 7 times that which is stored in the rest of the core. This illustrates that not all the energy is stored in the gap.

What's more, very often the air gap stores energy that is comparable to the energy stored by the rest of the core, in which case β would be closer to 1. This fact cannot be neglected, and the magnetic design should be done with this assumption.

In other words, we have options and tradeoffs to make when selecting the core. We can dissipate energy in the gap and deal with high temperature there. Or we can dissipate it more slowly across a relatively large magnetic core. Energy will be stored in both the gapped and distributed-gap cores, but at different expense.

Reference

"Designing Energy Storing Inductors Properly" by Gregory Mirsky, How2Power Today, January 2019.

About The Author

Gregory Mirsky, is a senior electrical engineer with Vitesco Technologies, a spinoff of Continental Automotive Systems, in Deer Park, Ill., which he joined in March 2015. In his current role, Gregory performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification.

He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

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