Optimize Inductor Design To Minimize Output-Filter Size

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In the converter PWM-switch CP (buck) and CA (boost) configurations, the PWM-switch inductor, L, is in series with the input and output ports. The output capacitor, C_o, is in parallel with the output port, as shown in Fig. 1. Actual L and C_o components have parasitic series resistance which causes power loss and these parasitics are not represented in the figure.

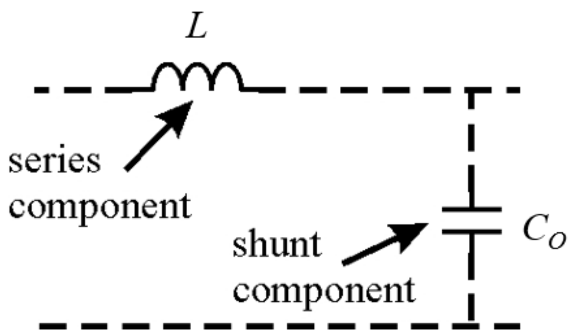

Fig. 1. The output filter circuitry in PWM-switch converters, with the power-transfer inductor from input to output port (switches omitted), and output capacitor shunting the output voltage port.

Both the inductor and the output storage capacitor are subject to either design or component selection by which the loss in each can be determined. It's common that they are the largest components in a converter. Their combined component size can be minimized by not allowing either component to become much larger than the other.

In this article, equations are derived to enable the inductor and capacitor to be similarly sized in order to minimize overall filter size (volume). We approach this problem from a thermal viewpoint and set out to equalize the inductor and capacitor power losses. From there we can determine the optimal value of L for a minimum volume of L and C_o.

The first version of the equation for optimal L includes size-dependent core parameters, which assumes a particular core has been chosen. So, this equation is further refined to express it without reference to these core-specific parameters as well as in another form that does assume a specific core, but uses more readily available core data sheet parameters. Each of these equations also assumes that the capacitor has been selected.

Toward the end of the article, another form of the equation will express the optimum value of C for the case where the inductor has already been designed or specified. But before that a design example will be presented to demonstrate the calculation of optimal L, given a specified core and capacitor. The article concludes with some discussion of how to use the derived equations, and how capacitor and core manufacturers could support the use of these equations in their data sheets.

Determining L And C Losses

With constant output voltage, the ripple component, $i_{L\sim}$ of inductor current is the capacitor current; $i_C = i_{L\sim}$. In the inductor, $i_{L\sim}$ causes core and eddy-current winding loss, minimized by winding design. Core loss depends on frequency, field density, and choice of core material and size which relates to field density. Core power-loss density, in mW/cm^3, is graphed in core catalogs for various core materials. If the winding area is not over-constrained, core loss is the determining factor of core size. Cores are designed so that if the window area is filled with winding, the winding loss, P_w and core loss, P_c result in a power-loss ratio of $\psi = P_w/P_c \approx 1$ for maximum power transfer.[1] Then total inductor loss is $P_L = P_c + P_w \approx 2 \cdot P_c$.

Cylindrical electrolytic capacitors have a shape similar to that of pot-based cores, including PQ, RM, and slabbed pot cores and thus have a comparable *thermal shape factor.*[2] Capacitors have both electrical (plate resistance) losses and dielectric loss from polarization in the dielectric material by displacement current—a loss corresponding to core hysteresis loss in inductors.

Electrolytic capacitor power-loss ratings are not given in catalogs as such; maximum current is given instead. Capacitor series resistance is also not usually given. If inductor and capacitor power loss densities are about the same and core size is determined by core loss, then equal parasitic resistances dissipating power result in equal component volumes.

By the equal-resistance criterion, for inductor core resistance, R_L and capacitor series resistance, R_C, average inductor and output power losses from ripple current are

$$\overline{P}_L = 2\cdot\overline{P}_c = \tilde{i}_{L\sim}^{\;2}\cdot R_L = \overline{P}_C = \tilde{i}_{L\sim}^{\;2}\cdot R_C$$

For the inductor, magnetic loss can be approximated by the Steinmetz "classical" loss equation,

$$\overline{P}_c = \left[\overline{p}_{c0}\cdot\left(\frac{\hat{B}_\sim}{\hat{B}_{\sim 0}}\right)^2\right]\cdot V$$

where $\overline{p}_{c0}$ is the operating-point core-loss average power density, $\hat{B}_\sim = \Delta B/2$ is the peak or amplitude of the magnetic-field density ripple, and $\hat{B}_{\sim 0}$ is its value at a given operating-point of $B_0 = \overline{B}$. $\overline{P}_c$ scales with $\overline{p}_{c0}$ with the given core material and constant switching frequency, f_s.

