Determining Design Power Over An Input Voltage Range (Part 3): Maximum Transformer Power

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Magnetic components operated as transformers, like inductors, have maximum power ratings. But as was the case with inductors, the maximum power handled by the transformer is not optimal for sizing the transformer. The same analysis which we applied to inductors in parts 1 and 2[1,2] can be extended to transformers for the three configurations of PWM-switch converters as we'll show here in this third and final installment in the series.

Once again, we'll derive expressions for design power relative to maximum average input power for each PWM-switch configuration. The main differences in the analysis with the transformer as the magnetic component relate to the differences in the waveforms seen by transformers versus those seen by inductors. This applies both to the shape of the waveforms and the influence of the transformer turns ratio.

Transformer Waveforms And Power

Transformers normally have no static current in the windings and the net per-cycle flux is zero to keep the average current zero. The voltage and current waveforms applied to transformer windings are typically symmetrical and bipolar: $\pm x(t)$. Unlike the PWM-switch inductor, which maintains a near-constant (unipolar) current, transformer voltage and current waveforms are approximately square-waves that vary in time (or D).

The transformer input (primary) power, $\overline{P}_p$, is given as constant over the input voltage and current range at

$$\overline{P}_p = \tilde{v}_p \cdot \tilde{i}_p$$

where ~ over the symbol of a voltage or current indicates its RMS value. The RMS value of a square-wave, $x(t)$, of duty-ratio, D, and peak value or amplitude, $\hat{x}$, is

$$\tilde{x} = \sqrt{D} \cdot \hat{x}$$

For CP (buck) and CL (buck-boost) converters, the peak transformer voltage is the on-time amplitude of the input voltage, V_g. The RMS voltage on the primary is

$$\tilde{v}_p = \sqrt{D} \cdot \hat{v}_p = \sqrt{D} \cdot V_g$$

The CP PWM-switch is in the secondary circuit, it is in the primary circuit for the CA, and for the CL spans both.

For the CA (boost) configuration, the inductor is in the primary circuit so that the voltage seen at the transformer primary is not V_g, but rather V_s', which is greater than V_g due to inductor flux balance. Therefore,

$$\tilde{v}_p = \sqrt{D'} \cdot V_s'$$

where V_s' is the secondary voltage referred to the primary; $V_s' = n \cdot V_s$, where ′ indicates referral to the other winding.

Applying the small-ripple approximation (large transformer magnetizing inductance), the current waveform is also nearly a square-wave of duty-ratio, D or D'. Transfer current is dominant and real, and magnetizing current is negligible. The voltage and current waveforms are nearly bipolar square-waves, as are typical of switched-converter transformers. Then the current waveform is of the same form as the voltage,

$$\tilde{i}_p = \sqrt{D} \cdot \hat{i}_p = \sqrt{D} \cdot I_g \text{, CP, CL} \;\; ; \;\; \tilde{i}_p = \sqrt{D'} \cdot \hat{i}_s' = \sqrt{D'} \cdot I_s' \text{, CA}$$

Primary Winding And Input Power

The average primary winding power is

$$\overline{P}_p = \tilde{v}_p \cdot \tilde{i}_p = (\sqrt{D} \cdot V_g) \cdot (\sqrt{D} \cdot I_g) = D \cdot (V_g \cdot I_g) = D \cdot P_g \text{, CP, CL}$$

$$\begin{aligned} \overline{P}_p &= \tilde{v}_p \cdot \tilde{i}_p = (\sqrt{D'} \cdot V_s') \cdot (\sqrt{D'} \cdot I_s') = D' \cdot (V_s' \cdot I_s') = D' \cdot P_p \\ &= (D' \cdot V_s') \cdot I_s' = V_g \cdot I_g = P_g \end{aligned} \text{, CA}$$

where P_g is the on-time amplitude of the input power.

For a high-efficiency (η) transformer, $P_s \approx P_p$, and for a lossless input circuit, transformer input power equals the power from the constant-V_g input source;

$$\overline{P}_g = V_g \cdot \bar{i}_g = V_g \cdot (D \cdot I_g) = D \cdot (V_g \cdot I_g) = D \cdot P_g \text{, CP, CL}$$

$$\overline{P}_g = V_g \cdot \bar{i}_g = V_g \cdot I_g = P_g \text{, CA}$$

Therefore, in contrast with the previous equations, $\overline{P}_g = \overline{P}_p$. The CP, CL input-source current waveform is a unipolar square-wave of duty-ratio, D while the primary winding current is bipolar. Each half-cycle has a current (and voltage) duration of D (CP, CL) or D' (CA).

