Understanding Core Magnetization In Current Transformers—Avoiding Saturation And Dangerous Output Voltage

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Current transformers are used as ac current sensors and power handling devices in current-fed power supplies. Very often current transformers operate in circuits where the primary current has a dc component. This dc magnetization adds more magnetic flux to the core, leaving less headroom for the ac magnetizing, which is often the only point of interest (i.e. it reflects what we're trying to measure, the ac current). Therefore the current transformer design or selection should be based on the maximum or peak operating flux density of the core.

It is important to also keep in mind that the input waveform for a current transformer is a rectangular-shaped current flowing through what's usually a one-turn primary winding. These conditions makes the analysis of a current transformer different from the analysis of a voltage transformer.

Since the primary winding of a voltage transformer connects to a voltage source having very low output impedance, the primary winding has multiple turns, and thus the transformer is immune to open load and is sensitive to shorted load.

On the other hand, the current transformer is supplied from a very high impedance current source and has just one turn (usually) on the primary winding. This makes the primary turn(s) a magnetizing winding. Meanwhile, the output is a parallel connection of the secondary winding and load, which makes the current transformer not invariant to the load. This creates very high voltage at no load and zero secondary volts with a load short.

Therefore, with a current transformer, the shorted load is a very light and safe mode of current transformer operation, while no-load operation may pose a danger to an operator or destroy equipment if the load resistor does not have a low enough value. So it's important to foresee if there might be a situation where the load disconnects occasionally.

In this article, we derive the equation for the magnetic flux density of the primary winding of a current transformer, and from this equation derive an expression for the maximum value of primary current that the transformer can support without going into saturation.

We next derive an equation for the secondary voltage that includes magnetic flux density. We can use this equation in evaluating whether the dc component applied to the primary is excessively contributing to the magnetic flux density of the core, and not leaving the transformer enough headroom before core saturation.

Then, in the next section, we derive an expression for the rectangular-shaped primary current waveform that leads us to an expression for the secondary voltage in a current transformer under no-load. As discussed above, this value is needed to ensure the safe design and application of the transformer since the output voltage is a derivative of the input current.

Finally, in the last section, we present a current transformer example that illustrates use of the equations for primary current and no-load secondary voltage. We also use the volt-second integral to determine magnetic flux density for this example, which then verifies the expression previously derived.

Magnetic Flux Density In The Current Transformer's Primary

To begin, let's define the circuit and magnetic field parameters that we'll need in this discussion:

$i_p(t)$: current flowing through the primary winding.

N_p: number of turns on the primary winding

H_m: magnetic field strength in the core

l_m: length of the median magnetic line in the core

B_m: magnetic flux density

I_p: amplitude of the input pulse or dc component of the input current pulse

$i_{pac}(t)$: input pulse as a function of time.

$v_s(t)$: secondary voltage as a function of time

A_e: cross sectional area of a magnetic core.

Next, we start our analysis by observing that current flowing in the primary winding creates a magnetic field in the core:

$$H_m(t) = i_p(t)\cdot\frac{N_p}{l_m} \tag{1}$$

This magnetic field, in its turn, creates a magnetic flux in the core whose density, $B_m(t)$, can be characterized in terms of the transformer's parameters, by applying the expression for magnetic field strength given in equation (1):

$$B_m(t) = \mu_0\cdot\mu_r\cdot H_m(t) = \mu_0\cdot\mu_r\cdot\left(i_p(t)\cdot\frac{N_p}{l_m}\right) \tag{2}$$

From equation (2), one can see that the flux density shape follows the primary current shape. If the primary current has a dc component I_p and an ac component $i_{pac}(t)$, the resulting magnetization of the core will be:

$$i_p(t) = I_p + i_{pac}(t) \tag{3}$$

and equation (2) turns into:

$$B_m(t) = \mu_0\cdot\mu_r\cdot\left[\left(I_p + i_{pac}(t)\right)\cdot\frac{N_p}{l_m}\right] \tag{4}$$

Current Limitation

Expression (4) allows for defining the maximum composite primary current value $I_{p\_tot\_max} = I_p + i_{pac\_max}(t)$ that should not saturate the current transformer core:

$$I_{p\_tot\_max} = \frac{B_{m\_sat}\cdot l_m}{N_p\cdot\mu_0\cdot\mu_r} \tag{5}$$

Be cautious—you can have an RMS value for $i_{pac}(t)$, but the amplitude value is higher. Core saturation is caused by the maximum value of primary current.