The inductor average magnetic power loss is $\overline{P}_c = \overline{p}_c \cdot V$ where V is the core volume. For optimal inductor design, the magnetic (core) and electrical (winding) losses are about equal and the total inductor loss is

$$\overline{P}_L = \overline{P}_c + \overline{P}_w \approx 2\cdot\overline{P}_c$$

The capacitor loss for $i_L(t)$ with CCM triangle-wave ripple, $i_{L\sim} = \Delta i_L$ is

$$\overline{P}_C = \tilde{i}_{L\sim}^{\;2}\cdot R_C = \frac{(\Delta i_L/2)^2}{3}\cdot R_C = \frac{\hat{i}_{L\sim}^{\;2}}{3}\cdot R_C$$

where the ripple component is RMS capacitor current, $\tilde{i}_C = \tilde{i}_{L\sim}$, and is the RMS ($\tilde{i}$) value of the i_L ripple ($i_{L\sim}$). Equating losses for approximately maximum power transfer at high efficiency,[1]

$$\overline{P}_L = \overline{P}_C \;\Rightarrow\; 2\cdot\overline{p}_{c0}\cdot\left(\frac{\hat{B}_\sim}{\hat{B}_{\sim 0}}\right)^2\cdot V = \frac{\hat{i}_{L\sim}^{\;2}}{3}\cdot R_C$$

The inductor current ripple amplitude that corresponds to the peak magnetic field density is $\hat{i}_{L\sim} = \Delta i_L/2 = \hat{i}_C/2$.

Next, refer $\Delta\lambda = L\cdot\Delta i_L$ to the field (with "Magnetic Ohm's Law") as

$$\Delta\lambda = N\cdot\Delta\phi = N\cdot\Delta B\cdot A = (N^2\cdot\mathcal{L})\cdot\Delta i_L \;\Rightarrow$$

$$\hat{B}_\sim = \frac{\Delta B}{2} = \frac{\mathcal{L}\cdot N\cdot(\Delta i_L/2)}{A},\;\; N = \sqrt{\frac{L}{\mathcal{L}}}$$

Then substituting *N* and simplifying,

$$\hat{B}_{\sim} = \frac{\sqrt{L \cdot \boldsymbol{\mathcal{L}}}}{A} \cdot \frac{\Delta i_L}{2} = \frac{\sqrt{L \cdot \boldsymbol{\mathcal{L}}}}{A} \cdot \hat{i}_{L\sim}$$

where the left factor is constant. Then $\hat{B}_{\sim} / \hat{B}_{\sim 0}$ is

$$\frac{\hat{B}_{\sim}}{\hat{B}_{\sim 0}} = \frac{\hat{i}_{L\sim}}{\hat{i}_{L\sim 0}}$$

Substitute this into the equal-loss equation;

$$2 \cdot \overline{p}_{c0} \cdot \left(\frac{\hat{i}_{L\sim}}{\hat{i}_{L\sim 0}} \right)^2 \cdot V = \frac{\hat{i}_{L\sim}{}^2}{3} \cdot R_C \Rightarrow \hat{i}^2_{L\sim 0} = \frac{6 \cdot \overline{p}_{c0} \cdot V}{R_C}$$

The optimal *L* for minimized volume of *L* and C_o is thus

$$L_{opt} = N^2 \cdot \boldsymbol{\mathcal{L}} = N^2 \cdot \frac{\boldsymbol{\mathcal{L}}^2}{\boldsymbol{\mathcal{L}}} = \left(\frac{(N \cdot \hat{i}_{L\sim 0}) \cdot \boldsymbol{\mathcal{L}}}{\hat{i}_{L\sim 0}} \right)^2 \cdot \frac{1}{\boldsymbol{\mathcal{L}}} = \left(\frac{\hat{\phi}_{L\sim 0}}{\hat{i}_{L\sim 0}} \right)^2 \cdot \frac{1}{\boldsymbol{\mathcal{L}}} = \frac{\hat{B}^2_{\sim 0} \cdot A^2}{\hat{i}_{L\sim 0}{}^2 \cdot \boldsymbol{\mathcal{L}}} \qquad (1)$$