For CA converters, input voltage, current, and power are constant, and for a boost push-pull converter, the primary current waveform is not a simple square-wave but has three levels. The following analysis applies to CCM forward (CP) and flyback (CL) converters and boost (CA) converters with the same transformer square-wave waveforms. Isolated Ćuk converters that have discrete transformers are CL converters that also have square-wave waveforms.

Transformer Design Power

The average power rating for transformer design is based on the RMS voltage and current maximums, $\tilde{v}_{p\max}$ and $\tilde{\imath}_{p\max}$. The transformer must sustain both, resulting in a transformer input *design power* of

$$\overline{P}_{pd} = \tilde{v}_{p\max} \cdot \tilde{i}_{p\max}$$

We found for inductor design power that the maximum $\bar{v}_L$ occurred at $V_{g\max}$ for CP and CL but for CA, it was within the V_g range (at $V_{g\max}/2$). For the transformer-based converter, the maximum $\tilde{v}_p$ similarly is found by either solving for it or else by plotting $\tilde{v}_p(V_g)$ and reading it from the graph of Fig. 1. All $\tilde{v}_p(V_g)$ on the graph monotonically increase and $D(V_g)$ decrease so that $\tilde{v}_{p\max} = \sqrt{D_{\min}} \cdot V_{g\max}$. The current and voltage waveforms are similar though their amplitudes are related inversely as

$$\tilde{i}_p = \frac{\overline{P}_p}{\tilde{v}_p} = \tilde{i}_g = \frac{\overline{P}_g}{\tilde{v}_p}$$