No-Load Mode

When the ac magnetic flux linkage $\Psi(t)$, created by the primary current, crosses the secondary winding, it generates the secondary voltage across the secondary winding:

$$v_s(t) = \frac{d}{dt}\Psi(t) = \frac{d}{dt}B_m(t)\cdot N_s\cdot A_e \qquad (6)$$

Plug in equation (4) for B_m and get

$$v_s(t) = N_s\cdot A_e\cdot\mu_0\cdot\mu_r\cdot\frac{N_p}{l_m}\cdot\left(\frac{d}{dt}i_{pac}(t)\right) \qquad (7)$$

From equation (7) we see that the expression for the secondary voltage has no dc component, but this does not mean that this component does not exist on the primary side and cannot saturate the current transformer's core. There might be situations when the primary current dc component is so large that it does not leave headroom for the ac component, and such a current transformer becomes inoperable.

Rectangular Shape Of Primary Current

A rectangular-shaped current can be described by means of exponential functions. Assume the pulse amplitude is I_{pac} and the value for τ_{0x} is a few orders of magnitude lower than the pulse duration t_d. We can then describe the front end (the rising edge of the current waveform) as:

$$i_{pacf}(t) = I_{pac}\cdot\frac{1}{1+e^{\frac{-t}{\tau_{0x}}}} \qquad (8)$$

We can obtain the full pulse equation as follows:

$$i_{pac}(t) = I_{pac}\cdot\left(\frac{1}{1+e^{\frac{-t}{\tau_{0x}}}} - \frac{1}{1+e^{\frac{-t+t_d}{\tau_{0x}}}}\right) \qquad (9)$$

It is easy to see that at t = 0, $i_{pac}(t) > 0$

$$i_{pac}(0) = I_{pac}\cdot\left(0.5 - \frac{1}{1+e^{\frac{t_d}{\tau_{0x}}}}\right) > 0 \qquad (10)$$

In order to mitigate this drawback, let's shift the pulse to the right and agree on what pulse amplitude can be considered as 0. This is necessary because this exponential function does not have zero value.

When shifted by θ, the primary current pulse is described by the following expression:

$$i_{pac}(t) = I_{pac}\cdot\left(\frac{1}{1+e^{\frac{-t}{\tau_{0x}}+\theta}} - \frac{1}{1+e^{\frac{-t+t_d}{\tau_{0x}}+\theta}}\right) \tag{11}$$

Equate the front end at t = 0 described by equation (8), to the pulse amplitude I_{pac} divided by some big number, σ:

$$I_{pac}\cdot\frac{1}{1+e^{\frac{0}{\tau_{0x}}+\theta}} = \frac{I_{pac}}{\sigma} \tag{12}$$

Solving (12) for θ, we get

$$\theta = \ln(\sigma - 1) \tag{13}$$

Using

$$\frac{d}{dt}\left[I_{pac}\cdot\left(\frac{1}{1+e^{\frac{-t}{\tau_{0x}}+\theta}} - \frac{1}{1+e^{\frac{-t+t_d}{\tau_{0x}}+\theta}}\right)\right] \rightarrow I_{pac}\cdot\left[\frac{e^{\theta-\frac{t}{\tau_{0x}}}}{\tau_{0x}\cdot\left(e^{\theta-\frac{t}{\tau_{0x}}}+1\right)^2} - \frac{e^{\theta-\frac{t-t_d}{\tau_{0x}}}}{\tau_{0x}\cdot\left(e^{\theta-\frac{t-t_d}{\tau_{0x}}}+1\right)^2}\right]$$

and equation (7), we obtain an expression for the secondary voltage at no load:

$$V_s(t) = N_{s1}\cdot A_e\cdot\mu_0\cdot\mu_r\cdot\frac{N_p}{l_m}\cdot\left[I_{pac}\cdot\left[\frac{e^{\theta-\frac{t}{\tau_{0x}}}}{\tau_{0x}\cdot\left(e^{\theta-\frac{t}{\tau_{0x}}}+1\right)^2} - \frac{e^{\theta-\frac{t-t_d}{\tau_{0x}}}}{\tau_{0x}\cdot\left(e^{\theta-\frac{t-t_d}{\tau_{0x}}}+1\right)^2}\right]\right] \tag{14}$$

Verification Of Results—Checking By Volt-Second Integral

Assume the following values for the current transformer.

N_s = 50

A_e = 64 mm^2

μ_r = 1300

$N_p = 1$

$l_m = 220$ mm

$I_{pac} = 2$ A

$\tau_{0x} = 0.15$ ns

$\sigma = 2 \times 10^2$

$t_d = 0.022$ μs

$\tau_f = t_d/2$

$\theta = \ln(\sigma - 1)$

Then, using the following expressions for primary current and no-load secondary voltage obtained above, we can plot these parameters for our current transformer example as shown in the figure.

$$i_{pac}(t) := I_{pac} \cdot \left(\frac{1}{1 + e^{\frac{-t}{\tau_{0x}} + \theta}} - \frac{1}{1 + e^{\frac{-t+t_d}{\tau_{0x}} + \theta}} \right)$$

$$V_s(t) := N_s \cdot A_e \cdot \mu_0 \cdot \mu_r \cdot \frac{N_p}{l_m} \cdot \left[I_{pac} \cdot \left[\frac{e^{\theta - \frac{t}{\tau_{0x}}}}{\tau_{0x} \cdot \left(e^{\theta - \frac{t}{\tau_{0x}}} + 1 \right)^2} - \frac{e^{\theta - \frac{t-t_d}{\tau_{0x}}}}{\tau_{0x} \cdot \left(e^{\theta - \frac{t-t_d}{\tau_{0x}}} + 1 \right)^2} \right] \right]$$