A and $\mathcal{L}$ are geometry- or size-dependent core parameters. Without choice of a core, size is unknown and size-dependent parameters need to be eliminated from the equation. Substituting for $i_{L\sim 0}{}^2$, this simplifies to

$$L_{opt} = \frac{R_C \cdot \hat{B}^2_{\sim 0} \cdot A^2}{6 \cdot \overline{p}_{c0} \cdot \boldsymbol{\mathcal{L}} \cdot V} = \frac{R_C \cdot \hat{B}^2_{\sim 0} \cdot A^2}{6 \cdot \overline{p}_{c0} \cdot \left(\frac{\mu \cdot A}{l} \right) \cdot (A \cdot l)} = \frac{R_C}{6 \cdot \mu} \cdot \left(\frac{\hat{B}^2_{\sim 0}}{\overline{p}_{c0}} \right)$$

where μ is the permeability of the inductor core at its operating-point. The expression for *L* can be cast into a more interpretable form as

$$L_{opt} = \tfrac{1}{3} \cdot R_C \cdot \frac{\tfrac{1}{2} \cdot \hat{B}^2_{\sim 0} / \mu}{\overline{p}_{c0}} = \tfrac{1}{3} \cdot R_C \cdot \frac{w^2_{L0}}{\overline{p}_{c0}} \qquad (2)$$

The numerator is the linear magnetic energy density at operating-point, 0 and the ratio is in units of time. R_C is in ohms (Ω) and ohms x seconds equals henries (Ω·s ≡ H), the unit of inductance.

As an alternative to μ, given as catalog core data are $\mathcal{L}$, *A*, and *l* if it has already been chosen. Substituting for μ,

$$L_{opt} = R_C \cdot \frac{\hat{B}^2_{\sim 0} / \mu}{6 \cdot \overline{p}_{c0}} = R_C \cdot \frac{\hat{B}^2_{\sim 0} / \boldsymbol{\mathcal{L}} \cdot (l / A)}{6 \cdot \overline{p}_{c0}} = R_C \cdot \frac{\hat{B}^2_{\sim 0} \cdot A}{6 \cdot \overline{p}_{c0} \cdot \boldsymbol{\mathcal{L}} \cdot l} \qquad (3)$$

As an example, the Micrometals iron-powder 26 material has μ_r = 75 and a power-loss density of

$$\overline{p}_c = \begin{cases} 400\,\mathrm{mW/cm^3}, 30\,\mathrm{mT} \\ 45\,\mathrm{mW/cm^3}, 10\,\mathrm{mT} \end{cases}, \quad f_s = 100\,\mathrm{kHz}$$

Substituting into the core-loss equation,

$$\overline{p}_c = \frac{\overline{P}_c}{V} = \overline{p}_{c0} \cdot \left(\frac{\hat{B}_\sim}{\hat{B}_{\sim 0}} \right)^2$$

from the first line of $\overline{p}_c$ data, the result is

$$\overline{p}_c = \left(400 \, \frac{\text{mW}}{\text{cm}^3} \right) \cdot \left(\frac{\hat{B}_\sim}{30 \, \text{mT}} \right)^2$$

This is an equation for a log-log plot of the general form

$$\frac{y}{y_0} = \left(\frac{x}{x_0} \right)^m$$

with log(y/y_0) plotted against log(x/x_0) as a line with slope, m. Applied to the magnetic loss data,

$$m = \frac{\log \overline{p}_{c2} - \log \overline{p}_{c1}}{\log \hat{B}_2 - \log \hat{B}_1} = \frac{\log(\overline{p}_{c2} / \log \overline{p}_{c1})}{\log(\hat{B}_2 / \log \hat{B}_1)} = \frac{\log(400/45)}{\log(30/10)} = 1.989 \approx 2$$

These two data points of the loss function show that the "classic" B exponent of 2 from the Steinmetz equation,

$$\overline{P}_c = \left[\overline{p}_{c0} \cdot \left(\frac{\hat{B}_\sim}{\hat{B}_{\sim 0}} \right)^2 \right] \cdot V$$

is a close approximation to the powdered-iron material manufacturer data. If 30 mT is chosen as the magnetic operating-point, then $\mu = \mu_r \cdot \mu_0$ = (75)·(1.26 µH/m) = 94.5 µH/m, and

$$L_{opt} = R_C \cdot \frac{\hat{B}_{\sim 0}^2 / \mu}{6 \cdot \overline{p}_{c0}} = R_C \cdot \frac{(30\,\text{mT})^2 / (94.5\,\mu\text{H/m})}{6 \cdot (400\,\text{mW/cm}^3)} = R_C \cdot (3.97\ \mu\text{s})$$

With a choice of capacitor and R_C, L_{opt} is calculated from the formula. As an example, for R_C = 0.25 Ω, L_{opt} = 1 µH.