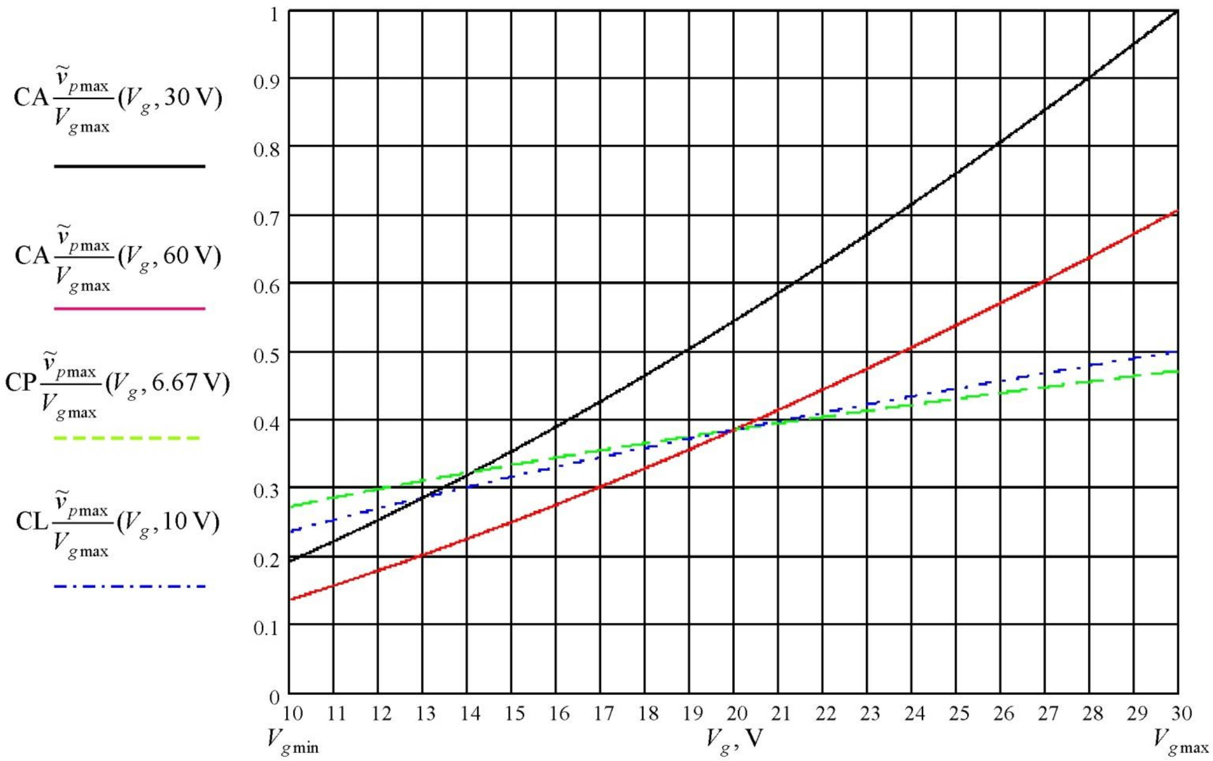

Fig. 1. Plots of $\tilde{v}_{p\max}/V_{g\max}$ for the three PWM-switch configurations over the V_g range. V_o for each is chosen for equal values of $\tilde{v}_{p\max}/V_{g\max}$ at the midrange V_g of V_{gmid} = 20 V.

For the three configurations,

$$\begin{aligned}\overline{P}_{pd} &= \tilde{v}_{p\max}\cdot\tilde{i}_{p\max} = (\sqrt{D_{\min}}\cdot V_{g\max})\cdot(\sqrt{D_{\max}}\cdot I_{g\max}) \\ &= \sqrt{D_{\min}\cdot D_{\max}}\cdot P_{g\max}\end{aligned}\text{, CP, CL}$$

$$\begin{aligned}\overline{P}_{pd} &= \tilde{v}_{p\max}\cdot\tilde{i}_{p\max} = (\sqrt{D'_{\min}}\cdot V_{s\max}')\cdot(\sqrt{D'_{\max}}\cdot I_{s\max}') \\ &= \sqrt{D'_{\min}\cdot D'_{\max}}\cdot P_{p\max} = \sqrt{D'_{\min}\cdot D'_{\max}}\cdot P_{g\max}\end{aligned}\text{, CA}$$

Because $\overline{P}_p = \overline{P}_g$, then so are their design values: $\overline{P}_{pd} = \overline{P}_{gd}$. To express design power in $\overline{P}_g$, recall from part 1 that

$$r = \frac{V_{g\max}}{V_{g\min}} = \frac{\bar{i}_{g\max}}{\bar{i}_{g\min}}\text{, constant } \overline{P}_g$$

and substitute $P_{g\max} = r\cdot P_g$ into what henceforth will be called $\overline{P}_{gd}$;

$$\overline{P}_{gd} = \sqrt{D_{\min}\cdot D_{\max}}\cdot\left(\frac{V_{g\max}}{V_{g\min}}\right)\cdot P_g\text{ , CP, CL}$$

$$\overline{P}_{gd} = \sqrt{D'_{\min} \cdot D'_{\max}} \cdot \left(\frac{V_{g\max}}{V_{g\min}}\right) \cdot P_g = \sqrt{(1-D_{\max})\cdot(1-D_{\min})} \cdot \left(\frac{V_{g\max}}{V_{g\min}}\right) \cdot P_g \text{ , CA}$$

These equations relate the design power, $\overline{P}_{gd}$ to maximum on-time power, P_g. P_g is related to the specified input power, $\overline{P}_g$ through previous equations for $\overline{P}_g$ (or $\overline{P}_p$). For the three configurations, maximum

$$\overline{P}_g = D_{\max} \cdot P_g \text{ , CP, CL ; } \overline{P}_g = P_g \text{ , CA}$$

Substituting for P_g,

$$\overline{P}_{gd} = \sqrt{D_{\min} \cdot D_{\max}} \cdot \left(\frac{V_{g\max}}{V_{g\min}}\right) \cdot P_g = \sqrt{D_{\min} \cdot D_{\max}} \cdot \left(\frac{V_{g\max}}{V_{g\min}}\right) \cdot \frac{\overline{P}_g}{D_{\max}} \text{ , CP, CL}$$

$$\overline{P}_{gd} = \sqrt{(1-D_{\max})\cdot(1-D_{\min})} \cdot \left(\frac{V_{g\max}}{V_{g\min}}\right) \cdot \overline{P}_g \text{ , CA}$$

The design-power ratios are

$$\frac{\overline{P}_{gd}}{\overline{P}_g} = \sqrt{\frac{D_{\min}}{D_{\max}}} \cdot \left(\frac{V_{g\max}}{V_{g\min}}\right) \text{ , CP, CL} \qquad \frac{\overline{P}_{gd}}{\overline{P}_g} = \sqrt{(1-D_{\max})\cdot(1-D_{\min})} \cdot \left(\frac{V_{g\max}}{V_{g\min}}\right) \text{ , CA}$$