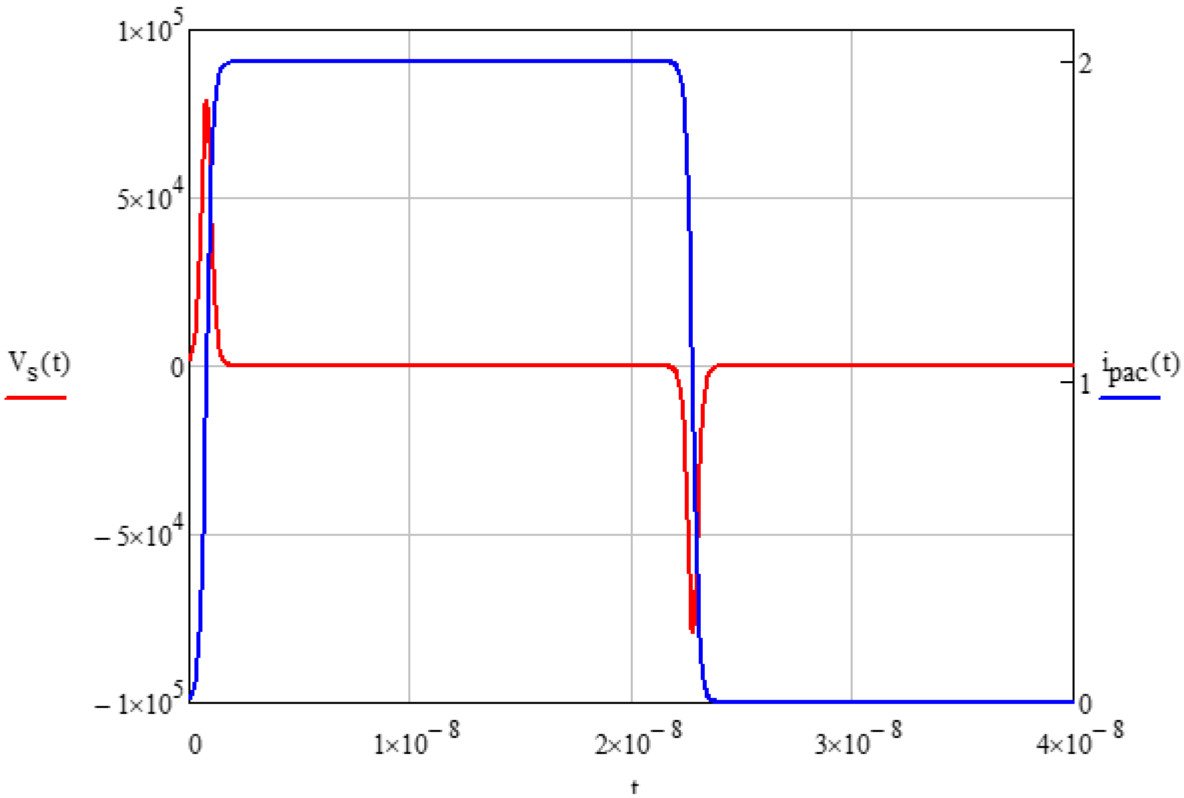

Figure. Primary current and secondary voltage under no load plotted for the example current transformer.

We see in the figure that the secondary voltage can go up to very high values (approximately 75 kV in this example), which may only be limited by the core saturation or internal or external voltage breakdown. Let's check if the primary current can saturate the current transformer core, and if it is reflected by the volt-second integral:

Secondary winding voltage can be expressed this way:

$$V_s = \frac{d}{dt}\Psi = \frac{d}{dt}\left(N_s \cdot A_e \cdot B\right)$$

or

$$V_s = N_s \cdot A_e \cdot \left(\frac{d}{dt}B\right) \qquad (15)$$

Hence

$$V_s \cdot dt = N_s \cdot A_e \cdot dB$$

Integrating both parts of this expression, we get:

$$\int_{0}^{\tau_f} V_s \, dt = N_s \cdot A_e \cdot \int_{0}^{B} 1 \, dB$$

which means that

$$B = \frac{\int_{0}^{\tau_f} V_s \, dt}{N_s \cdot A_e} \qquad (16)$$

This means that a volt-second integral reflects the core magnetization and characterizes the core status, created by magnetizing current.

Plugging in equation (14) for V.s and considering half of the pulse for integrating since the whole pulse causes both magnetizing and de-magnetizing processes, we get:

$$B := \frac{\displaystyle\int_{0}^{\tau_f} N_s \cdot A_e \cdot \mu_0 \cdot \mu_r \cdot \frac{N_p}{l_m} \cdot \left[I_{pac} \cdot \left[\frac{e^{\theta - \frac{t}{\tau_{0x}}}}{\tau_{0x} \cdot \left(e^{\theta - \frac{t}{\tau_{0x}}} + 1 \right)^2} - \frac{e^{\theta - \frac{t - t_d}{\tau_{0x}}}}{\tau_{0x} \cdot \left(e^{\theta - \frac{t - t_d}{\tau_{0x}}} + 1 \right)^2} \right] \right] dt}{N_s \cdot A_e} = 0.015\,T \qquad (17)$$

Checking Core Saturation By Primary Current

Using equation (4) and assuming a dc current component equal to 0, I_p = 0 A, we calculate

$$B := \frac{\left(I_p + I_{pac} \right) \cdot N_p}{l_m} \cdot \mu_0 \cdot \mu_r = 0.015\,T \qquad (18)$$

In this example, we see that the unloaded current transformer does not saturate during idle operation because the primary current is sufficiently low. But we also observe that the secondary winding can generate a very high voltage when the primary current changes abruptly.

About The Author

Gregory Mirsky, is a senior electrical engineer with Vitesco Technologies, a spinoff of Continental Automotive Systems, in Deer Park, Ill., which he joined in March 2015. In his current role, Gregory performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification.

He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature *superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.*

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

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