This exercise in component size minimization is an example of an optimization not usually considered. Inductor design involves multiple criteria leading to multiple kinds of optimizations. The optimization given here places no further constraints on the magnetics design, especially if the output capacitor is chosen from R_C instead so that inductor design remains unconstrained. The minimization equation, solved for either L or R_C, results in two design formulas:

$$L = R_C \cdot \frac{\hat{B}_{\sim 0}^2}{6 \cdot \mu \cdot \overline{p}_{c0}} = R_C \cdot \frac{\hat{B}_{\sim 0}^2 \cdot A}{6 \cdot \overline{p}_{c0} \cdot \boldsymbol{\mathcal{L}} \cdot l} \qquad (4)$$

$$R_C = L \cdot \frac{6 \cdot \mu \cdot \overline{p}_{c0}}{\hat{B}_{\sim 0}^2} = L \cdot \frac{6 \cdot \overline{p}_{c0} \cdot \boldsymbol{\mathcal{L}} \cdot l}{\hat{B}_{\sim 0}^2 \cdot A} \qquad (5)$$

The first expression in both formulas is core-size independent. The second expression assumes that the core has already been chosen from magnetics design.

Constraint of R_C might conflict with control-loop response optimization because R_C affects the placement of an important zero in the *s*-domain response of the voltage-control loop. However, it can guide dynamics design toward a value of R_C that will also reduce converter size.

Closure

The derived design formulas for minimizing converter inductor and output capacitor size are based on the assumption that the thermal resistance of inductor and capacitor are comparable, that inductor current ripple equals capacitor current, and that overall size is minimized when power loss in equivalent resistances of inductor and capacitor are the same. These assumptions should be verified for components of any particular design.

Hopefully, in future capacitor catalogs such component parameters as series resistance (ESR) as a function of frequency and current, and thermal resistance will be given. (Ideally, the thermal shape factor, Ξ_θ will be given by both capacitor and magnetic-core manufacturers in their parts data.) Consequently, this optimization is somewhere between an approximate quantitative method and a rationale for calculating minimum-size inductance and capacitance. Hopefully, the logic of it can guide minimization of component sizes. Another rationale on the same general topic is given in reference [3].

References

1. It is widely believed that maximum power is transferred through a transformer or an inductor when the core and winding losses are equal, but this is only approximately true and is exact only for no power loss. The details are worked out in my book, *Power Magnetics Design Optimization (PMDO)* at www.innovatia.com for which PDF copies are free upon request through this website or by emailing me at dennisf at innovatia dot com. For high-efficiency (low core loss) transductors, equal losses are approximately optimal.

2. "How To Thermally Model Magnetic Components" by Dennis Feucht, How2Power Today, March 2019. As explained in this article, the allowable power-loss density of a core is the thermal shape factor, Ξ_θ multiplied by the power-loss density of the worst-case thermal shape, the sphere. Ξ_θ is thus the thermal improvement over a sphere of a given core shape, independent of its size.

3. "The Optimum Value of *r*" in the book *Switching Power Supplies A to Z* by Maniktala, Sanjaya, Newnes-Elsevier, 2006, pages 77 - 79. In this section, Maniktala's "ripple ratio", $r = 2 \cdot \gamma$ where γ is the average ripple factor, $\gamma = (\Delta x/2)/\bar{x} = \hat{x}_{\sim}/\bar{x}$, the ripple amplitude over the average. At the CCM-DCM boundary it is 1. ($\gamma \leq 1$ is CCM; $\gamma > 1$ is DCM.) Maniktala recognizes the importance of γ in converter design and argues for an optimum value of $\gamma = 0.2$. Compare this to the author's derivation in the section, "Maximum Power with Large Ripple" in *PMDO*, pages 247 - 251. (See reference [1] above for weblink.)

About The Author

Dennis Feucht has been involved in power electronics for 35 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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