$D_{\min}$ and $D_{\max}$ can also be expressed for a given configuration in V_g and V_o. The D formulas based on circuit transfer ratios in parts 1 and 2 for the PWM-switch inductors are valid with the inclusion of the transformer turns ratio, $n = N_p / N_s$. V_o is referred to the primary winding as $V_o' = n \cdot V_o$.

$$D = \frac{V_o'}{V_g} \text{ , CP ; } D = \frac{V_o'}{V_o' + V_g} \text{ , CL ; } D = \frac{V_o' - V_g}{V_o'} \text{ , CA}$$

Substituting for $D_{\min}$ and $D_{\max}$ of the CP configuration,

$$\frac{\overline{P}_{gd}}{\overline{P}_g} = \sqrt{\frac{D_{\min}}{D_{\max}}} \cdot \left(\frac{V_{g\max}}{V_{g\min}}\right) = \sqrt{\frac{V_o'/V_{g\max}}{V_o'/V_{g\min}} \cdot \frac{V_{g\max}^2}{V_{g\min}^2}} = \sqrt{\frac{V_{g\max}}{V_{g\min}}} \text{ , CP}$$

For the CL configuration,

$$\frac{\overline{P}_{gd}}{\overline{P}_g} = \sqrt{\frac{D_{\min}}{D_{\max}}} \cdot \left(\frac{V_{g\max}}{V_{g\min}}\right) = \sqrt{\frac{n \cdot V_o + V_{g\min}}{n \cdot V_o + V_{g\max}}} \cdot \left(\frac{V_{g\max}}{V_{g\min}}\right) \text{ , CL}$$

For the CA configuration, $D' = V_g / n \cdot V_o$. Substituting,

$$\frac{\overline{P}_{gd}}{\overline{P}_g} = \sqrt{\frac{V_{g\max}}{V_{g\min}}} \cdot \left(\frac{V_{g\max}}{n \cdot V_o}\right) \text{ , CA}$$

Plots of $\tilde{v}_{p\max}/V_{g\max}$ and $D(V_g, V_o)$—or D' in the case of CA—for the three PWM-switch configurations are shown in Figs. 1 and 2.

The PWM-switch inductor table in part 2, "Inductor Design Power" contains a column of D expressions for the three configurations which are modified in the transformer table below by the substitution of V_o' for V_o (where V_o' is expressed as $n \cdot V_o$). $\overline{P}_{gd}/\overline{P}_g$ is the multiplier of the maximum input power that gives the design power rating for the transformer. The range, $r \geq 1$.

Table 1. Transformer design power for PWM-switch configurations.

Configuration	D $(n = N_p/N_s)$	$\dfrac{\overline{P}_{gd}}{\overline{P}_g}$
CA	$\dfrac{n \cdot V_o - V_g}{n \cdot V_o}$	$\sqrt{\dfrac{V_{g\max}}{V_{g\min}} \cdot \left(\dfrac{V_{g\max}}{n \cdot V_o}\right)}$
CP	$\dfrac{n \cdot V_o}{V_g}$	$\sqrt{\dfrac{V_{g\max}}{V_{g\min}}}$
CL	$\dfrac{n \cdot V_o}{n \cdot V_o + V_g}$	$\sqrt{\dfrac{n \cdot V_o + V_{g\min}}{n \cdot V_o + V_{g\max}} \cdot \left(\dfrac{V_{g\max}}{V_{g\min}}\right)}$

