Interstitial Wire Interleaving Packs More Conductor Into Magnetic Windings

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One of the limitations on winding efficiency is the *packing factor*, k_p, the fraction of winding area that is conductor. Ideally the conductive part of a winding—the copper or aluminum part of the wire turns—would completely fill the area allotted to a particular winding, and the current and power density would then be maximum. However, gaps between round wires result in areas not conducting current.

This article proposes a way to reduce gaps and thus increase the winding *fill factor*, k_f by filling these gaps with smaller wires. After reviewing the various packing factor components that quantify winding density, we'll analyze how to determine the optimum wire size for the smaller wire in square-layered and hexagonal-layered winding configurations. We'll also determine the improvement in fill factor obtained in each case, and assess the relative benefit of adding the extra wire in one winding configuration versus the other.

Limits On Winding Density

To keep wire turns from shorting, wires or foil must be insulated, and this reduces k_p from 1 to a value of $k_p < 1$ resulting from wire spacing caused by insulation. If the turns of insulated wire are not adjacent to each other, then the additional spacing also contributes to this reduction in k_p called the *porosity* packing factor, k_{pw}.

Contributing to packing factor are the geometric gaps between wires caused by their shapes when packing together. They reduce k_p by the *fill factor*, k_f. Then the winding $k_p = k_{pw} \cdot k_{pf}$. Additionally, if the wire is bundled, the bundle has a fill factor, the *bundle packing factor*, in which the porosity and bundle fill factor are lumped together to give the total bundle packing factor as $k_{pw} \cdot k_{pb}$ where k_{pb} is the bundle fill factor. Then the round or oval bundles pack together with spaces in the *interstices* between the bundles where there are gaps, and this is the fill factor, k_f of the turns of bundles. Overall,

$$k_p = (k_{pw} \cdot k_{pb}) \cdot k_{pf}$$

There is a fourth factor that affects bundle packing factor, the *twist factor*, k_{tw}. As a bundle of wire is twisted tighter—that is, its length of a twist cycle, or *pitch* is reduced—the bundle expands slightly from twisting and

$$k_p = (k_{pw} \cdot k_{pb} \cdot k_{tw}) \cdot k_{pf}$$

While twisted bundles can greatly reduce the eddy-current effect—and in particular, the proximity effect of increased resistance caused by the magnetic-field influence of adjacent turns or layers of wire—they can also reduce k_p substantially, forcing the magnetics designer to choose a larger core with a larger winding window. (For more on all of the packing factor terms, see the references.)

In the quest for higher power density in magnetics design, we are driven to imagine how we might overcome these basic limitations on packing factor. The most common one—and a very good one—is to use foil instead of wire. Foil has advantages over wire in reducing eddy-current resistance. It is stretched as a single continuous conductor over a wide distance in each layer which increases k_p substantially.

However, at typical switching frequencies, foil must be layered as a stack of thin conductors to minimize eddy currents, with insulation between conductors. This increases conductor separation and decreases k_{pw}. Additionally, to connect to foil, a wire is usually soldered across its width at each end and the wire diameter wastes winding window height.

The author has been studying how foil might be twisted, like wire bundles, to reduce the proximity effect of multiple layers, but there is generally no established method for twisting foil layers. Consequently, tradeoffs between wire and foil conductors leave the designer to contemplate this decision.

Square-Layered Windings

One possible way to alleviate the packing factor problem with wire is to fill in the gaps between adjacent wires in a layered winding with smaller-sized wire, as shown in Fig. 1.

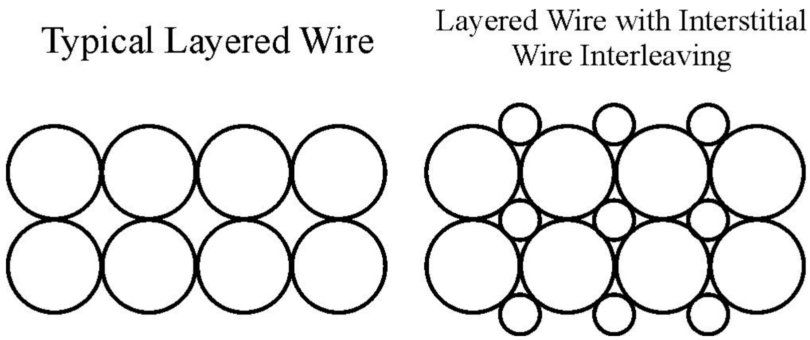

Fig. 1. Comparison of ordinary layered wire in a winding (left) and the same winding with additional, interstitial wire (right) that increases the winding packing factor.

The winding configuration in Fig. 1 is square in that layers align rather than fit into the grooves of adjacent layers and form a hexagonal configuration (which will be discussed shortly). The square configuration is hard to maintain while winding wire, but it can be constructed as structured bundling by wire manufacturers. Additionally, the smaller wires can be inserted in the gaps between wires, as shown on the right of Fig. 1. The relative sizes of the two wires can be derived as follows, following the geometric construction of the square shown in Fig. 2.

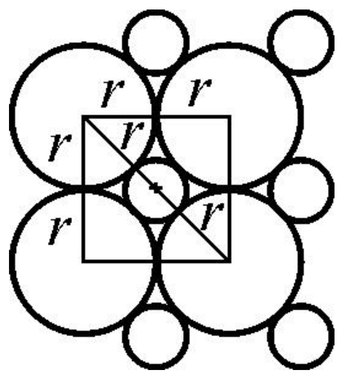

Fig. 2. Closer view of winding with square constructed on the adjacent wire centers of wire of radius, r.

The squares, when replicated for adjacent sets of four wires, pack together with a packing factor of one—perfect packing. Each square has an area of

$$A_{sq} = (2 \cdot r)^2 = 4 \cdot r^2$$

Every large wire has an area within the square of a quarter of a circle, and there are four of them in the square, or

$$A_{cw} = 4 \cdot \left(\frac{\pi}{4} \cdot r^2 \right) = \pi \cdot r^2$$

Then the packing factor of the large wire alone is

$$k_p = \frac{A_{cw}}{A_{sq}} = \frac{\pi \cdot r^2}{4 \cdot r^2} = \frac{\pi}{4} \approx 0.785$$

The diameter of the interstitial wire is the square diagonal minus 2 large-wire radii. Then the small-wire radius is

$$r_{iw} = \tfrac{1}{2} \cdot (\sqrt{2} \cdot (2 \cdot r_{cw}) - 2 \cdot r_{cw}) = (\sqrt{2} - 1) \cdot r_{cw} \approx (0.414) \cdot r_{cw}$$

The diagonal of a square of side, $2 \cdot r_{cw}$ is $\sqrt{2} \cdot (2 \cdot r_{cw})$. Subtracting from this length the radii of the diagonally-opposed circular sections within the square, $2 \cdot r_{cw}$, the diameter of the interstitial wire results, and when divided by two gives the insulated wire radius, r_{iw}.

The area of the smaller wire is

$$A_{iw} = \pi \cdot [(\sqrt{2} - 1) \cdot r]^2 = (\sqrt{2} - 1)^2 \cdot (\pi \cdot r^2) = (3 - 2 \cdot \sqrt{2}) \cdot (\pi \cdot r^2)$$

This area adds to the area of the larger wires to result in an improved fill factor of

$$k_f = \frac{\pi \cdot r^2 + (3 - 2 \cdot \sqrt{2}) \cdot (\pi \cdot r^2)}{4 \cdot r^2} = \pi \cdot \left(1 - \frac{\sqrt{2}}{2}\right) \approx 0.9202$$

This enhanced k_f is 0.9202/0.785 = 1.172, or 17.2% greater than without the interstitial wire. This k_f does not include the porosity caused by the wire insulation, and the more complete $k_p = k_{pw} \cdot k_f$. To be more exacting, if the porosity values of the two wire sizes differ significantly, each can be applied.

The large wire contributes k_{pw} because a full circumference of it is within the square. The smaller wire is wholly within the square and its k_{pw} applies to the whole wire. The differences in wire size also affect how much of the area the insulation of each occupies. Some additional geometry results in a more refined k_p. However, insulation is thin on most wires, and $k_p \approx k_f$ will be only slightly high in value.