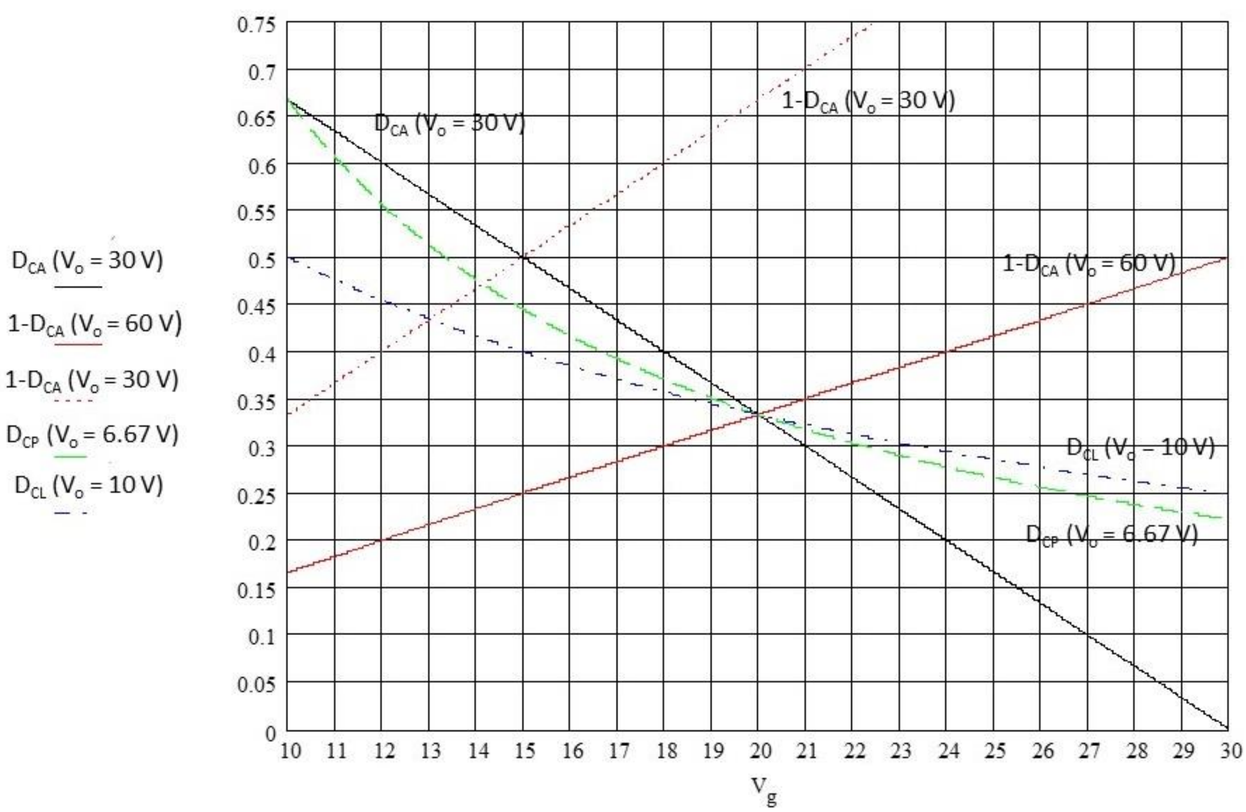

Fig. 2. $D(V_g, V_o)$ for CP and CL and $D'(V_g, V_o)$ for CA over the V_g range, corresponding to the plots in Fig. 1.

Using the same values of the V_g range, [10 V, 30 V] and V_o as for the inductor in parts 1 and 2 with n = 1, $\overline{P}_{gd}\,/\,\overline{P}_g$ calculates for the CA ($n \cdot V_o$ = 60 V) to be 0.866; for the CP, 1.732; and for the CL (at the midrange V_g value of $n \cdot V_o$ = 10 V), 2.121. So, clearly the buck-boost CL configuration is more demanding of transformer design-power as it is for the inductor-based converter. On the other hand, the boost CA transformer shares power-handling with the series inductor. Plots of $D(V_g, V_o)$, D' (for CA) shown in Fig. 2 correspond to those of $\tilde{v}_{p\max}\,/V_{g\max}$ in Fig. 1.

The analysis provided here and in the previous parts quantifies how a wide input-voltage range for a converter brings with it the disadvantage of oversized magnetics components. The high voltage requires more turns to constrain the flux while at low voltage, larger wire is required to carry the current. The result is more turns of larger wire, a larger winding window, and a larger core than for a fixed-V_g power-transfer circuit. This series of articles has presented the design formulas for calculating the power handling requirements of inductors and transformers in the three PWM-switch configurations.

Reference

1. "Determining Design Power Over An Input Voltage Range (Part 1): Maximum Inductor Power" by Dennis Feucht, How2Power Today, November 2020.
2. "Determining Design Power Over An Input Voltage Range (Part 2): Inductor Design Power" by Dennis Feucht, How2Power Today, February 2021.

About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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