The relative size of the two wires differ in AWG, a logarithmic scale, by 0.414 or

$$\boxed{\Delta 7.66 \text{ AWG} \approx \Delta 8 \text{ AWG}}$$

A winding of #20 AWG wire could therefore have within its gaps #28AWG wire.

Hexagonal-Layered Windings

However, the more likely layering is hexagonal for round wires, and if wound manually with some care to sequence the wires in each layer (or by using a winding machine that does this) the question then arises as to what the interstitial wire size is for the hex-layer configuration. The geometric construction for deriving it is shown in Fig. 3 for two adjacent wires of the upper layer and one wire of the lower layer fitting into the groove between the upper wires.

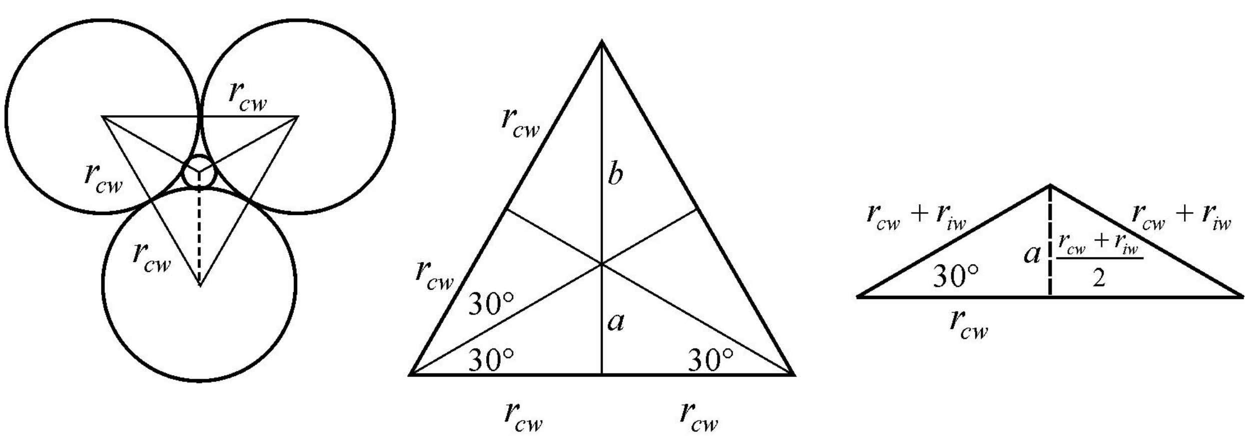

Fig. 3. Interstitial wire between layers of round wire of the hexagonal layer configuration (left); the triangle, inverted and bisected (center); and the lower triangle detail (right).

As the enlargement of the winding fragment to the left in Fig. 3 illustrates, the interstitial space is much smaller and the advantage of adding interstitial wire is less. The triangle with vertices at the circle centers, expanded and inverted in Fig. 3 center, has bisecting segments of length,

$$c = a + b = (2 \cdot r_{cw}) \cdot \sin(60°) = (2 \cdot r_{cw}) \cdot \frac{\sqrt{3}}{2} = \sqrt{3} \cdot r_{cw}\ ,\ b = r_{cw} + r_{iw}$$

Formed from bisections is the lower triangle in the center of Fig. 3, expanded on the right. From it,

$$a = r_{cw} \cdot \tan(30°) = \frac{r_{cw}}{\sqrt{3}} = \frac{\sqrt{3}}{3} \cdot r_{cw}$$

Solving for r_{iw},

$$b = r_{cw} + r_{iw} = c - a = \sqrt{3} \cdot r_{cw} - \frac{\sqrt{3}}{3} \cdot r_{cw} = \frac{2}{3} \cdot \sqrt{3} \cdot r_{cw} \Rightarrow$$

$$r_{iw} = (\tfrac{2}{3} \cdot \sqrt{3} - 1) \cdot r_{cw} \approx (0.1547) \cdot r_{cw}$$

From the Fig. 3 lower-triangle detail on the right,

$$a = (r_{cw} + r_{iw}) \cdot \sin(30°) = (r_{cw} + r_{iw}) \cdot \tfrac{1}{2}$$

$$a = \tfrac{1}{2} \cdot (r_{cw} + r_{iw}) = \frac{\sqrt{3}}{3} \cdot r_{cw} \Rightarrow r_{iw} = 2 \cdot \left(\frac{\sqrt{3}}{3} - \frac{1}{2} \right) \cdot r_{cw} = (\tfrac{2}{3} \cdot \sqrt{3} - 1) \cdot r_{cw} \approx (0.1547) \cdot r_{cw}$$

The alternative derivation agrees with the previous r_{iw}. The size difference, in radii, of the two wires is greater than for the square-configured layering. In the case of the hexagonal-layered winding configuration, the relative size of the two wires is about

Δ16.2 ≈ Δ17 AWG

The area of the triangle is

$$A_{tr} = \tfrac{1}{2} \cdot (2 \cdot r_{cw}) \cdot (\sqrt{3} \cdot r_{cw}) = \sqrt{3} \cdot r_{cw}^2$$

Acute triangles as part of the winding hexagonal pattern, pack like squares with a packing factor of one. The three sectors of circles within the triangle are a sixth the area of a circle. Three of them have the area of half a circle, or $\tfrac{1}{2} \cdot \pi \cdot r_{cw}^2$. The area of the interstitial wire is

$$A_{iw} = \pi \cdot [(\tfrac{2}{3} \cdot \sqrt{3} - 1) \cdot r_{cw}]^2$$

Then the fill factor of the hexagonally-layered winding is

$$k_f = \frac{\tfrac{1}{2} \cdot \pi \cdot r_{cw}^2 + A_{iw}}{A_{tr}} = \frac{(\pi \cdot r_{cw}^2) \cdot [\tfrac{1}{2} + (\tfrac{2}{3} \cdot \sqrt{3} - 1)^2]}{\sqrt{3} \cdot r_{cw}^2} = \frac{\pi}{\sqrt{3}} \cdot \left(\frac{1}{2} + \frac{7}{3} - \frac{4}{3} \cdot \sqrt{3} \right) = \frac{\pi}{3 \cdot \sqrt{3}} \cdot \left(\frac{17}{2} - 4 \cdot \sqrt{3} \right) \approx 0.9503$$

With the interstitial strand of wire added, k_f is increased by 0.9503/0.907 ≈ 1.048 or 4.8%. This increase is only about 4.8%/17.2% ≈ 28% of the increase in k_f of that for the square-configured layering. With the smaller wire added to the hex-configured layering, k_f is only increased by 0.9503/0.9202 ≈ 1.033, or 3.27% higher with the interstitial wire over that of the square configuration with interstitial wire. This is not much of an improvement for the hex configuration but clearly the improvement obtained by adding the interstitial wire to the square configuration is significant.

Conclusions

Configuring windings with two sizes of wire, the smaller fitting into the interstices of the larger to increase packing factor, is an intriguing if not also an appealing idea. The two sizes of wire are bifilar wound and have a high coupling coefficient. In forward or boost push-pull (especially Weinberg) converters, the smaller wire can be used as a deflux winding, to return leakage energy to the supply or output. The additional winding area that otherwise would be wasted is put to use as a deflux winding, as though "free area" were supplied by the primary winding.

A challenge is in how to construct windings with both sizes of wire. If multiple windings are configured as an *N*-filar bundle twisted together to form a multifilar winding, both (or multiple) sizes of wire can be twisted together. Although this does not guarantee that the smaller wires will fill the voids between the larger wires, it can reduce bundle packing factor, k_{pb} and thereby accomplish the same purpose.

References

The following works by the author discuss packing factor and related concepts:

1. "How To Calculate Winding Packing Factor," How2Power Today, November 2016 issue.
2. "A New Method Of Winding Design Optimization (Part 1): Window Geometry And Eddy Current Plots," How2Power Today, September 2017 issue.
3. "The Geometry Of Twisted Wire Bundles," How2Power Today, July 2018 issue.
4. "Interbundle Penetration Of Wire Bundles Improves Their Packing Factor," How2Power Today, November 2018 issue.

About The Author

Dennis Feucht has been involved in power electronics for 35 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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